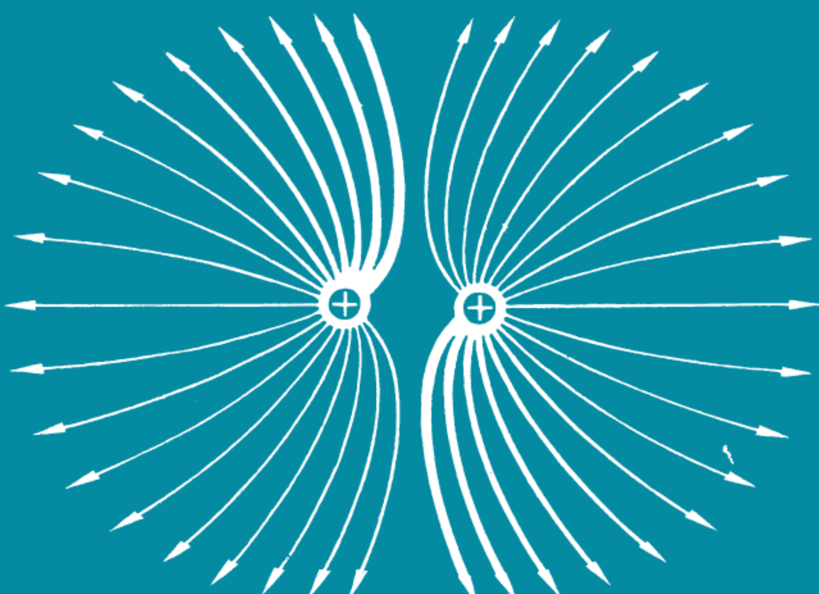
R. A. Gladkova, V. E. Dobronravov, L. S. Zhdanov and F. S. Tsodikov

PHYSICS PROBLEMS

for the Technician

Edited by R. A. Gladkova

Mir Publishers Moscow



PHYSICS PROBLEMS FOR THE TECHNICIAN
Edited by R. A. Gladkova

to supplement

PHYSICS FOR THE TECHNICIAN
by L. S. Zhdanov

It is said that solving problems is not only essential, but one of the best ways to learn and appreciate physics. Into this amazing world of physics of today, both classic and modern, this collection of carefully selected problems and worked examples ushers you smoothly and expertly. No wonder—behind this book is the enormous experience of its authors, who have been teaching general physics for decades at secondary technical and vocational schools of the Soviet Union. The book has been tested, rewritten, and retested, and this constant improvement made it into a classic in the field.

Examples at the beginning of each section are worked in detail. These are chosen so that working with the book on his own the student could overcome all the difficulties encountered in solving the problems.

Questions cover the major concepts and their applications providing explanations of physical phenomena in daily life.

Problems range from straightforward ones to real challengers.

Answers to the questions stimulate creative thinking, and answers to difficult problems contain hints and references to a typical example or earlier problem.

Other Mir Publishers' Books

PROBLEMS IN GEOMETRY

P. MODENOV, D.Sc.

The book describes certain general methods of solving the problems in elementary geometry. Designed for mathematics teachers and senior pupils at secondary schools. Some of the material (complex numbers in plane geometry, inversion, bundles of circles, etc.) is beyond the secondary-school curriculum.

Contents. Vector Algebra. Analytic Geometry. Complex Numbers in Plane Geometry. Inversion. Basic Definitions, Theorems and Formulas. Symbols Used. Appendix: Basic Formulas for Reference. Bibliography. Index.

Other Mir Publishers' Books

**NUCLEAR PHYSICS
AND NUCLEAR REACTORS**

V. LEVIN, Cand.Sc.

The textbook is based on lectures for polytechnical students. It contains some basic facts of atomic physics and quantum mechanics necessary for a clear understanding of the subject matter: the essentials of atomic nucleus structure and radioactivity, the interaction with matter and detection of ionizing radiations, acceleration methods, nuclear reactions, and properties of cosmic rays. A special chapter deals with the physics of neutrons and describes the sources and properties of neutrons, neutron detection, nuclear reactions induced by neutrons, and nuclear fission chain reactions. The second part of the book outlines the technical and scientific aspects of atomic power engineering and considers the design, physics, and performance of nuclear reactors. The book also considers in some detail the economic aspects of nuclear power. It concludes with a brief description of some research, experimental and power reactors.

PHYSICS PROBLEMS

for the Technician

MENDELEEV'S PERIODIC

Periods	Series	Groups of I	II	III	IV	V
I	1	H 1 Hydrogen 1.00797				
II	2	Li 3 Lithium 6.939	Be 4 Beryllium 9.0122	5 B Boron 10.811	6 C Carbon 12.01115	7 N Nitrogen 14.0067
III	3	Na 11 Sodium 22.9898	Mg 12 Magnesium 24.312	13 Al Aluminium 26.9815	14 Si Silicon 28.086	15 P Phosphorus 30.9738
IV	4	K 19 Potassium 39.102	Ca 20 Calcium 40.08	Sc 21 Scandium 44.956	Ti 22 Titanium 47.90	V 23 Vanadium 50.942
	5	29 Cu Copper 63.546	30 Zn Zinc 65.37	31 Ga Gallium 69.72	32 Ge Germanium 72.59	33 As Arsenic 74.9216
V	6	Rb 37 Rubidium 85.47	Sr 38 Strontium 87.62	Y 39 Yttrium 88.905	Zr 40 Zirconium 91.22	Nb 41 Niobium 92.906
	7	47 Ag Silver 107.870	48 Cd Cadmium 112.40	49 In Indium 114.82	50 Sn Tin 118.69	51 Sb Antimony 121.75
VI	8	Cs 55 Cesium 132.905	Ba 56 Barium 137.34	La 57 Lanthanum 138.91 *58-71	Hf 72 Hafnium 178.49	Ta 73 Tantalum 180.948
	9	79 Au Gold 196.967	80 Hg Mercury 200.59	81 Tl Thallium 204.37	82 Pb Lead 207.19	83 Bi Bismuth 208.980
VII	10	Fr 87 Francium [223]	Ra 88 Radium [226]	Ac 89 Actinium [227] **90-103	Ku 104 Kurchatovium [261]	105

* LANTHANI

Ce 58 Cerium 140.12	Pr 59 Praseodymium 140.907	Nd 60 Neodymium 144.24	Pm 61 Promethium [145]	Sm 62 Samarium 150.35	Eu 63 Europium 151.96	Gd 64 Gadolinium 157.25

** ACTINI

Th 90 Thorium 232.038	Pa 91 Protactinium [231]	U 92 Uranium 238.03	Np 93 Neptunium [237]	Pu 94 Plutonium [242]	Am 95 Americium [243]	Cm 96 Curium [247]

TABLE OF THE ELEMENTS

Elements

VI	VII	VIII			
					He 2 Helium 4.0026
8 O Oxygen 15.9994	9 F Fluorine 18.9984				Ne 10 Neon 20.183
16 S Sulphur 32.064	17 Cl Chlorine 35.453				Ar 18 Argon 39.948
Cr 24 Chromium 51.996	Mn 25 Manganese 54.9380	Fe 26 Iron 55.847	Co 27 Cobalt 58.9332	Ni 28 Nickel 58.71	
34 Se Selenium 78.96	35 Br Bromine 79.904				Kr 36 Krypton 83.80
Mo 42 Molybdenum 95.94	Tc 43 Technetium [99]	Ru 44 Ruthenium 101.07	Rh 45 Rhodium 102.905	Pd 46 Palladium 106.4	
52 Te Tellurium 127.60	53 I Iodine 126.905				Xe 54 Xenon 131.30
W 74 Tungsten 183.85	Re 75 Rhenium 186.2	Os 76 Osmium 190.2	Ir 77 Iridium 192.2	Pt 78 Platinum 195.09	
84 Po Polonium [210]	85 At Astatine [210]				Rn 86 Radon [222]

DES

Tb 65 Terbium 158.92	Dy 66 Dysprosium 162.50	Ho 67 Holmium 164.930	Er 68 Erbium 167.26	Tm 69 Thulium 168.934	Yb 70 Ytterbium 173.04	Lu 71 Lutetium 174.97

DES

Bk 97 Berkelium [247]	Cf 98 Californium [249]	Es 99 Einsteinium [254]	Fm 100 Fermium [253]	Md 101 Mendelevium [256]	(No) 102 (Nobelium) [256]	(Lr) 103 (Lowrencium) [257]

ИЗДАТЕЛЬСТВО • МИР •

СБОРНИК ЗАДАЧ И ВОПРОСОВ ПО ФИЗИКЕ

для средних специальных учебных заведений

Под общей редакцией
Р. А. ГЛАДКОВОЙ

Издательство «Наука» Москва

R. A. Gladkova, V. E. Dobronravov,
L. S. Zhdanov, and F. S. Tsodikov

PHYSICS PROBLEMS

for the Technician

Edited by R. A. Gladkova

Translated from the Russian
by
Alexander Repyev

Mir Publishers Moscow

First published 1982
Revised from the 1980 Russian edition

На английском языке

PREFACE

The present fifth edition of "Physics Problems" is the product of almost seven years of improvement by a small group of authors, who drew on their wide experience of teaching physics at secondary technical and vocational schools.

Since the first edition of the book in Russian, it has been translated into a number of languages of the peoples of the Soviet Union and into German.

"Physics Problems for the Technician" is meant to supplement the textbook "Physics for the Technician" by L. S. Zhdanov, with which it is coordinated in topic coverage and format, this principle of exposition of standard material making for the solid grasping of concepts of physics by students at higher and secondary technical colleges.

The problem selection in the book allows the standard physics course to be presented at a higher conceptual and methodological level.

To help the student work with the book and to facilitate the assimilation of the material each section begins with some typical examples worked in detail. The examples are carefully chosen so that when working with the "Physics Problems" on their own the students could overcome all the difficulties encountered in working the problems.

We have tried to achieve a solid balance in the number and difficulty of problems with the aim of assisting the student to better understand the principal concepts and ideas of modern and develop problem-solving techniques. The questions cover the major concepts and their applications providing explanations of physical phenomena in daily life.

Answers to the questions in most cases stimulate creative thinking on the part of the students and encourage original

reasoning. Answers to difficult problems contain hints and references to a typical example or earlier problem.

The SI units are used throughout, and in full accordance with Standard SMEA 1052-78 for terminology and symbols of physical units. The book uses the graphical representation freely. The Appendices contain the tables of physical quantities with principal formulas to be used in solving the problems.

The authors consider that "Physics Problems" may be used by the students of extramural polytechnics, those sitting for their exams to universities, high school pupils and for self-study.

Our best wishes to the readers. Enjoy the book!

The authors

CONTENTS

INTRODUCTION

SEC. 1. DENSITY

Example 1. A block of gold-silver alloy with density $1.4 \cdot 10^4$ kg/m^3 has a mass of 0.40 kg. With the alloy volume being equal to the sum of volumes of its constituent parts, what is the percentage and mass of gold of alloy?

Given: $m = 0.40$ kg is the alloy mass, $\rho = 1.4 \cdot 10^4$ kg/m^3 is the alloy density, $\rho_1 = 1.93 \cdot 10^4$ kg/m^3 is the gold density, and $\rho_2 = 1.05 \cdot 10^4$ kg/m^3 is the silver density.

Determine: m_1—the gold mass of the alloy;

$x = \frac{m_1}{m} \cdot 100\%$—the percentage gold of alloy.

Solution. The gold mass of the alloy, m_1, may be found knowing the gold density ρ_1 and gold volume V_1:

$$m_1 = \rho_1 V_1 \tag{1}$$

In order to determine the gold volume V_1, we will make use of the fact that the alloy mass m is equal to the sum of constituent masses of gold and silver, and the alloy volume is equal to the sum of volumes of its constituents (as given), i.e.

$$m = m_1 + m_2$$
$$V = V_1 + V_2$$

Considering that $m = \rho V$, $m_1 = \rho_1 V_1$, and $m_2 = \rho_2 V_2$, we write

$$\rho V = \rho_1 V_1 + \rho_2 V_2$$

Substituting $V_2 = V - V_1$ into the above relationship gives

$$\rho V = \rho_1 V_1 + \rho_2 (V - V_1)$$

Hence

$$V_1 = \frac{\rho V - \rho_2 V}{\rho_1 - \rho_2} = \frac{(\rho - \rho_2) V}{\rho_1 - \rho_2} \tag{2}$$

Substituting the value of V_1 of Eq. (2) into Eq. (1) we obtain

$$m_1 = \rho_1 \frac{(\rho - \rho_2) V}{\rho_1 - \rho_2}$$

Recalling that $V = m/\rho$, we arrive at

$$m_1 = \frac{(\rho - \rho_2) \rho_1 m}{(\rho_1 - \rho_2) \rho}$$

The gold of the alloy in terms of percent will be

$$x = \frac{m_1}{m} \cdot 100\,\%$$

Substituting the numerical values we obtain

$$m_1 = \frac{(1.4 \cdot 10^4\ \mathrm{kg/m^3} - 1.05 \cdot 10^4\ \mathrm{kg/m^3}) \cdot 1.93 \cdot 10^4\ \mathrm{kg/m^3} \cdot 0.40\ \mathrm{kg}}{(1.93 \cdot 10^4\ \mathrm{kg/m^3} - 1.05 \cdot 10^4\ \mathrm{kg/m^3}) \cdot 1.4 \cdot 10^4\ \mathrm{kg/m^3}} = $$

$$= 0.22\ \mathrm{kg}$$

$$x = \frac{0.22}{0.40} \cdot 100\,\% = 55\,\%$$

Answer. The gold mass is $m_1 = 0.22$ kg, the gold mass in the alloy accounts for 55% of the alloy mass total.

1.1. What is the mass and weight of air in a room, if the floor area is 20 m^2, and the height is 3.0 m?

1.2. A wooden casting pattern has a mass of 4.0 kg. If the wood density is 500 kg/m^3, what is the mass of the brass casting, neglecting a brass shrinkage in cooling?

1.3. A bundle of 2.0 mm^2 copper wire has a mass of 20 kg. How can the wire length be determined without unwinding the bundle? Compute it.

1.4. Gold can be flattened to as thin as 0.10 μm. What surface can be plated with a gold sheet of mass 2.0 g?

1.5. An iron rod with length 2.0 m and cross-sectional area 4.0 cm^2 has a mass of 6.28 kg. Determine the density of iron.

1.6. The volumes being equal, the mass of an iron block is by 12.75 kg higher than that of aluminium. Compute the masses of the iron and aluminium blocks.

1.7. What amount of petroleum is transported during a

1.0 hour period over a 0.50 m-dia pipe-line with petroleum travelling at 1.0 m/s?

1.8. A cast-iron casting with an external volume of 3.1 dm^3 has a mass of 21 kg. Are there any voids in it? If any, what is their volume?

1.9. The standard kilogram is made of an alloy containing 90 per cent platinum, and the balance iridium. Assuming the alloy volume being equal to the sum of constituent volumes, calculate the alloy density, and the standard volume.

1.10. An alloy contains 2.92 kg tin and 1.46 kg lead. What is the density of the alloy, if its volume is equal to the sum of constituent volumes?

1.11. A body floats in water submerged 3/4 of its volume. Determine the density of matter constituting the body.

1.12. An iron block floats in a liquid submerged 0.574 of its volume. What is the density of the liquid? Identify the liquid using Table III.

1.13. What fraction of the volume of a body of density ρ_1 remains under the surface when the body floats in a liquid of density ρ_2? $\rho_1 < \rho_2$.

1.14. In water a body weighs 1.147 times less than in air. What is the density of the matter of the body?

1.15. A 270 g aluminium cylinder suspended from a load gauge by a string is fully submerged in a liquid. Calculate the density of the liquid if the load gauge reads 1.66 N.

CHAPTER I

MOLECULAR PHYSICS AND HEAT

SEC. 2. ELEMENTS OF KINETIC THEORY OF GASES

Example 2. Compute the number of molecules contained in 1 kg of carbon dioxide; determine the mass of one molecule. Determine at standard conditions of temperature and pressure (STP) the number of molecules in 1 m^3 and 1 cm^3 of gas, and the average intermolecular spacing.

Given: $\mu = 44 \cdot 10^{-3}$ kg/mol is the mass of 1 mol of CO_2, $\rho_0 = 1.98$ kg/m^3 is the CO_2 density at STP, $N_A = 6.023 \times \times 10^{23}$ mol^{-1} is Avogadro's number.

Determine: n_m—the number of molecules in a unit mass of gas;
m—the mass of one molecule;
n_0—the number of molecules in a unit volume of the gas;
d_0—the average intermolecular spacing at STP.

Solution. We obtain the number of molecules in a unit mass from the relationship

$$n_m = \frac{N_A}{\mu} = \frac{6.023 \cdot 10^{23}\ \text{mol}^{-1}}{44 \cdot 10^{-3}\ \text{kg/mol}} \approx 1.37 \cdot 10^{25}\ \text{kg}^{-1}$$

For the mass of one molecule we have

$$m = \frac{\mu}{N_A} = \frac{44 \cdot 10^{-3}\ \text{kg/mol}}{6.023 \cdot 10^{23}\ \text{mol}^{-1}} \approx 7.31 \cdot 10^{-26}\ \text{kg}$$

The number of molecules in 1 m^3 of volume at STP is

$$n_0 = \frac{N_A}{\mu}\rho_0 = \frac{6.023 \cdot 10^{23}\ \text{mol}^{-1} \cdot 1.98\ \text{kg/m}^3}{44 \cdot 10^{-3}\ \text{kg/mol}} \approx 2.7 \cdot 10^{25}\ \text{m}^{-3}$$

and in 1 cm^3 there are fewer molecules by a factor of 10^6,

In the gas the average spacing between molecules is

$$d_0=\sqrt[3]{\frac{\mu}{N_A\rho_0}}=\sqrt[3]{\frac{mN_A}{N_Amn_0}}=\sqrt[3]{\frac{1}{n_0}}$$

$$=\sqrt[3]{\frac{1}{2.7\cdot10^{25}}\ \mathrm{m}^3}\approx3.3\cdot10^{-9}\ \mathrm{m}$$

Answer. The number of molecules in a unit mass of gas is about $1.37\cdot10^{25}$ kg^{-1}; the mass of a CO_2 molecule is about $7.31\cdot10^{-26}$ kg; the number of molecules in a unit volume is about $2.7\cdot10^{25}$ m^{-3} or $2.7\cdot10^{19}$ cm^{-3}; the average spacing between the gas molecules at STP is about $3.3\cdot10^{-9}$ m.

Example 3. Calculate the average kinetic energy of translational motion of one molecule of helium having a density of 0.12 kg/m^3 at pressure 100 kPa.

Given: p = 100 kPa is the gas pressure, ρ = 0.12 kg/m^3 is the gas density, $\mu = 4.0\cdot10^{-3}$ kg/mol is the molar mass of gas, $N_A = 6.02\cdot10^{23}$ is Avogadro's number.

Determine: $\bar{\varepsilon}$—the average kinetic energy of translational motion of a gas molecule under given conditions.

Solution. We will use the fundamental equation of the kinetic theory of gases:

$$p=\frac{2}{3}n\bar{\varepsilon}$$

where p is the pressure of gas, n is the number of molecules in a unit volume. We determine the number of molecules in a unit volume by the formula

$$n=\frac{N_A}{\mu}\rho$$

Hence

$$\bar{\varepsilon}=\frac{3}{2}\frac{p}{n}=\frac{3}{2}\frac{p\mu}{\rho N_A}$$

Substituting numerical values gives

$$\bar{\varepsilon}=\frac{3}{2}\cdot\frac{10^5\ \mathrm{Pa}\cdot4.0\cdot10^{-3}\ \mathrm{kg/mol}}{6.023\cdot10^{23}\ \mathrm{mol}^{-1}\cdot0.12\ \mathrm{kg/m^3}}=8.3\cdot10^{-21}\ \mathrm{J}$$

Answer. The average kinetic energy of one gas molecule under given conditions is $8.3\cdot10^{-21}$ J.

Note. The conditions in this example allow us to find the r.m.s. molecule velocity

$$\bar{v}_{r.m.s.}=\sqrt{\frac{2\bar{\varepsilon}N_A}{\mu}}$$

and the gas temperature

$$T=\frac{p}{nk}$$

where k is the Boltzmann constant.

Example 4. Calculate at STP and 100°C the r.m.s. velocity and energy of translational motion of carbon dioxide molecules. Determine the molecular free path at STP, if the number of collisions of each molecule with others in 1 s is on the average equal to $9.12\cdot10^9$.

Given: $T_0 = 273$ K is the initial temperature of the gas, $p_0 = 1.013\cdot10^5$ Pa is the standard atmospheric pressure, $T = 373$ K is the gas temperature, $\mu = 44\cdot10^{-3}$ kg/mol is the molar mass of the gas, $\bar{z}_0 = 9.12\cdot10^9\ s^{-1}$ is the average number of collisions of each molecule with others in 1 s, $R = 8.314$ J/(mol·K) is the molecular gas constant, $\rho_0 = 1.98$ kg/m³ is the CO_2 density at STP, $k = 1.38\times\times10^{-23}$ J/K is the Boltzmann constant.

Determine: (1) $\bar{v}_0$, $\bar{v}$—the r.m.s. velocities of gas molecules;

(2) $\bar{\varepsilon}_0$, $\bar{\varepsilon}$—the average energies of translational motion of molecules;

(3) $\bar{l}_0$—the molecular free path.

Solution. (1) The r.m.s. velocity of gas molecules at STP will be

$$\bar{v}_0=\sqrt{\frac{3p_0}{\rho_0}}=\sqrt{\frac{3\cdot1.013\cdot10^5\ \text{Pa}}{1.98\ \text{kg/m}^3}}\approx392\ \text{m/s}$$

The r.m.s. velocity of molecules at a given temperature T is given by the relationship

$$\bar{v}=\sqrt{\frac{3RT}{\mu}}=\sqrt{\frac{3\cdot8.314\ \text{J/(mol·K)}\cdot373\ \text{K}}{44\cdot10^{-3}\ \text{kg/mol}}}=460\ \text{m/s}$$

(2) The average total kinetic energy of one gas molecule and all the mass of the gas are, respectively

$$\bar{\varepsilon}=\frac{1}{2}ikT\quad \bar{E}=\frac{1}{2}ipV$$

where k is the Boltzmann constant; p, V, T are the state parameters of gas, i is the number of degrees of freedom. At $i = 3$ the above expression for $\bar{E}$ defines the average energy of translational motion of molecules of all gases (for a single-atom gas it also determines the total kinetic energy of its molecules); at $i = 5$ or $i = 6$ the total kinetic energy for two-atom and three-atom gases, respectively. Substituting numerical data into the above formula we obtain

$$\bar{\varepsilon}_0 = \frac{3 \cdot 1.38 \cdot 10^{-23}\ \mathrm{J/K} \cdot 273\ \mathrm{K}}{2} = 5.65 \cdot 10^{-21}\ \mathrm{J}$$

$$\bar{\varepsilon} = \frac{3 \cdot 1.38 \cdot 10^{-23}\ \mathrm{J/K} \cdot 373\ \mathrm{K}}{2} = 7.72 \cdot 10^{-21}\ \mathrm{J}$$

(3) The molecular free path at STP is

$$\bar{l}_0 = v/\bar{z}_0$$

where v is the arithmetic average velocity of gas molecules:

$$v = \sqrt{8RT/\pi\mu} = 0.92\bar{v}_0$$

Hence

$$\bar{l}_0 = \frac{0.92 \cdot 392\ \mathrm{m/s}}{9.12 \cdot 10^{9}\ \mathrm{s^{-1}}} = 4.0 \cdot 10^{-8}\ \mathrm{m}$$

Answer. The r.m.s. velocities of CO_2 molecules at STP and at 373 K are 392 and 460 m/s, respectively. The average energies of translational motion of gas molecules at 373 and 273 K are $7.72 \cdot 10^{-21}$ and $5.65 \cdot 10^{-21}$ J. The mean free path of CO_2 molecules at STP is $4.0 \cdot 10^{-8}$ m.

Example 5. Determine the density of oxygen at 300 K and $1.6 \cdot 10^5$ Pa. Calculate the mass of 200 m^3 of oxygen under these conditions.

Given: $V = 200.0$ m^3 is the gas volume, $T = 300$ K is the gas temperature, $p = 1.6 \cdot 10^5$ Pa is the gas pressure, $\rho_0 = 1.43$ kg/m^3 is the oxygen density at STP, $p_0 = 1.013 \times 10^5$ Pa is the standard atmospheric pressure, $\mu = 32 \times 10^{-3}$ kg/mol is the molar mass of oxygen, $R = 8.314$ J/(mol·K) is the molar gas constant.

Determine: ρ—the density of oxygen;
m—the mass of 200 m^3 of oxygen under the given conditions.

Solution. The problem is solved using the equation of state for an ideal gas and the relationships for density

$$\frac{pV}{T}=\frac{p_0V_0}{273}$$

$$\rho=m/V, \quad \rho_0=m/V_0$$

Substituting and rearranging give

$$\rho=\frac{273\cdot\rho_0}{p_0}\cdot\frac{p}{T} \quad m=\rho V$$

Substituting the numerical values we arrive at the density of oxygen under specified conditions

$$\rho=\frac{273\ \mathrm{K}\cdot 1.43\ \mathrm{kg/m^3}}{1.013\cdot 10^5\ \mathrm{Pa}}\cdot\frac{1.6\cdot 10^5\ \mathrm{Pa}}{300\ \mathrm{K}}\approx 2.05\ \mathrm{kg/m^3}$$

and the mass of 200 m^3 of oxygen under the same conditions will be

$$m = 2.05\ \mathrm{kg/m^3}\cdot 200\ \mathrm{m^3} = 410\ \mathrm{kg}$$

The problem allows of a simpler solution by the use of the Mendeleev-Clapeyron equation

$$pV=\frac{m}{\mu}RT$$

Hence

$$m=\frac{\mu}{R}\frac{pV}{T} \quad \rho=\frac{\mu}{R}\frac{p}{T}$$

Substituting the numerical values we arrive at the same results

$$m=\frac{1.6\cdot 10^5\ \mathrm{Pa}\cdot 200\ \mathrm{m^3}\cdot 32\cdot 10^{-3}\ \mathrm{kg/mol}}{8.314\ \mathrm{J/(mol\cdot K)}\cdot 300\ \mathrm{K}}=410\ \mathrm{kg}$$

$$\rho=2.05\ \mathrm{kg/m^3}$$

Answer. The density of oxygen at temperature 300 K and pressure $1.6\cdot 10^5$ Pa is 2.05 kg/m^3; the mass of 200 m^3 of oxygen under these conditions is 410 kg.

Note I. From the ideal gas equation of state

$$p_1V_1/T_1 = p_2V_2/T_2$$

we have:

at $V_1=V_2$ $\quad p_1/p_2=T_1/T_2$ (isochoric process)

at $p_1=p_2$ $\quad V_1/V_2=T_1/T_2$ (isobaric process)

at $T_1=T_2$ $\quad p_1V_1=p_2V_2$ (isothermal process)

Note II. The equation of state for a gas may also describe the state of a solution of a crystalline matter. Here to the gas pressure p there corresponds the osmotic pressure in the solution (cf. Problems 6.7-6.9).

Note III. If a part α of all the molecules with a given mass m is dissociated into ions in the solution, then the total number of all the molecules and ions will be $(1-\alpha)+$ $+2\alpha=1+\alpha$. Hence

$$pV=(1+\alpha)\frac{m}{\mu}RT$$

which is also valid for inorganic matter (cf. Problems 2.71 and 2.72)

Molecular Motion. Diffusion and Osmosis. Gas Pressure

2.1. Account for the fact that the intensity of the Brownian motion, diffusion, and osmosis grows with temperature.

2.2. Figure 2.2 shows a form of the apparatus detecting the traces of illuminating gas, methane, and other gases lighter than air in the ambient air. A porous air-filled cylinder A is connected with a U-tube B containing mercury. A mercury contact C makes the network of an electric ring D. Describe the action of the apparatus.

2.3. Under weightlessness there are no convective air flows, which is a necessary condition for sustaining the burning. But in this case also a candle or a match would burn for a while with weak, bleak, spherical flame. Provide account for the phenomenon.

2.4. Steel cementation is a process yielding a solid hardened layer over the surface of products made of mild steel. What is the physical phenomenon underlying the cementation process?

2.5. Soldering according to the Luchikhin method consists in the following: (a) cleaning the steel or iron surfaces to be soldered, (b) placing a thin copper foil between the latter, and (c) heating to 1080°C. Such a joint is much stronger than conventional copper soldering. Explain why.

2.6. In order to "weld" one piece of iron to another, both are heated to white heat in a furnace, placed one over the

other on an anvil and subjected to mighty strikes of a forge hammer. Explain why this results in a strong connection.

2.7. One of the ways to "weld" an iron part to another is as follows. The parts are placed one over the other in the cold state and subjected to strong compression (squeezing). Explain why this gives rise to a strong connection.

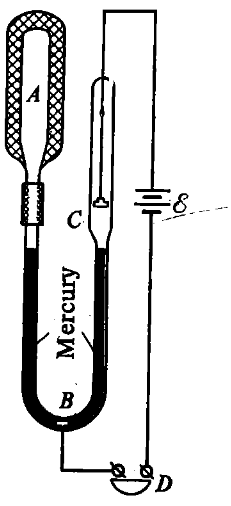


Fig. 2.2

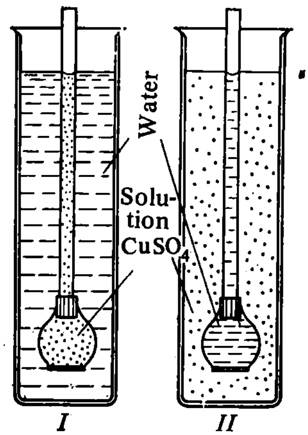


Fig. 2.8

2.8. Figure 2.8 depicts an arrangement to illustrate osmosis, i.e. slow penetration of a solvent (water) into the solution of a substance through a semi-permeable interface (film). How will the liquid level change in the narrow tube in cases *I* and *II*? How is the porous interface, the "bottom" of the inner vessel, going to behave?

Molecular Velocities. Dimensions and Mass

2.9. Why is the Brownian motion in a fluid chaotic and why is it the more spectacular, the smaller the particles?

2.10. What are the segments of broken lines in tables and figures showing the Brownian motion?

2.11. Figure 2.11 gives the cross-section of a device designed to measure directly the velocities of molecular thermal motion. A silver-plated platinum wire stretched along the common axis of the cylinders A and B with diameters 12 and 240 mm is heated by an electric current. Silver molecules that evaporate from its surface travel in a vacuum past the slot in cylinder A to form on the surface of cylinder B a fogging, i.e. a strip of silver. When the device is set in fast rotation about the axis of the cylinders, there occurs such a shift of the strip that its middle part M appears at a position K displaced by $MK = l$. Calculate the average velocity of molecules if the displacement l at a wire temperature of 1173 K is as large as 7.6 mm, and the rotational speed of cylinders is 2800 rpm.

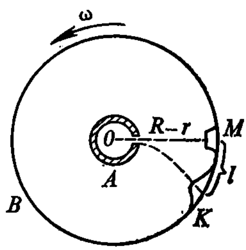


Fig. 2.11

2.12. In the Stern experimental arrangement, the apparatus described in the above problem rotates at 3000 rpm and the silver strip displacement is 9.5 mm. What is the velocity of silver atoms?

2.13. In the Stern experiment the silver strip appears somewhat blurred, as at this temperature molecular velocities are dissimilar. From measurements of the silver fogging (silver strip) thickness at various places along the strip, the percentage of molecules within any given velocity range can be determined. The measurements yield the following table:

Velocity range, m/s	Percentage of molecules, %	Velocity range, m/s	Percentage of molecules, %
0-100	1.4	600-700	9.2
100-200	8.1	700-800	4.8
200-300	16.7	800-900	2.0
300-400	21.5	900-1000	0.6
400-500	20.3	more than 1000	0.3
500-600	15.1		

From this table construct a velocity distribution diagram for silver molecules (at 1173 K). How will the diagram configuration change in decreasing the width of velocity range? Describe the behaviour of the broken line that bounds the bars of the diagram?

2.14. What is the number of molecules at STP in 1 g of nitrogen; 1 g of carbon dioxide; in 1 m^3 of oxygen?

2.15. What is the number of atoms at STP in 1 g of helium; in 1 g of fully dissociated nitrogen; in 1 m^3 of argon?

2.16. How many particles are there in 1 g of oxygen that is dissociated by half?

2.17. Calculate the average intermolecular spacing in an ideal gas at STP.

2.18. Determine the mass of molecules of oxygen, carbon dioxide, water vapour, and ammonia.

Molecular Free Path. Internal Friction

2.19. Figure 2.19 shows schematically an apparatus designed to measure directly the mean free path l of gas or vapour molecules using the molecular beam method. A fraction of molecules evaporating from the surface of a heated silver ball A placed into a cylinder B pass in the form of a narrow beam through a diaphragm D. Silver molecules scatter when colliding with molecules of rarefied gas in the cylinder. On a screen S_1 there precipitate n_1 molecules, on a screen S_2 during the same time and with screen S_1 removed, n_2 molecules. The n_1/n_2 ratio is obtainable weighing screens S_1 and S_2 before and after the experiment, or measuring the thickness of the silver fogging. To determine the mean free path we make use of the relationship

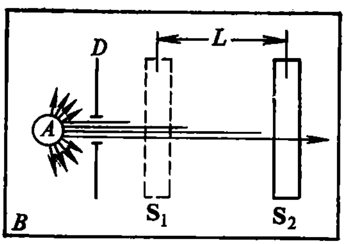


Fig. 2.19

$$l = \frac{L}{M \log (n_1/n_2)} \cdot \frac{p_{ex}}{p}$$

where L is the distance between the screens S_1 and S_2 in metres, $M = 2.302$ is the coefficient of conversion from common

logarithms to natural ones, n_1 and n_2 are the amounts of silver molecules on the screens, p_{ex} is the gas pressure at which the experiment is conducted, p is a specified gas pressure. Compute l for silver molecules at STP if the experiment is performed at a pressure of 0.800 Pa, a distance of $L = 3.0$ mm, and $n_1/n_2 = 1.2$.

2.20. The mean free path of a hydrogen molecule at STP is $1.12 \cdot 10^{-7}$ m, for nitrogen under the same conditions it amounts to $6.0 \cdot 10^{-8}$ m. What is the number of collisions of each molecule with the others per second for hydrogen and nitrogen?

2.21. Calculate the effective diameters of hydrogen and nitrogen molecules making use of the data of problem 2.20.

2.22. Figure 2.22 shows the sectional view of the apparatus used to measure the dynamic viscosity (internal friction coefficient) of gases. Between two concentric cylinders A and B there is a gas, the outer cylinder fully enclosing the inner one. If the outer cylinder is set into fast rotation, then any element ΔS of the wall of the inner cylinder will be subjected to a tangential force called the internal friction force $F_{fr} = -\eta \frac{\Delta v}{\Delta r} \Delta S$, where η is the dynamic viscosity of the gas occupying the space between the cylinders, $\Delta v/\Delta r$ is a measure of the decreasing of linear velocity per unit length (radius). Calculate the dynamic viscosity for air and carbon dioxide, if the internal friction force F_{fr} is $6.2 \cdot 10^{-3}$ and $5 \cdot 10^{-3}$ N, respectively, the cylinder diameters are 210 and 200 mm, their height is 290 mm, and the outer cylinder rotates at 900 rpm.

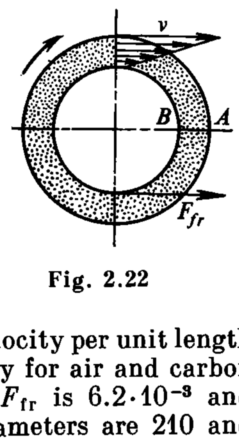


Fig. 2.22

2.23. Determine the rotational velocity of the outer cylinder of the preceding problem, if for oxygen the coefficient η is found to be $1.92 \cdot 10^{-5}$ Pa·s, and the friction force acting on the inner cylinder is $4.07 \cdot 10^{-3}$ N. Calculate the dynamic viscosity for hydrogen, if at the same internal friction force the rotational velocity of the outer cylinder is 1200 rpm.

Atmospheric Pressure. Gas Pressure in Closed Volume

2.24. What is the construction and principle of the mercury barometer?

2.25. The upper end of the tube of a well-type liquid column barometer is fitted to one arm of the weight beam

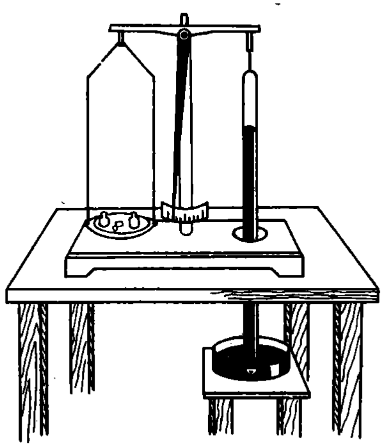


Fig. 2.25

(Fig. 2.25); and two counterweights are placed on a cup suspended from the other arm. What are they compensating? Will the equilibrium be disturbed, if the atmospheric pressure changes?

2.26. Inside a spaceship the standard atmospheric pressure is maintained, though the air in the ship is weightless like all the other bodies aboard. Amount for the phenomenon.

2.27. How do the mercury barometer and aneroid barometer behave under weightlessness? May the readings of a water-level gauge glass be relied upon here?

2.28. How high were one to rise vertically from the ground surface for the atmospheric pressure to decrease by 133.3 Pa? By what amount would it drop at an elevation of 150 m, neglecting the variation of temperature and density of air with height?

2.29. The value of atmospheric pressure at a given height may be found from the graph (Fig. 2.29) or calculated by the formula $p = p_0 \cdot 10^{-0.06h}$, where p_0 is the atmospheric pressure at sea level, h is the altitude above sea level in km.

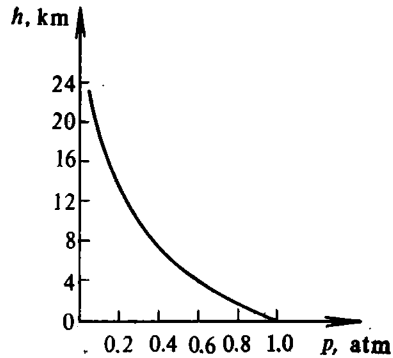


Fig. 2.29

Assuming $p_0 = 1$ atm, what is the pressure at altitudes 1, 2, 10, and 22 km? Ignore the variation of temperature with altitude.

2.30. Calculate the force with which the atmosphere at standard pressure acts on Magdeburg hemispheres, if the diameter of the hemispheres is 100 mm. The pressure of air that remained inside the hemispheres after pumping is assumed to be 2.67 kPa.

2.31. Helium is almost twice as heavy as hydrogen. By how many times is the lift of a hydrogen-filled balloon larger than that of the same balloon filled with helium?

2.32. Illuminating gas* is almost eight times heavier than hydrogen. By how many times is the lift of a hydrogen-filled balloon larger than that of the same balloon filled with illuminating gas?

---

* Illuminating gas is a mixture of combustible gases, mainly hydrogen, methane, carbon oxide, etc. Produced in thermal processing of solid fuels, e.g. coal coking. Used as a fuel in household and industry, and formerly to illuminate streets and houses, and also to fill balloons.

2.33. On a boiler the pressure gauge* reads $1.1 \cdot 10^6$ Pa; the orifice area closed by a relief valve is 400 mm^2. What is the force which steam exerts on the valve?

2.34. Compute the steam pressure in a boiler, if from the arm of the relief valve (Fig. 2.34) a 2.0 kg weight is suspended; the orifice of the valve has an area of 2.5 cm^2; distances from the hole centre to the lever axis and to the weight suspension point being 20 and 200 mm, respectively.

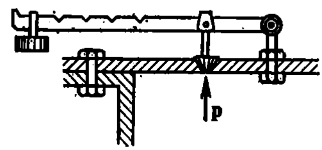


Fig. 2.34

Basic Equation of Kinetic Theory of Gases

2.35. The r.m.s. velocity of the molecules of acetylene in a closed cylinder is 500 m/s. The density of the gas is 18 kg/m^3. Calculate the energy of translational motion for one molecule and the total energy of all the molecules. What is the pressure of the gas, if its mass is 7.2 kg?

2.36. A 10^{-3} m^3 cylinder contains nitrogen under a pressure of 200 kPa, and 1 cm^3 of the gas is known to contain $4.3 \cdot 10^{19}$ molecules. What are the energy of translational motion of one molecule and the total energy of all the molecules? Calculate the r.m s. velocity of the molecules and the density of gas.

2.37. Determine the r.m.s. velocity of the molecules of a gas with density 1.8 kg/m^3 at 1.5 atm.

2.38. A certain amount of gas at 300 K exerts a pressure of $1.2 \cdot 10^5$ Pa, and at 240 K, $9.6 \cdot 10^4$ Pa, the volume of the gas remaining constant. Determine the temperature coefficient of gas pressure, if the reference is the pressure at a temperature of 273 K (0°C). Compute the gas pressure at 273, 373, and 180 K. Calculate the temperature, at which the pressure of the same amount of gas is $8 \cdot 10^4$ Pa.

2.39. To observe the Brownian motion a solution of India ink (density $1.2 \cdot 10^3$ kg/m^3) in alcohol is used. Determine

* The industrial pressure gauge is graduated so that to read the pressure of gas in a balloon or steam in a boiler in excess of the atmospheric pressure.

the velocities of particles with diameters 0.5 and 1 μm. What are diameters of particles moving at a velocity of 0.5 m/s? The temperature in both cases is 18°C.

2.40. The surface of a metallic mirror is silver-plated by an evaporation technique. What is the rate of growth (z/t) of the thickness of the layer, if the pressure of silver molecules is 0.105 Pa, and the average kinetic energy of each molecule is $2.25 \cdot 10^{-20}$ J?

2.41. Determine the pressure at which 1 m^3 of gas contains $2.4 \cdot 10^{26}$ molecules, the gas temperature being 60°C.

2.42. At what temperature will 1 cm^3 of gas contain $1.0 \cdot 10^{19}$ molecules, if the gas pressure is $1.0 \cdot 10^4$ Pa? How will the pressure change, if, with the volume and temperature constant, half the molecules is replaced by molecules of a heavier gas?

2.43. Under laboratory conditions a high vacuum is achieved, i.e. a minor pressure equal to $1.33 \cdot 10^{-9}$ Pa. Assuming the temperature to be 293 K, how many molecules will remain in this case in 1 m^3 and 1 cm^3 of gas?

Equation of State of Ideal Gas

2.44. A gas at pressure $8.1 \cdot 10^5$ Pa and temperature 12°C occupies a volume of 855 l. What will the pressure be, if the same mass of gas at a temperature of 320 K occupies 800 l?

2.45. At a pressure of 6.0 atm and temperature of 293 K a gas occupies a volume of 586 l. Determine the volume, occupied by the same mass of gas at 248 K and $4.0 \cdot 10^5$ Pa.

2.46. The volume of a gas at pressure $7.2 \cdot 10^5$ Pa and temperature 288 K is 0.60 m^3. At what temperature will the same mass of gas occupy a volume of 1.6 m^3, if the pressure is $2.25 \cdot 10^5$ Pa?

2.47. A certain mass of gas at 1.25 atm and 300 K occupies a volume of 0.60 m^3. What is the volume of the gas at STP?

2.48. A gas at $3.2 \cdot 10^4$ Pa and 290 K occupies a volume of 87 l. What is the volume of the gas at STP?

2.49. What pressure is created by 40.0 l of oxygen at a temperature of 103°C, if at STP the same mass of gas occupies a volume of 13.65 l? What is the mass of the gas?

2.50. At what temperature will the pressure of 240 l of hydrogen be 1.25 atm, if at STP the same mass of gas occupies a volume of 364 l? Determine the mass of the gas.

2.51. Before sending a sounding balloon aloft the pressure of gas in it at 17°C is $1.16 \cdot 10^5$ Pa. How and to what extent will the volume of the balloon change at an altitude, for which the observed temperature and pressure of atmospheric air are —30°C and $8.5 \cdot 10^4$ Pa, respectively? We will assume the pressure exerted by the elastic envelope of the balloon to be constant and equal to $5 \cdot 10^3$ Pa, the temperature of the gas to be equal to the ambient temperature.

2.52. What is the weight of the air occupying a volume of 150 l at a temperature of 288 K and pressure of $1.5 \cdot 10^5$ Pa?

2.53. A pump of a laboratory kerosene burner draws in 35 cm^3 of air per cycle; in the reservoir, the space that contains no kerosene is 0.45 l. What pressure will be achieved in the reservoir after 20 cycles, if the air temperature in it has increased from 286 to 325 K?

2.54. A rubber chamber contains air at a temperature of 300 K and standard atmospheric pressure. How deep must the chamber be submerged in water at 277 K for the volume of the former to be reduced by one half?

2.55. Determine the density of air at an altitude of 8.7 km over sea-level at —47°C, if at sea-level we have STP (cf. problem 2.29).

2.56. What is the density of carbon dioxide at a pressure of 93.3 kPa and temperature of 250 K, and the density of hydrogen at $6.0 \cdot 10^5$ Pa and 293 K?

2.57. Illuminating gas is fed over a pipe-line at a pressure of 4.0 atm and a temperature of 300 K, so that the pipeline with cross-sectional area of 8.0 cm^2 delivers 8.4 kg of gas during 20 min. What is the velocity of travel of the gas over the pipe-line?

2.58. A 1.2 l sealed ball containing 9.0 g of water is heated. If the ball walls are rated at a pressure of not higher than 40 atm, at what temperature will it burst?

2.59. A forcing pump draws in 2 l of air per piston stroke at an atmospheric pressure and temperature of 22°C. Then the pump delivers the air to a 0.15 m^3 reservoir, that was formerly vented to atmosphere. How many pump cycles are

required for the pressure in the reservoir to attain 4.0 atm at a temperature of 27°C?

2.60. Determine the mass of carbon dioxide in a 40 l cylinder at a temperature of 288 K and a pressure of 50 atm.

2.61. A cylinder having a volume of 25.6 l contains 1.04 kg of nitrogen at 35 atm. Determine the temperature of the gas.

2.62. To determine the molar gas constant the following experiment was carried out (Fig. 2.62): into a 5.0 l vessel 0.88 g of dry ice was placed, thereafter the vessel was sealed. The difference of levels in the manometer tube was 73.6 mm (partial pressure of carbon dioxide), and the equilibrium temperature was 295 K. From the experimental results determine the value of the molar gas constant.

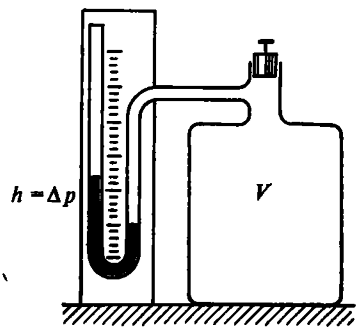


Fig. 2.62

2.63. Determine the temperature of a fuel-air mixture in a cylinder of an internal-combustion engine at the compression stroke, from the following data: the mixture pressure in the cylinder before the compression is 0.75 atm, at the end of the compression 8.4 atm, the mixture temperature before the compression is 315 K, the compression ratio is 6.3*.

2.64. The air pressure in a diesel engine at the beginning of the compression stroke is 0.85 atm, at the end 34 atm, and the temperature changes from 323 to 923 K. Determine the compression ratio. How does it compare with the figure given in the preceding problem? Explain such a dramatic difference.

2.65. A cylinder contained an ideal gas at $4.0 \cdot 10^7$ Pa and 300 K. Then 3/5 of the gas were released from the cyl-

* The compression ratio is the maximum-to-minimum ratio of volumes occupied by gas in the engine cylinder at the extremes of the stroke.

inder with the result that the temperature dropped to 240 K. Under what pressure is the remaining gas in the cylinder?

2.66. A 30 l cylinder contained oxygen at a pressure of 72 atm and temperature of 264 K. Then a part of the gas from the cylinder was released, and in a short while the temperature of gas within the cylinder increased to 290 K, and the pressure dropped to 29 atm. What was the amount of oxygen released?

2.67. For a submarine to surface, the water-filled tanks are blown with compressed air fed from a 30 l cylinder containing air under $1.47 \cdot 10^7$ Pa at 285 K. The blowing is carried out at a depth of 25 m, the air assuming the temperature of ambient water, i.e. 277 K. What is the volume of water to be driven out of the tanks, if the density of sea water is 1030 kg/m^3?

2.68. In a demonstration of the Stern experiment (see Fig. 2.11) a silver wire is heated to 827°C. The diameter of the outer cylinder is 240 mm, and that of the inner one, 12 mm. With what frequency is it necessary to rotate the cylinders in order to obtain a silver strip with its centre line displaced by 9.5 mm?

2.69. Up to what temperature was a copper wire heated in the Stern experiment (see Fig. 2.11), if the cylinders with diameters of 285 and 19 mm were rotated at 3000 rpm, and the displacement of copper strip centre was found to be 8.4 mm?

2.70. A glass tube sealed at one end contains hydrogen "locked" by a 10 cm column of mercury. Initially the tube had its open end up, and the gas inside had a temperature of 16°C. What was the length of the hydrogen column under the given conditions, if after the tube was positioned with its open end down and the gas heated to 39°C, the mercury column shifted by 7 cm? The atmospheric pressure is 10^5 Pa.

2.71. Some atmospheric air penetrated into a 300 l reservoir containing 16 g of hydrogen. Compute the mass of this air, if at 6°C in the reservoir the established pressure was $9.3 \cdot 10^4$ Pa.

2.72. Determine the pressure developed by 22 g of gas whose molecules have a degree of dissociation of 60 per

cent and which occupies 500 l at a temperature of 63°C. Consider the cases of oxygen and carbon dioxide ($2\ CO_2 \rightleftarrows 2\ CO + O_2$).

SEC. 3. SPECIAL APPLICATIONS OF THE MENDELEEV-CLAPEYRON EQUATION

Example 6. A mass of air occupying at a temperature of 27°C and a pressure of 2 atm a volume of 120 l is subjected to heating. Determine the gas temperature, if the heating is: (1) isochoric with the pressure increased by 0.56 atm, and (2) isobaric with the gas volume increased to 150 l. What is the mass of the gas?

Given: $T_1 = 300$ K is the initial temperature, $p_1 = 2 \cdot 1.013 \cdot 10^5$ Pa and $p_2 = 2.56 \cdot 1.013 \cdot 10^5$ Pa are the initial and final pressures, $V_1 = 120 \cdot 10^{-3}$ m^3 and $V_2 = 150 \times 10^{-3}$ m^3 are the initial and final volumes of the gas, $\mu = 29 \cdot 10^{-3}$ kg/mol is the molar mass of air, $R = 8.314$ J/(mol·K) is the molar gas constant.

Determine: T_2—the final temperature of the gas in both cases;

m—the mass of the gas.

Solution. The final temperature T_2 is to be found from the equations for isochoric and isobaric processes, each of which involves two thermodynamic parameters only,

$$(1)\quad p_1/p_2 = T_1/T_2 \quad \text{and} \quad (2)\quad V_1/V_2 = T_1/T_2'$$

The mass of gas, m, is to be obtained from the Mendeleev-Clapeyron equation

$$pV = \frac{m}{\mu} RT$$

Substituting numerical values into the equations and performing some algebra we obtain

$$(1)\quad T_2 = \frac{2.56}{2} \cdot 300\ \text{K} = 384\ \text{K}$$

$$(2)\quad T_2' = \frac{150}{120} \cdot 300\ \text{K} = 375\ \text{K}$$

$$m = \frac{2 \cdot 1.013 \cdot 10^5\ \text{Pa} \cdot 120 \cdot 10^{-3}\ \text{m}^3 \cdot 29 \cdot 10^{-3}\ \text{kg/mol}}{8.314\ \text{J/(mol} \cdot \text{K)} \cdot 300\ \text{K}} \approx 0.283\ \text{kg}$$

Answer. The final temperature of the gas with isochoric process is 111°C, and with isobaric process 102°C. The mass of the gas is 0.283 kg.

Example 7. A piston pump has a cylinder with a volume of 0.50 l. The pump is connected to a 3.0 l reservoir containing air at standard atmospheric pressure. Determine the air pressure in the reservoir after 5 working strokes of the piston for (1) delivery and (2) evacuation.

Given: $V_1 = 0.50$ l $= 0.50 \cdot 10^{-3}$ m^3 is the displacement volume of the pump, $V_2 = 3.0$ l $= 3.0 \cdot 10^{-3}$ m^3 is the reservoir volume, $n = 5$ is the number of working strokes of the piston, $p_0 = 1.013 \cdot 10^5$ Pa is the initial air pressure in the reservoir.

Determine: p_d and p_e—the pressure of air in the reservoir after n strokes of the piston under delivery and evacuation working conditions.

(1) *Solution.* With delivery operation, after n working strokes the pump draws in from atmosphere the air volume $V_n = nV_1$ at pressure p_0; this mass of air is to be fed to the reservoir to cause there the partial pressure p_n; as we neglect any variations in the temperature, we have from Boyle's law

$$p_n V_2 = p_0 n V_1$$

hence

$$p_n = p_0 \frac{V_1}{V_2} n$$

The required air pressure in the reservoir will be

$$p_d = p_n + p_0 = p_0 \left(\frac{V_1}{V_2} n + 1 \right)$$

Substituting numerical values we get

$$p_d = 1.013 \cdot 10^5 \text{ Pa} \left(\frac{0.5 \cdot 10^{-3} \text{ m}^3 \cdot 5}{3 \cdot 10^{-3} \text{ m}^3} + 1 \right) = 1.86 \cdot 10^5 \text{ Pa}$$

Answer. With delivery operation the air pressure in the reservoir after 5 piston strokes is $1.86 \cdot 10^5$ Pa.

(2) *Solution.* If at the beginning of the first working stroke the air in the reservoir occupied the volume V_2 at a pressure of p_0, then with evacuation operation by the end of the first piston stroke the same air mass will occupy the volume $V_2 + V_1$ at a pressure of p_1. Ignoring again any temperature

variation, we have from Boyle's law

$$p_1 (V_2 + V_1) = p_0 V_2$$

hence

$$p_1 = \frac{V_2}{V_2 + V_1} p_0$$

At the beginning of a second piston stroke the volume and pressure of air in the cylinder are V_2 and p_1, respectively, and at the end of the stroke these are $V_2 + V_1$ and p_2, hence

$$p_2 = \frac{V_2}{V_2 + V_1} p_1$$

or

$$p_2 = \left(\frac{V_2}{V_2 + V_1}\right)^2 p_0$$

Reasoning along the same lines we obtain that by the end of the nth working stroke

$$p_n = \left(\frac{V_2}{V_2 + V_1}\right)^n p_0$$

Making substitutions we get

$$p_e = \left(\frac{3 \cdot 10^{-3}}{3 \cdot 10^{-3} + 0.5 \cdot 10^{-3}}\right)^5 \cdot p_0 = \left(\frac{6}{7}\right)^5 \cdot 1.013 \cdot 10^5 \text{ Pa} =$$
$$= 0.48 \cdot 10^5 \text{ Pa}$$

Answer. With evacuation operation the air pressure in the reservoir after 5 strokes is $0.48 \cdot 10^5$ Pa.

Note. It is easily seen that a knowledge of values of V_2 and V_1 is not necessary, it would suffice to know their ratio $V_2/V_1 = k$. In fact, in the first case

$$\frac{V_1}{V_2} = \frac{1}{k}$$

and in the second case

$$\frac{V_2}{V_2 + V_1} = \frac{V_2/V_1}{V_2/V_1 + 1} = \frac{k}{k+1}$$

Example 8. Using the data of Example 6 and its answers, determine the amount of heat taken up by a gas (air) and the change in its internal energy; calculate the work performed by the gas at constant pressure.

Given: $T_1 = 300$ K is the initial temperature, $p_1 = 2 \cdot 1.013 \cdot 10^5$ Pa is the initial pressure, $V_1 = 120 \cdot 10^{-3}$ m^3, $V_2 = 150 \cdot 10^{-3}$ m^3 are the initial and final gas volumes, $c_p = 1.0 \cdot 10^3$ J/(kg·K) is the specific heat of air at constant pressure, $\mu = 29 \cdot 10^{-3}$ kg/mol is the molar mass of air, $R = 8.314$ J/(mol·K) is the molar gas constant, $T_2 = 384$ K and $T_2' = 375$ K are final air temperatures at constant volume and constant pressure, respectively, $m = 0.283$ kg is the initial mass of air.

Determine: Q_V and Q_p—the amounts of heat taken up by the gas at constant volume and constant pressure;
A —the work performed by the gas;
ΔU_V and ΔU_p —the change of internal energy in both cases.

Solution. At constant volume

$$Q_V = c_V m \, \Delta T_1$$

and at constant pressure

$$Q_p = c_p m \, \Delta T_2$$

where c_V and c_p are specific heats of air at constant volume and pressure; for air $c_p/c_V = 1.4$; $\Delta T_1 = T_2 - T_1$, and $\Delta T_2 = T_2' - T_1$.

The work done by the gas at constant pressure is to be obtained from the formula

$$A_p = p_1 \Delta V = \frac{m}{\mu} R \Delta T$$

With constant volume, the work done by the gas will be

$$A_V = 0$$

According to the first law of thermodynamics

$$\Delta U_p = Q_p - A_p \quad \text{(constant pressure)}$$

$$\Delta U_V = Q_V \quad \text{(constant volume)}$$

Making substitutions and calculations we obtain for isochoric process

$$Q_V = \Delta U_V = \frac{10^3}{1.4} \text{ J/(kg·K)} \cdot 0.283 \text{ kg} \cdot 84 \text{ K} = 17 \text{ kJ}$$

for isobaric process

$$Q_p = 10^3\ \text{J/(kg}\cdot\text{K)}\cdot 0.283\ \text{kg}\cdot 75\ \text{K} = 21.2\ \text{kJ}$$

$$A_p = 2\cdot 1.013\cdot 10^5\ \text{Pa}\cdot 30\cdot 10^{-3}\ \text{m}^3 = 6.08\ \text{kJ}$$

$$\Delta U_p = 21.2\ \text{kJ} - 6.08\ \text{kJ} = 15.1\ \text{kJ}$$

Answer. The gas takes up 17 kJ of heat at constant volume and 21.2 kJ at constant pressure; at constant volume the gas does no work, i.e. $A_V = 0$, and at constant pressure $A_p = 6.08$ kJ; the change in the internal energy in the first case is 17 kJ, and in the second 15.1 kJ.

Isochoric Process

3.1. Why are bulbs of electric lamps filled with nitrogen under low pressure (under 0.5 atm)? Why does a heated medical cup stick to the body?

3.2. The gas pressure at 293 K is $1.07\cdot 10^6$ Pa. What will the pressure of the gas be, if the latter is heated at constant volume to 423 K? Cooled at constant volume down to 250 K?

3.3. A gas is kept in a cylinder at a temperature of 288 K and a pressure of 18 atm. At what temperature will the gas pressure be 15.5 atm? The cylinder volume is assumed to be constant.

3.4. In fabrication, an electric lamp is filled with nitrogen at pressure $5.065\cdot 10^4$ Pa and temperature 288 K. If the pressure in a lighted lamp has grown to $1.1\cdot 10^5$ Pa, what is the temperature of gas in it?

3.5. At 33°C the pressure gauge on a gas cylinder reads 2.4 atm. At what temperature will it read 2 atm? What will it read at a temperature of —66°C? The gas mass and cylinder volume are thought of as constant (cf. problem 2.33).

3.6. A pressure gauge on a gas cylinder reading 2.8 atm reduced its reading by 1 atm in lowering the temperature by 85°C. What are the temperatures in both cases, the process being isochoric?

3.7. A pressure gauge on an oxygen cylinder reads 2.3 atm at an ambient temperature of 24°C. When placed in a shed with temperature —12°C, the pressure gauge reads 1.9 atm. Is there any gas leakage during the time elapsed between these two readings of pressure?

3.28. Two different states of one and the same mass of gas are represented in the VT-diagram by points *1* and *2* (Fig. 3.28). Which of these points corresponds to the higher pressure? Indicate in the figure the graphs for isochoric and two isobaric processes. Which of these two last processes occurs at the higher pressure?

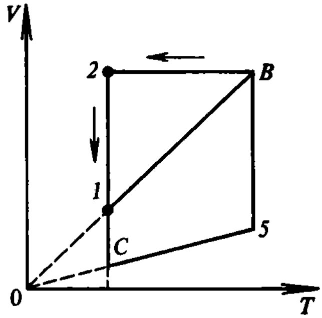


Fig. 3.28

3.29. A hollow glass ball with an internal volume of 10 cm^3 and a narrow neck is heated to 400°C. Then the neck is put in mercury having a room temperature of 16°C. Determine the mass of mercury drawn into the ball.

Isothermal Process

3.30. Justify Boyle's law on the basis of the kinetic theory of gases.

3.31. Volume of air in a cylinder has been reduced five-fold by sharply lowering the piston. Could air pressure in the cylinder be considered to have increased five-fold, the temperature being constant?

3.32. (1) Calculate the density of a mixture of hydrogen and nitrogen at STP, if we take equal (a) volumes, (b) masses of gases. (2) The same for a mixture of nitrogen with chlorine.

3.33. A balloon filled with a gas at standard atmospheric pressure has reached a layer of air with pressure 66.7 kPa. By what factor will the volume of the balloon increase? Temperature changes and the influence of elasticity of the envelope are to be neglected.

3.34. A 12 l vessel containing a gas under $4.0 \cdot 10^5$ Pa is connected with another fully evacuated vessel having a volume of 3 l. Determine the final pressure for isothermal process.

3.35. A vessel containing a gas at pressure $1.4 \cdot 10^5$ Pa is connected with an empty 6.0 l vessel. Thereafter in both

vessels there establishes a pressure of $1.0 \cdot 10^5$ Pa. If the process is isothermal, calculate the volume of the first vessel.

3.36. A rubber chamber contains air at 780 mm Hg. The chamber is compressed so that its volume is reduced by 2/5 of the former value. Assuming the temperature and mass to be constant, what is the new pressure?

3.37. A vessel contains a gas under a pressure of 6.0 atm. When 3/8 of the contents of the vessel have been released, what will the new pressure be, if the temperature is considered to be unchanged?

3.38. A tube used to test Boyle's law contains a mercury column 100 mm in length. When the tube is positioned vertically with its open end upward, the length of the air column in the sealed end is 80.0 mm. If the tube is turned with its open end downward, what is the length of the air column? If placed horizontally? The atmospheric pressure is 740 mm Hg.

3.39. A tube used to test Boyle's law contains a mercury column 75.0 mm in length. When the tube is positioned vertically with its sealed end downward, the length of air column in this end is 120 mm; with horizontal position of the tube the length of the air column is 132 mm. Determine the atmospheric pressure. Calculate the length of air column when the tube has its sealed end downward.

3.40. A mercury barometer gives false indications because of a bubble of air that got into the space over the mercury. At a pressure of 760 mm Hg the barometer reads 740, and at 727.5 mm Hg the reading is 710. Find the length of the barometer tube (from the upper end to the mercury level in the cup).

3.41. An air bubble in water rises to the surface. At what depth is its volume half of that at the surface? What is the bubble volume near the surface, if at a depth of 3.0 m it is 5.0 mm^3? The atmospheric pressure is standard, the water temperature is taken to be independent of depth.

3.42. A thin-walled Bunsen beaker of a volume of 300 cm^3 and a mass of 100 g is immersed in water with its bottom up. What is the minimum depth at which it stops floating up and starts sinking? The atmospheric pressure is standard, the water temperature is taken to be independent of depth.

3.43. Calculate the mass of hydrogen occupying a volume

of 4.0 l at $7.6 \cdot 10^4$ Pa; of air occupying a volume of 0.60 l at 5.0 atm; of carbon dioxide occupying a volume of 1.5 m^3 at $1.8 \cdot 10^5$ Pa. In all the cases the temperature is 0°C.

3.44. What is the volume occupied by 12 g of nitrogen at a pressure of 30 atm and temperature of 0°C?

3.45. A football of volume 2.5 l is pumped with air by a pump drawing in 0.150 l of atmospheric air per cycle at standard pressure. What is the pressure in the football after 50 cycles, if at first it was empty?

3.46. A burner consumes 70.5 g of illuminating gas per hour. What should the capacity of the gas cylinder be to contain enough gas at 100 atm to last the burner for 12 hours? The temperature remains unchanged at 0°C?

3.47. A 12 l automobile tyre is to be pumped to a pressure of 3.5 atm. Determine the number of pumpings to be done by the pump delivering 500 cm^3 of air per pumping at standard pressure, if the tyre was at first empty; half-filled with air; fully filled with air at standard atmospheric pressure.

3.48. What pressure arises in the tank of an air brake of a tram car after 250 cycles of the pump? Given that the tank volume is 30 l, and the pump draws in 600 cm^3 of air per cycle at standard pressure. Temperature variations are to be neglected.

3.49. In the tank of a compressor the pressure is equal to atmospheric. In the forcing pump the cylinder volume is less than that of the tank by a factor of 40. How many pumpings are required by the compressor piston for the pressure in the tank to reach 4 atm, if the temperature is unchanged?

3.50. A 2 l cylinder is filled with gas under a pressure of $5.5 \cdot 10^5$ Pa; another cylinder of volume 5 l is filled with the same gas under $2 \cdot 10^5$ Pa, the cylinders being connected by a tube with a tap. If the process is isothermal what will the pressure in the cylinders be if the tap is opened? Will the result be different if the first cylinder is filled with air, and the second with nitrogen?

3.51. The air pressure in a vessel is 102.4 kPa. The volume of the cylinder of an exhaust pump is one third that of the vessel. Assuming that the temperature is constant, what is the pressure in the vessel after three piston strokes? After four strokes?

3.52. The volume of a vessel is 3000 cm^3, the volume of the cylinder of an exhaust pump is 200 cm^3. After 48 piston strokes a pressure of 4.53 kPa is reached in the vessel. If the temperature is unchanged, what is the initial gas pressure in the vessel?

3.53. A horizontal cylinder closed at both ends is separated by two fastened pistons into three sections. The pressure and volume of gas in each section are $2 \cdot 10^5$ Pa and 36 cm^3, $6 \cdot 10^4$ Pa and 60 cm^3, $5 \cdot 10^4$ Pa and 104 cm^3, respectively. If the temperature remains constant, determine the pressure and volume of the gas in each section after the pistons have been set free.

3.54. Under the piston of a vertically positioned cylinder there is 300.0 cm^3 of gas. The piston mass is 6.75 kg, its area 25 cm^2. On the piston weights are placed and the former descends compressing the gas to a volume of 212 cm^3. Find the mass of the weights. The process is isothermal, at standard atmospheric pressure.

3.55. A vertically placed cylinder contains a quantity of gas under the piston. The mass of the piston is 3.0 kg, its area is 20 cm^2. On the piston acts a force of 490 N to lower it to a height of 13 cm relative to the cylinder bottom. What was the initial volume of the gas? The atmospheric pressure is standard, the temperature is constant.

3.56. Draw schematically the isothermal process in the coordinates p, V; p, T, and V, T.

3.57. Draw a graph of isothermal process for the case of $pV = 40$. Determine the change in the volume of a given mass of gas, if at constant temperature the pressure is increased by $1/n$ (e.g., by $^1/_4$) of its initial value; if the pressure is decreased in a like manner.

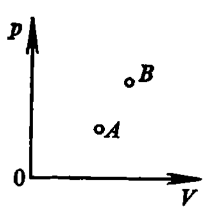


Fig. 3.58

3.58. What is represented in the pV-diagram of Fig. 3.58 by the points A and B and hyperbolas drawn through these points?

Internal Energy of Ideal Gas. Work in Changing Its Volume

3.59. Calculate the r.m.s. velocity and total kinetic energy of carbon dioxide molecules at a temperature of 223 K.

3.60. Determine the r.m.s. velocities and translational energies of hydrogen molecules at temperatures 173, 273, and 423 K.

3.61. At what temperatures have oxygen molecules a r.m.s. velocity of 200 m/s and 700 m/s? How will the internal energy of 1 kg of gas change in transition from the higher of the temperatures to be found to the lower one?

3.62. Determine the change in the internal energy (rather in its kinetic part) of 1 g of argon and 1 mole of hydrogen on heating by 160 K; and also of 500 l of ammonia gas (NH_3) on raising the pressure by $4.0 \cdot 10^3$ Pa. How will the energy of chaotic (thermal) motion of each molecule change in the process?

3.63. Calculate the average energy of chaotic (thermal) motion of the molecules of helium and oxygen at 77°C: (a) for the translational motion of the molecules only, (b) also considering the rotation of molecules. At what temperature will the internal energy of 1.5 kg of ozone (O_3) be $2.33 \cdot 10^5$ J?

3.64. Two different states of one and the same mass of gas are represented in the pV-diagram by the points A and B (see Fig. 3.58). Which of these points corresponds to the higher temperature? What processes can be used to transfer gas from one state into another?

3.65. A quantity of gas is transferred from a state indicated in the VT-diagram of Fig. 3.28 by the point *1* first to the state B, then to the state *2*, and is thereafter returned to the initial state *1*, the transition routes being shown by the segments *1B*, *B2*, and *21*. Represent the process in the pV-diagram; show at what transitions the gas releases heat, and at what absorbs.

3.66. A quantity of an ideal gas is subjected to a closed reversible process indicated in the VT-diagram of Fig. 3.28 by a closed loop *B2C5B*. Show this process in the pV-diagram, indicate in what stages of the process the gas acquires heat, and in what stages it loses heat.

3.67. What work is done by a gas in expanding at a constant pressure of 3 atm from a volume of 3 l to 18 l? What work is done by 6 kg of air in expanding in isobaric heating from 5 to 150°C?

3.68. A gas expands isothermally from a volume of 2 l to 12 l. The initial pressure is $1.2 \cdot 10^6$ Pa. Draw the graph of the process and from the graph determine the work done by the gas.

3.69. Determine schematically the work done by a gas in isothermal expanding from a volume of 2 l to 10 l, if the initial pressure is $1.6 \cdot 10^6$ Pa; and the work done with the gas by external forces in isothermal compression from a volume of 8 m^3 to 1 m^3, if the initial pressure of gas is 1 atm.

3.70. Air occupying in the cylinder under the piston a volume of 1.5 l under 1.2 atm is subjected to strong heating, the temperature growing proportionally to the volume squared: $T = \alpha V^2$. Draw the graph of the process in the coordinates p, V. From the curve determine the work done by the air in increasing the volume to 9 l. Find the value of the coefficient α, if the mass of the air is 58 g.

SEC. 4. INTERNAL ENERGY OF A BODY AND WAYS OF CHANGING IT. HEAT AND WORK. FIRST LAW OF THERMODYNAMICS

Example 9. An iron weight of mass 0.26 kg at a temperature of 100°C is placed into a brass calorimeter of mass 0.15 kg containing 0.20 kg of water at 15°C. Determine the common new temperature, ignoring the heat losses.

Given: $m_w = 0.26$ kg is the mass of the weight; $T = 373$ K is the initial temperature of the weight, $m_{wat} = 0.20$ kg is the mass of the water, $T_1 = 288$ K is the initial temperature of water and calorimeter, $m_c = 0.15$ kg is the mass of the calorimeter, $c_w = 460$ J/(kg·K), $c_{wat} = 4187$ J/(kg·K), $c_{cal} = 380$ J/(kg·K) are specific heats of iron, water, and brass, respectively.

Determine: θ—the final temperature of all the three bodies.

Solution. We derive the equation of the heat balance. The heat released by the weight will be

$$Q_w = c_w m_w (T - \theta)$$

The heat absorbed by the water is

$$Q_{wat} = c_{wat} m_{wat} (\theta - T_1)$$

The heat absorbed by the calorimeter is

$$Q_{cal} = c_{cal} m_{cal} (\theta - T_1)$$

From the energy conservation law we get

$$Q_w = Q_{wat} + Q_{cal}$$

or

$$c_w m_w (T - \theta) = (c_{wat} m_{wat} + c_{cal} m_{cal}) (\theta - T_1)$$

From the equation of the heat balance we obtain the final temperature

$$\theta = \frac{c_w m_w T + (c_{wat} m_{wat} + c_{cal} m_{cal}) T_1}{c_w m_w + c_{wat} m_{wat} + c_{cal} m_{cal}}$$

Making substitutions we get

$$\frac{460 \cdot 0.26 \cdot 373 + (4187 \cdot 0.2 + 380 \cdot 0.15) \cdot 288\ (\mathrm{J/kg \cdot K}) \cdot \mathrm{kg \cdot K}}{460 \cdot 0.26 + 4\,187 \cdot 0.2 + 380 \cdot 0.15\ (\mathrm{J/kg \cdot K}) \cdot \mathrm{kg}} = 298\ \mathrm{K}$$

Answer: The final temperature is 298 K (25°C).

Example 10. A steel shell travelling at a speed of 200 m/s, hits an earth bank and sticks there. By how many degrees will its temperature rise, if 60 per cent of its kinetic energy went into its heating?

Given: $v_0 = 200$ m/s is the initial speed of the shell, $v_f = 0$ is the final speed of the shell, $k = 60$ per cent $= 0.6$ is the percentage or share of kinetic energy of the shell that went into its heating, $c = 460$ J/(kg·K) is the specific heat of steel.

Determine: ΔT is the change in the shell temperature.

Solution. The heating consumed $^1/_2\ kmv_0^2$ of the total kinetic energy of the shell.

The increase in the internal energy of the shell is

$$cm\,\Delta T$$

We now derive the equation of heat balance

$$cm\Delta T = k\frac{mv_0^2}{2}$$

From the above equation we have

$$\Delta T = \frac{kv_0^2}{2c}$$

Substituting the numerical values gives

$$\Delta T = \frac{0.6 \cdot 40{,}000 \text{ m}^2/\text{s}^2}{2 \cdot 460 \text{ J/(kg} \cdot \text{K)}} \approx 26 \text{ K}$$

Answer. The shell temperature rose by about 26 K.

Example 11. In drilling, in a copper cylinder, of a bore of diameter 25 mm the cylinder heats by 43 K. The input torque is 16.2 Nm; 70 per cent of the energy input converts into the internal energy of the cylinder. Compute the pitch of a drill.

Given: $d = 25 \cdot 10^{-3}$ m is the bore diameter, $\Delta T = 43$ K is the rise in the cylinder temperature, $M = 16.2$ Nm is the torque of drilling, $k = 70$ per cent $= 0.70$ is the percentage or fraction of energy that went into the heating of cylinder, $\rho = 8{,}900$ kg/m³ is the copper density, $c = 380$ J/(kg·K) is the specific heat of copper.

Determine: p—the pitch of the drill.

Solution. The required pitch p can be found by dividing the cylinder height h by the rotational speed of drill, n, needed to drill the cylinder through:

$$p = h/n.$$

In drilling there evolves the heat

$$Q = cm\,\Delta T = cSh\rho\,\Delta T$$

where $S = \pi d^2/4$.
Hence

$$h = Q/cS\rho\,\Delta T$$

The quantity n is to be found from the relationship for the work A done in the drilling of the cylinder

$$A = M 2\pi n$$

hence

$$n = A/2\pi M$$

Substituting the values for h and n into the expression for p and considering that, as given, $Q/A = k$, we have

$$p = \frac{2\pi M}{\rho S c \Delta T} \cdot \frac{Q}{A} = \frac{2\pi M k}{\rho S c \Delta T}$$

And finally

$$p = \frac{2\pi M k}{\rho\,(\pi d^2/4)\,c\Delta T} = \frac{8Mk}{\rho d^2 c \Delta T}$$

Substituting gives

$$p = \frac{8 \cdot 16.2\ \text{Nm} \cdot 0.70}{380\ \text{J/(kg} \cdot \text{K)} \cdot 625 \cdot 10^{-6}\ \text{m}^2 \cdot 8\,900\ \text{kg/m}^3 \cdot 43\ \text{K}}$$

$$\approx 0.0010\ \text{m} \approx 1.0\ \text{mm}$$

Answer. The pitch of the drill is about 1.0 mm.

Example 12. For how many kilometres of car travel will 40 l of petrol last, if the car weighs 35.3 kN, the total resistance to motion is 0.050 of the weight, the engine efficiency is 18 per cent? The motion is uniform.

Given: $V = 0.040\ \text{m}^3$ is the petrol volume, $P = 35{,}300$ N is the car weight, $F = 0.050\ P$ is the force of resistance to motion, $\eta = 0.18$ is the efficiency of the engine, $q = 4.6 \times \times 10^7$ J/kg is the specific heat of combustion of petrol, $\rho = 700\ \text{kg/m}^3$ is the density of petrol.

Determine: s—the distance covered.

Solution. The distance covered is to be found by the formula for the work done by the engine

$$s = A/F$$

The engine does the work A using a part (Q_1) of the total energy Q obtained in burning the fuel

$$\eta = Q_1/Q$$

hence

$$Q_1 = Q\eta$$

The energy liberated in the burning of fuel is

$$Q = qm$$

where $m = \rho V$.
Hence

$$A = Q_1 = Q\eta = qm\eta = q\rho\,V\eta$$

The pull with uniform motion is equal to the force of resistance to motion, F, that is given to be 0.050 of the car weight, i.e.

$$F = 0.050P$$

The derived expressions are substituted for A and F in the relationship for s

$$s = \frac{q\rho\eta V}{0.050P}$$

After substitutions and calculations we arrive at

$$s = \frac{4.6 \cdot 10^7\ \mathrm{J/kg} \cdot 700\ \mathrm{kg/m^3} \cdot 0.040\ \mathrm{m^3} \cdot 0.18}{0.050 \cdot 35{,}300\ N} \approx 130{,}000\ \mathrm{m} \approx 130\ \mathrm{km}$$

Answer. The petrol will last for about 130 km.

Changes in Internal Energy. Heat Exchange

4.1. Why is insular climate more moderate and smooth than that of mainland?

4.2. In deserts at day-time temperature rises very high, and at night it drops below the freezing point. Why?

4.3. It is common knowledge that at high altitudes (800-1000 km) the velocities of molecules of gases constituting the atmospheric air correspond to temperatures about 2000°C. Why does the envelope of a satellite travelling at these altitudes not melt?

4.4. What is the new temperature of water after mixing 39 l of water at 20°C and 21 l of water at 60°C?

4.5. What is the temperature of water after mixing 6 kg of water at 42° C, 4 kg of water at 72° C and 20 kg of water at 18° C?

4.6. How many litres of water at 95° C are to be added to 30 l of water at 25° C to obtain water at 67° C?

4.7. A boiler contains 40 m³ of water at a temperature of 225° C. What amount of water at 9° C is to be added to have a common temperature of 200° C? Neglect variation of water density in raising the temperature.

4.8. How many litres of water at 20 and 100° C are to be mixed to obtain 300 l of water at 40° C?

4.9. A quantity of 60 kg of water at 90°C and 150 kg of water at 23°C are mixed; 15 per cent of heat released by the hot water is used up in the heating of environment. What is the final temperature of the water?

4.10. A piece of tin heated to 507 K is placed into a vessel containing 2.35 kg of water at 20°C; the temperature of the water in the vessel steps up by 15 K. Assuming that there is no water evaporation, what is the mass of the tin?

4.11. A steel drill of mass 0.090 kg heated in hardening to 840°C is placed into a vessel containing machine oil at

20°C. What amount of oil is to be taken in order that its final temperature be not higher than 70°C?

4.12. Figure 4.12 shows schematically the heat exchange process for the case where a metal block heated to t_2 °C is placed in a calorimeter containing water at temperature t_1 °C ($t_1 < t_2$). Explain the meaning of individual sections of the curve. What would an equal slope of segments KN and KL of the curve signify?

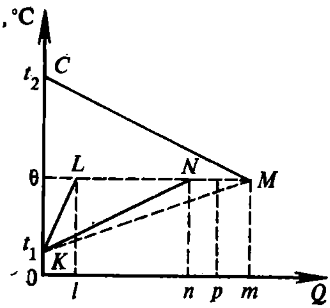


Fig. 4.12

4.13. A steel part preheated to 500°C is placed into a vessel containing 18.6 l of water at 13°C and heats to 35°C. Ignoring evaporation of water, what is the mass of the part?

4.14. A preheated block of mass 0.20 kg is placed in a vessel containing 0.80 kg of kerosene at temperature 15°C. The final temperature of the kerosene is 20°C. What is the initial temperature of the block?

4.15. A 0.30 kg plate heated preliminarily to 85°C is placed in an aluminium calorimeter of mass 42 g in which there is 0.25 kg of water at 22°C. The final temperature in the calorimeter is 28°C. Determine the specific heat of the plate material.

4.16. Into a glass bulb of mass 50 g containing 185 g of water at 20°C some mercury is added at temperature 100°C, and the temperature of water in the bulb rises to 22°C. What is the mass of the mercury?

4.17. To determine the temperature of a furnace a 0.30 kg steel bolt heated in it is placed into a copper vessel with mass 0.20 kg containing 1.27 kg of water at 15°C. The temperature of water steps up to 32°C. What is the temperature of the furnace?

4.18. A tin cylinder of mass 0.60 kg preheated to 100°C is placed into a 29.5 g aluminium calorimeter confining kerosene at 20°C. What amount of kerosene does the calorimeter contain, if the final temperature is 29.5°C and the heat losses into environment are 15 per cent?

4.19. A 0.15 kg heated steel cutter is placed into an alu-

minium vessel of mass 0.10 kg containing 0.60 kg of machine oil at 15°C, the oil heats to 48°C. The heat losses into environment are taken to be 25 per cent. What was the temperature of the cutter?

4.20. Water is fed to a hot-water heating reflector at a temperature of 80°C over tubing with sectional area 500 mm^2 at a speed of 1.2 cm/s, and at the outlet of the reflector the water has a temperature of 25°C. How much heat is absorbed by the heated room during 24 hours?

4.21. Water comes to hot-water radiators at 341 K and goes out at 313 K. In a $6 \times 5 \times 3$ m^3 heated room the initial air temperature is 279 K, and 40 l of water pass through the radiators. The heat losses through the walls, windows, and floor amount to 50 per cent. What is the new room temperature?

4.22. What amount of water is to pass through hot-water radiators for the air in a room with dimensions $10 \times 6 \times 3.5$ m^3 to heat from 10 to 22°C? The water temperature in the radiators drops by 25°C. The heat losses through the walls, windows, and floor are taken to be 60 per cent.

Heat and Work. Adiabatic Process

4.23. It is common knowledge that the specific heat of a gas at constant pressure (c_p) is substantially different from that at constant volume (c_V). Which of the two specific heats is the larger? Justify.

4.24. What is the principle of metal cutting by a friction saw that is essentially a steel disk without any teeth?

4.25. Why does the stirring of hot tea with a spoon result in the cooling of the tea? (In Joule's experiment a similar action leads to the liquid being heated.)

4.26. Flour comes hot from under millstones. Bread also comes hot from the oven. What is the reason behind each of these cases of increase in the internal energy of the body (flour, bread)?

4.27. Why does an air layer adjacent to the body of a spaceship travelling in the Earth's atmosphere warm up intensively? Why does a major part of meteors fail to reach the Earth's surface?

4.28. Water drops from a height of 1200 m, 60 per cent of gravity work goes into the heating of the water. How much will the water heat?

4.29. From what height is a piece of tin to fall in order that when it hits the ground it might warm up to 373 K? Up to the melting point? Assume that the heating of the tin consumes 40.0 per cent of gravity work, and its initial temperature is 273 K.

4.30. Two balls of equal mass, made of copper and aluminium, are dropped from 1000 m. Which of the two will heat the more and by what amount? The heat losses are to be neglected.

4.31. A steel bit of an air hammer develops a percussion energy of 37.5 J at 1000 strokes per minute. Determine the power developed by the hammer. How much will the bit temperature increase after 3 minutes of work, if 15 per cent of the total energy goes into its heating, the bit mass being 1.8 kg?

4.32. A 1.5 kg block of steel heats in stamping when subjected to a stroke by a hammer of mass 400 kg; the velocity of the hammer at the instant of hit is 7.0 m/s, and 60 per cent of the hammer energy is used up in the heating of steel. What is the rise in the temperature of the block?

4.33. A steel hammer of mass 12 kg drops on 0.20 kg iron plate lying on an anvil. The height of the drop of the hammer is 1.5 m. Assuming that the plate heating consumes 40 per cent of the kinetic energy of the hammer, calculate how much the plate heats after 50 strokes of the hammer.

4.34. Using a mechanical hammer of weight 58.8 kN an iron forging of mass 205 kg is processed. 35 strokes heat the forging from 283 to 291 K. What is the velocity of the hammer at the moment of hitting the forging, given that 70 per cent of the hammer energy is used up in the heating of the forging?

4.35. A 12.5 t tram car travelling at a speed of 28.8 km/h slows down and stops. How much will its 8 cast-iron brake shoes heat, if the mass of each shoe is 9.0 kg and the heating consumes 60 per cent of the kinetic energy of the car?

4.36. In drilling metal by a manual drill the drill bit of mass 50 g heats by 70.5 K in 3 min of continuous opera-

tion. Assuming that the bit heating takes 15 per cent of the energy input, determine the power developed in drilling.

4.37. One of the ways of manufacturing wire is the extrusion method. A heated stock is placed in a cylinder having an orifice with a diameter corresponding to that of wire. Then the stock is subjected to pressure exerted by a piston entering the cylinder (Fig. 4.37). How much will the temperature of a 1 kg copper stock increase in 5.0 s, if the power input in extrusion is 4.41 kW, and 60 per cent of energy is taken up by the heating?

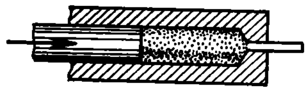


Fig. 4.37

4.38. What is the change in the temperature of a $2.0 \times \times 8.0$ cm^2 copper plate when cutting a thread in it with pitch 0.50 mm, if the torque input is 7.6 Nm? Ignore heat losses.

4.39. Determine the torque in cutting a 0.75 mm pitch thread in a steel nut, if in the course of the cutting time the nut heats by 50 K. The section of the nut is taken to be a circle of 20 mm in diameter. Ignore heat losses.

4.40. An amount of 1.43 kg of air at 0°C occupies a volume of 0.5 m^3. The air took up a quantity of heat and expanded at constant pressure to a volume of 0.55 m^3. Find the work done, the heat absorbed, the changes in the internal energy and temperature of the air.

4.41. A cylinder with a base area of 0.25 m^2 contains under the piston 0.022 kg of hydrogen at temperature 0°C and pressure $2.5 \cdot 10^5$ Pa. The gas is heated at constant pressure gaining 17.2 kJ of heat. Determine the work done in the process, the change in temperature within the cylinder, and the piston displacement.

4.42. Air at temperature 0°C and pressure 1.82 atm occupies under the piston of a cylinder a volume of 0.2 m^3. What is the work done in the expanding at constant pressure of the air heated by 30 K, and what is the heat absorbed in the process? Find the piston displacement, if at first the piston was at a distance of 500 mm from the cylinder bottom.

4.43. Give examples of adiabatic and near-adiabatic processes. What sets the adiabatic process apart from any other

iso-process? Can an adiabatic process in a gas occur without any thermal insulation between the gas and the environment? Plot a graph of the isothermal process $pV = C_1$ in the coordinates p, V. In the same diagram draw a graph of the adiabatic process $pV^\gamma = C_2$ for the same amount of the same gas ($\gamma = c_p/c_V = 1.4$).

4.44. Air at temperature —13°C and pressure 1.5 atm is subjected to adiabatic compression to reduce its volume 12-fold. Determine the final pressure and temperature of the gas, and the work done in compression of 1 kg of the gas.

4.45. The pump of a compressor delivers at —14°C 0.4 l of atmospheric air per piston stroke to a 20 l cylinder. How many piston strokes are required to obtain a pressure of 5.8 atm? What is the final temperature of the air in the cylinder? What work is done in compressing the air? The process is adiabatic.

Heat of Combustion

4.46. What amount of kerosene is to be burnt to have 50 l of water heated from 20°C to boiling? The heater efficiency is 35 per cent.

4.47. The heating of 1.8 kg of water from 18°C to boiling with a burner of 25 per cent efficiency requires 92 g of fuel. What is the specific heat of combustion of the fuel?

4.48. Determine the heater efficiency that consumes 80 g of kerosene to heat 3.0 l of water by 90 K.

4.49. What amount of water can be heated from 288 K to the boiling point with a gas burner of 40 per cent efficiency, if 100 l of natural gas is burnt?

4.50. How long before 1.55 l of water heats from 293 to 373 K, if the burner consumes 0.30 kg of alcohol per hour, its efficiency being 24 per cent?

4.51. A 9.0 g bullet leaves the rifle barrel at a speed of 850 m/s, the mass of the gun powder charge being 4.0 g. What is the efficiency of the shot?

4.52. The efficiency of a melting furnace is 20 per cent. What amount of coal of brand A-II must be burnt to heat 3.0 t of gray cast iron from 283 K to the melting point?

4.53. What amount of aluminium can be heated from 283 K to the melting point in a melting furnace of 26 per cent efficiency, if 25 kg of petrol is burnt?

4.54. How much heat is lost per 24 hours through the walls and windows of a room with stove heating, if to hold the air temperature in it at a constant level 10 kg of A-I coal is required to be burnt? The stove efficiency is taken to be 35 per cent.

4.55. The exterior walls of a flat have a total area of 45 m^2, a thickness of 0.60 m, and a thermal conductivity coefficient of 0.80 W/(m·K) (stock brick). The temperature inside the flat is 295 K, outside 268 K. What amount of firewood is it required to be burnt to maintain the above temperature drop constant during a 24 hour period? The stove efficiency is 40 per cent, the heat lost through the walls accounts for 3/4 of the total heat loss.

Efficiency of Thermal Engines

4.56. The engines of two tractors are diesels, one develops a power of 220 kW and consumes 0.23 kg of diesel fuel per 1 kW per hour; the other develops a power of 40 kW and consumes 0.30 kg of diesel fuel per 1 kW per hour. What is the efficiency of each engine?

4.57. A 55 kW automobile engine consumes 0.31 kg of petrol per 1 kW per hour. What is the efficiency of the engine?

4.58. Determine the fuel consumption per 1 kW per hour for a steam turbine of efficiency 30 per cent (petroleum); for a car of efficiency 24 per cent (petrol); for a locomobile of efficiency 6 per cent (straw).

4.59. Find the efficiency of a bus engine consuming 63 kg of ligroine in 2.5 hours of operation at an average power of 70 kW.

4.60. Determine the fuel mass required for a diesel locomotive to cover a distance of 1000 km with an average speed of 72 km/h, if the locomotive has two sections with 735 kW diesels of 28 per cent efficiency.

4.61. A steam locomotive of 1470 kW has an efficiency of 7.5 per cent. How much A-I coal is it required for the locomotive to cover 1000 km at an average speed of 54 km/h?

4.62. Determine the fuel consumption per 100 km by a car at a speed of 90 km/h, if the engine efficiency is 27 per cent, and its actual power amounts on the average to 0.4 of maximum power (72 kW).

4.63. Consider an outboard engine of power 13.2 kW and efficiency 15 per cent. How many kilometres will it travel with 20 l of petrol at a speed of 30 km/h?

4.64. A river motor ship has a diesel rated at 70 kW at an efficiency of 30 per cent. During a trip the diesel consumes 0.12 t of fuel. What was the duration of the trip, if the time spent on stops was 2 hours?

4.65. A producer gas engine of power 50 kW and an efficiency of 18 per cent is installed on a truck. Determine the mass of fuel (wood blocks) required for the truck to cover 100 km at a speed of 36 km/h.

4.66. For what distance is the capacity (46 l) of an automobile fuel tank designed, if the speed is 85 km/h, efficiency 25 per cent, and actual power on the average is 0.35 the maximum power (55 kW)?

4.67. How many kilometres will 8 l of petrol last for a motorcycle engine developing 8.8 kW at 70 km/h with an efficiency of 21 per cent?

4.68. A major power plant of a sea-going motor ship includes two 800 kW diesels. Given that the fuel consumption is 245 g/(kW·h) determine the efficiency of the engines and fuel consumed in a trip for one week.

4.69. An automobile travels 300 km, the mass of the automobile is 5.0 t, the engine efficiency 22 per cent, and the resistance to motion is 0.050 times the weight of the automobile. Determine the pull of the engine, its power at 108 km/h, and the fuel consumed.

4.70. What kind of fuel is used in a thermopower plant consuming 33.0 kg of fuel for 1.5 h of operation at an efficiency of 0.20 and actual power of 25.2 kW? What is its specific heat of combustion?

4.71. An engine of a jet aircraft of 20 per cent efficiency develops a pull of 88.2 kN flying at a velocity of 1800 km/h. Determine the kerosene consumption during 1 hour of flight and the developed power.

SEC. 5. PROPERTIES OF REAL GASES AND VAPOURS. ATMOSPHERIC WATER VAPOUR

Example 13. A 600 W electric stove heats in 35 minutes 2.0 l of water from 293 to 373 K with 200 g of water turned into vapour. Determine the stove efficiency.

Given: $m_w = 2.0$ kg is the mass of water heated by the stove to the boiling point, $T_1 = 293$ K, $T_2 = 373$ K are the initial and final temperatures of water, $m = 0.20$ kg is the mass of evaporated water, $P = 600$ W is the electric power of the stove, $t = 2100$ s is the time of operation of the stove, $c_w = 4187$ J/(kg·K) is the specific heat of water, $r = 2.26 \times \times 10^6$ J/kg is the heat of vaporization for water.

Determine: η—the efficiency of the electric stove.

Solution. By definition, the efficiency of a heater is

$$\eta = (Q_1/Q)\, 100\%$$

where $Q_1 = c_w m_w (T_2 - T_1) + rm$ is the quantity of heat used to heat the water and evaporize a part of it, $Q = Pt$ is the thermal energy consumed by the electric stove.

Substitution for Q_1 and Q gives

$$\eta = \frac{c_w m_w (T_2 - T_1) + rm}{Pt}$$

And substituting numerical values we get

$$\eta = \frac{4\,187 \text{ J/(kg·K)} \cdot 2 \text{ kg} \cdot 80 \text{ K} + 2.26 \cdot 10^6 \text{ J/kg} \cdot 0.2 \text{ kg}}{600 \text{ W} \cdot 2\,100 \text{ s}} \approx 0.89$$

Answer. The efficiency of the electric stove is about 89 per cent.

Example 14. What quantity of water may be cooled down from 20 to 0°C by cooling it in 200 g of ethyl ether with the initial temperature of 20°C evaporizing under lower pressure? The heat of vaporization of ether is regarded as temperature independent, the efficiency of the apparatus is 80 per cent.

Given: $\eta = 0.8$ is the efficiency of the apparatus, $m = = 0.2$ kg is the mass of the ethyl ether, $T_1 = 293$ K is the initial temperature of the ether and water, $c = 2330$ J/(kg·K), $c_w = 4187$ J/(kg·K) are specific heats of ethyl ether and water, respectively, $r = 3.52 \cdot 10^5$ J/kg is the heat of

vaporization of ether, $T_2 = 273$ K is the final temperature of ether and water.

Determine: m_W—the mass of the water.

Solution. The problem is solved using the heat balance equation. The ether evaporating under lower pressure absorbs the amount of heat rm. In cooling from T_1 to T_2 the quantity of heat absorbed will be given by $rm—cm (T_1—T_2)$. Due to imperfections of the thermal insulation heat will be also derived from ambient air, and, as stated, the coolant water will yield only a part $Q_1 = \eta [rm - cm (T_1 - T_2)]$ of heat required to evaporate the ether. By virtue of the energy conservation law we have

$$\eta m [r - c (T_1 - T_2)] = c_W m_W (T_1 - T_2)$$

Substituting the data gives

$$m_W = \frac{0.8 \cdot 0.2 \text{ kg} [3.52 \cdot 10^5 \text{ J/kg} - 2\,330 \text{ J/(kg}\cdot\text{K)} \cdot 20 \text{ K}]}{4\,187 \text{ J/(kg}\cdot\text{K)} \cdot 20 \text{ K}} \approx 0.58 \text{ kg}$$

Answer. The mass of water is about 0.58 kg.

Example 15. On lowering the temperature from 16 to 10°C each cubic metre of air evolves 1.5 g of water. What was the relative humidity of air at 16°C?

Given: $t_1 = 16$°C is the initial air temperature, $t_2 = 10$°C is the final air temperature, $m/V = 1.5 \cdot 10^{-3}$ kg/m^3 is the amount of water evolved from each cubic metre of air.

Determine: φ—the relative humidity of air at 16°C.

Solution. As stated, the air at temperature 283 K is saturated with water vapour. From the table we find the density of vapour, ρ_1, saturating the air at 10°C. Next we add to this figure the quantity of water derived from each cubic metre of air (m/V) to obtain the vapour density in the air at 16°C, i.e. the absolute humidity of air $\rho_a = \rho_1 + m/V$. Now we look up in the table for the vapour density saturating the air at 16°C and calculate the relative air humidity $\varphi = \rho_a/\rho_s$.

Substituting the numerical values into the relationship for φ and performing the appropriate calculations we obtain

$$\varphi = \frac{9.4 \cdot 10^{-3} \text{ kg/m}^3 + 1.5 \cdot 10^{-3} \text{ kg/m}^3}{13.6 \cdot 10^{-3} \text{ kg/m}^3} \cdot 100\,\% = 80\,\%$$

Answer. The relative air humidity at 16°C is 80%.

Vaporization and Condensation

5.1. Why do wet washing on a line and mowed grass dry faster in a breezy weather?

5.2. Why is the temperature of water in open water bodies in summer always lower than that of ambient air?

5.3. Why does a swimmer just out of water feel cold and especially so in a windy weather?

5.4. Why does one feel cooler in the rain?

5.5. Hot weather is endured harder in a marshy locality than in a dry one. Explain why?

5.6. Account for the fact that rubber dress makes heat harder to endure.

5.7. Can a solid body evaporate?

5.8. Why does water quench fire? What will put away a fire quicker: hot or cold water?

5.9. What is one of the advantages of a gas-filled lamp as compared with a vacuum lamp?

5.10. Figure 5.10 shows curves of the temperature variations for three different liquids heated with burners which provide to these liquids equal quantities of heat during equal periods of time. Masses of the liquids are equal. Using Tables V and VII compare specific heats of the liquids and identify them.

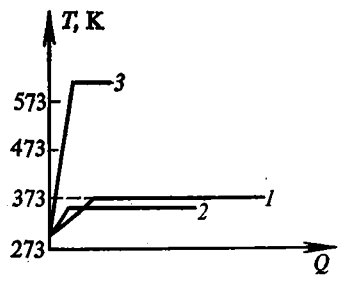


Fig. 5.10

5.11. There are 200 g of alcohol and 200 g of ether at a temperature of 293 K. What quantity of heat is it required to change these liquids into vapour?

5.12. How much heat is liberated in condensation of 200 g of water vapour at a temperature of 100°C and in cooling the water obtained to 20°C?

5.13. Into a vessel in which there is 400 g of water at a temperature of 17°C, 10 g of vapour at 100°C that turns to water is introduced. Determine the final temperature of the water. Ignore the heat capacity of the vessel and heat losses.

5.14. A 50 g aluminium calorimeter contains 250 g of

water at 289 K. What amount of vapour with a temperature of 373 K is to be added to the calorimeter for the temperature of the water in it to rise to 363 K?

5.15. To determine the specific heat of vaporization of water the following experiment is conducted: vapour at a temperature of 373 K is introduced into an aluminium calorimeter of mass 52 g, containing 0.25 kg of water at 282 K

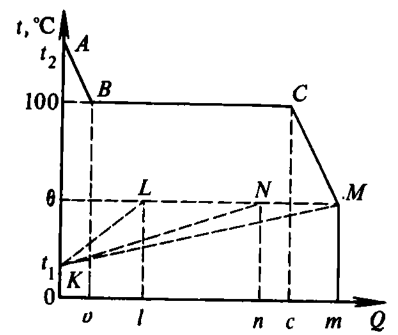


Fig. 5.16

After the injection of vapour the calorimeter is found to contain 0.259 kg of water at a temperature of 303 K. From the data obtained calculate the specific heat of vaporization of water. What fraction of the energy consumed by the vaporization is consumed in doing an external work (overcoming atmospheric pressure). The specific volume of water vapour at 373 K is taken to be 1.65 m^3/kg.

5.16. Shown in Fig. 5.16 is a heat exchange curve for the case where water vapour at $t_2 > 100$°C is fed to a calorimeter containing water at t_1°C. Explain the meaning of individual sections of the curve.

5.17. Into a vessel with 30 l of water 1.85 kg of water vapour is added at 100°C. Upon vapour condensation the water temperature in the vessel increases to 37°C. Find the initial temperature of the water. Ignore the heat capacity of the vessel.

5.18. A copper calorimeter of mass 0.70 kg at a tempera-

ture of 285 K contains 0.80 l of water. What will the new temperature be, if 0.050 kg of vapour at 373 K is introduced into the calorimeter?

5.19. Through a coil pipe of a heater containing 12 l of water at 12°C water vapour is passed at 100°C. At the outlet the water (condensate) has on the average a temperature of 60°C. What quantity of vapour is to be passed through the coil to have the temperature of the water in the heater rise to 50°C?

5.20. A hot-water boiler of 75 per cent efficiency contains 208 l of water at a temperature of 15°C. How much vapour at 104°C must be passed through the coil pipe of the boiler to heat the water to 92°C? The temperature of water at the outlet of the coil pipe is also taken to be 92°C.

5.21. Vapour enters the coil pipe of a heater at a temperature of 100°C, the temperature of condensate at the outlet of the coil pipe is 90°C. During an hour's time 2.0 m^3 of water with the initial temperature of 8°C passes through the heater, whereas 360 l of condensate flows out of the coil pipe. What is the new temperature of the water, if the heater has an efficiency of 80 per cent?

5.22. A 561 g aluminium block heated to 200°C is immersed into 400 g of water at 16°C. In the process, some water evaporates and the rest heats to 50°C. What is the mass of the water evaporated?

5.23. What mass of distilled water may be produced, if 20 kg of wood is burnt in the furnace of a still? The efficiency of the still is 35 per cent, the initial temperature of water is 279 K.

5.24. A still of 33 per cent efficiency contains 20 l of water at 283 K. What amount of distilled water may be obtained, if 2.0 kg of petroleum is burnt in the furnace?

5.25. In a distiller there is 30 l of water at 281 K. To produce 5.0 l of distilled water 1.6 m^3 of natural gas is spent. What is the efficiency of the distiller?

5.26. A vessel containing water is heated with an electric stove from 20°C to boiling in 20 minutes. How much time is required to change 20 per cent of water into vapour with the same stove efficiency and operating conditions?

5.27. The efficiency of the ammonia-gas refrigerator is 75 per cent. How much ammonia evaporates in the refrig-

erator tubes to cool 0.86 kg of water from 293 K to the freezing point?

5.28. What is the efficiency of a refrigerator, if to cool 2.0 kg of water from 282.5 K to the freezing point it is necessary to evaporate 73.0 g of Freon?

5.29. Into a 3.0 l chamber vented to atmosphere a piece of solid carbon dioxide ("dry ice") of mass 1.0 g is placed at STP, and immediately thereafter the chamber is closed. What is the new pressure in it after all the "dry ice" has evaporated? Ignore a temperature change within the chamber.

Saturated and Nonsaturated Vapour. Critical State

5.30. Can water boil in a pan floating in another pan with boiling water? Explain.

5.31. There is some liquid in a U-pipe with sealed ends (Fig. 5.31). Is there any way of knowing whether the space over the liquid contains saturated vapour only or a mixture of the saturated vapour with air?

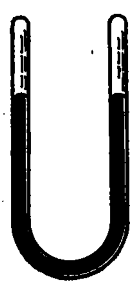


Fig. 5.31

5.32. How is superheated vapour produced? What water vapour is obtained at a temperature of 485 K and a pressure of $1.6 \cdot 10^6$ Pa? At 433 K and $6.18 \cdot 10^5$ Pa?

5.33. The pressure of water vapour at 15°C is 1280 Pa, the volume 5.76 l. What is the vapour pressure at 27°C and 8.0 l?

5.34. Saturated water vapour with the initial temperature of 20°C is separated from the liquid and heated to 30°C at constant volume. Determine the pressure of the vapour. How is this vapour termed?

5.35. Water vapour at a temperature of 40°C and a pressure of 1117 Pa has been cooled to 7°C. What is the new vapour pressure? What vapour is it? What will occur with further isochoric cooling down to 2°C?

5.36. Why do vegetables and fruit dry faster in rarefied air (in a vacuum apparatus)? What amount of water will be released as a result of the drying of a batch of fruits in

the chamber of a vacuum apparatus? The pumping (air and water vapour) is carried out at a rate of 70 l/min for 1 hour at a constant pressure (986.4 Pa); after the pumping the pressure drops almost to zero. The temperature in the chamber is to be regarded as constant and equal to 7°C.

5.37. Determine the density of water vapour at temperatures of 10, 29, 70°C, if the vapour tension at these temperatures is 1227, 4000, and 31,400 Pa, respectively.

5.38. Determine the water vapour pressure at temperatures of 18, 29, and 50°C, if the vapour density is 15.4, 25.8, and 83.2 g/m^3, respectively.

5.39. A mass of 0.90 g of unsaturated water vapour is compressed isothermally under the piston of a cylinder at

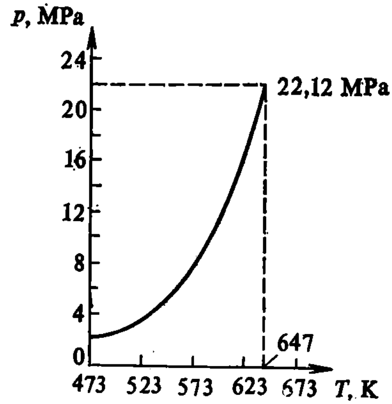


Fig. 5.42

a temperature of 29°C. What will the vapour volume be at the start of condensation?

5.40. A mass of 276 mg of unsaturated water vapour is subjected to isothermal compression. When the volume occupied by the vapour had reduced to 12 l, vapour condensation sets in. What is the temperature of the process?

5.41. In a cylinder under the piston there is 2.0 g of ammonia gas at —55°C. The gas is compressed isothermally, and at a pressure of 2.7·10^4 Pa the saturation begins. What

is the volume of the gas at this instant? How much ammonia will condense, if the volume is reduced to 6 l?

5.42. Figure 5.42 depicts the variation of the pressure of saturated water vapour with temperature. Determine from the curve what the aggregate state of water is at a temperature of 575 K and pressures of 3 and 14 MPa; at a pressure of 10 MPa and temperatures of 525 and 625 K; at a temperature of 655 K and pressures of 220 and 250 atm.

5.43. What is the aggregate state of nitrogen at a temperature of 123 K and a pressure of $3.5 \cdot 10^6$ Pa? Oxygen at 153 K and pressure $4.8 \cdot 10^6$ Pa? Ether at 463 K and $4.0 \cdot 10^6$ Pa?

5.44. Using oxygen boiling at standard pressure (boiling point 90 K), is it possible to cool nitrogen down to critical temperature? Neon? Argon?

5.45. In what way is it possible to isolate from air its components? In what succession are they to separate out? Dry atmospheric air contains (by volume): 78.08 per cent nitrogen, 20.95 per cent oxygen, 0.93 per cent argon, 0.03 per cent carbon dioxide, the balance helium, neon, krypton, and xenon.

Atmospheric Water Vapour

5.46. Why does only the indoor side of window panes sweat in cold weather?

5.47. Why does one begin to feel damp as the air temperature in the room drops?

5.48. Explain the formation of dew and fog?

5.49. Why is the dew the most abundant after a hot day?

5.50. Why is there no night dew with dense clouds?

5.51. In 6 m^3 of air with a temperature of 19°C there is 51.3 g of water vapour. What are the absolute and relative humidities?

5.52. The temperature of air is 20°C, the dew point is 12°C. Find the absolute and relative humidities of the air.

5.53. The air temperature is 23°C, the relative humidity is 45 per cent. Determine the absolute humidity of the air and the dew point.

5.54. Under what condition may the relative air humidity increase despite a decrease in absolute humidity?

5.55. When will the sensation of dampness be stronger: in air containing 15 g/m^3 of vapour at a temperature of 30°C or in air with 4 g/m^3 of vapour at 2°C?

5.56. Find the relative air humidity if the dry- and wet-bulb thermometers of a psychrometer read 29 and 22°C; 15 and 9°C; 25 and 21°C; 20 and 18°C, respectively. Compare the found values of relative humidity with the readings of Lambrecht hygrometer, if its dew-point indications at the same moments and place are 18; 2; 19; and 17°C, respectively.

5.57. The relative air humidity in a room is 43 per cent, the temperature is 19°C. What does the wet-bulb thermometer of a psychrometer read?

5.58. The relative humidity of air is 73 per cent. What do the dry- and wet-bulb thermometers of a psychrometer read, if the difference in their indications is 2°C? 4°C?

5.59. At 25°C the relative humidity of air in a room is 70 per cent. How much water will each cubic metre yield on lowering the temperature down to 16°C?

5.60. In the evening on the shore of a lake at a temperature of 18°C the relative humidity of air is 75 per cent. At what temperature might one expect a fog in the morning?

5.61. The relative humidity of air at 22°C is 60 per cent. Will there be any dew in lowering the temperature down to 16°C? To 11°C? If so, what amount of water will separate out of each cubic metre of air?

5.62. At 6°C the relative humidity of air is 55 per cent. Will any hoar frost appear with the temperature lowered to —1°C? To —3°C? If so, how much water does each cubic metre of air yield?

5.63. As the temperature lowers from 27 to 10°C each cubic metre of air gives out 8 g of water. What is the relative humidity of air at 27°C?

5.64. At 15°C the relative humidity of air in a $6 \times 4 \times 3$ m^3 room is 80 per cent. How much water will evolve from the air as the temperature drops to 10°C? By how many degrees is it necessary to heat the air for the relative humidity to be equal to 60 per cent?

5.65. The air temperature is 27°C, the relative humidity is 54 per cent. What will the change in atmospheric pres-

sure be, if at constant temperature the relative humidity increases to 70 per cent? Decreases down to 45 per cent?

5.66. What amount of water may evaporate in a $10 \times 8 \times 4.5$ m^3 room, if the air temperature is 22°C, and the relative humidity is 70 per cent? If the temperature is 25°C, and the dew point is 11°C?

5.67. A cylinder of capacity $V = 25$ l contains $m = 4.0$ g of water and the air saturated with water vapour at $t_1 = 11$°C and $p_1 = 8.652 \cdot 10^4$ Pa. The temperature is raised to $t_2 = 80$°C at constant volume. Determine the pressure p_2 and the relative humidity of air φ in the cylinder.

5.68. In a storehouse with volume 1500 m^3 during a night there establishes a temperature of 12°C at relative humidity 75 per cent. It is required to raise the temperature to 22°C while simultaneously lowering the humidity down to 60 per cent. What is to be done to this end?

5.69. A closed volume of 2 m^3 contains 97.6 g of water and at the top some saturating vapour of density 51.2 g/m^3 and pressure 7.36 kPa. The volume is increased to 10 m^3 at constant pressure. Determine the final pressure of the vapour, its temperature and relative humidity.

5.70. A variable-capacity reservoir—sylphon—resembling a bellows or an accordion, at first contained 4 l of air at temperature 25°C, humidity 62.5 per cent and pressure 98 kPa. Into the vessel a small amount of water is introduced, the vessel is sealed hermetically and expanded to a volume of 20 l at constant temperature, the humidity growing in the process to 80 per cent. What is the mass of water and final pressure of air?

5.71. A quantity of air occupies a volume of 1200 l at a temperature of 22°C and relative humidity of 75 per cent. What will the relative humidity be if the temperature is increased to 100°C and the volume decreased to 48.0 l?

SEC. 6. PROPERTIES OF LIQUIDS

Example 16. A cork cube, 2 cm on a side, floats on the surface of water. Determine the depth of its immersion in water, the wetting is regarded as complete.

Given: $a = 0.02$ m is the length of a cube side, $\rho_c = 240$ kg/m^3 and $\rho_w = 1000$ kg/m^3 are the densities of

cork and water, $\sigma = 0.072$ N/m is the surface tension of water, $g = 9.8$ m/s^2 is the free fall acceleration, $\theta = 0°$ is the contact angle (wetting angle).

Determine: Δh—the depth of immersion of the cube.

Solution. Let us look at the forces acting on the cube. The vertical forces are: the gravitational force $G = a^3\rho_c g$ and the force opposing to the resultant of the surface tension forces $F = -4a\sigma$, and an upward (buoyancy) force $F_A = a^2\Delta h\rho_w g$. We derive the equation for the force equilibrium

$$-a^3\rho_c g - 4a\sigma + a^2\Delta h\rho_w g = 0$$

hence

$$\Delta h = \frac{\rho_c}{\rho_w}a + \frac{4\sigma}{\rho_w g}\frac{1}{a}$$

Now we substitute the numerical values into the above relationship for Δh:

$$\Delta h = \frac{240\ \text{kg/m}^3}{1\,000\ \text{kg/m}^3}\cdot 0.02\ \text{m} + \frac{4\cdot 0.072\ \text{N/m}}{1\,000\ \text{kg/m}^3\cdot 9.8\ \text{m/s}^2\cdot 0.02\ \text{m}}$$
$$= 6.2\cdot 10^{-3}\ \text{m} = 0.62\ \text{cm}$$

Answer. The depth of immersion in water of a cube is 0.62 cm.

Example 17. Consider a glass capillary of 0.20 mm in diameter containing some water. The contact angle is 30°, the temperature of water is 20°C. Calculate the rise of water and the work done in the process by the surface tension forces. What kind of work is done?

Given: $d = 2.0\cdot 10^{-4}$ m is the diameter of the capillary, $\theta = 30°$ is the contact angle of the media water-glass-air, $\sigma = 0.072$ N/m is the surface tension of water at 20°C, $\rho = 1.0\cdot 10^3$ kg/m^3 is the density of water, $g = 9.8$ m/s^2 is the free fall acceleration.

Determine:
h—the rise of water in the capillary,
A—the work of the surface tension forces.

Solution. The rise of water is to be found from the formula

$$h = \frac{4\sigma}{\rho g d}\cos\theta$$

We determine the work A by multiplying the vertical component of the resultant surface tension force $F_{s.t} = \pi\sigma d$ by the rise h

$$A = F_{s.t} h \cos\theta = \frac{\pi\sigma d \cdot 4\sigma}{\rho g d}\cos^2\theta = \frac{4\pi\sigma^2}{\rho g}\cos^2\theta$$

The potential energy $\Pi = mg\frac{h}{2}$ of the water column accounts for half the work by the surface tension forces, which is readily seen if we eliminate m and h from the formula for Π:

$$\Pi = \rho S g \frac{h^2}{2} = \frac{2\pi\sigma^2}{\rho g}\cos^2\theta$$

The other half of the work is spent to overcome the friction and is lost to heat.

Making substitutions for h and A we arrive at

$$h = \frac{4 \cdot 7.2 \cdot 10^{-2}\ \text{N/m} \cdot 0.866}{1.0 \cdot 10^3\ \text{kg/m}^3 \cdot 9.8\ \text{m/s}^2 \cdot 2.0 \cdot 10^{-4}\ \text{m}} \approx 0.13\ \text{m}$$

$$A = \frac{4 \cdot 3.14 \cdot (7.2 \cdot 10^{-2}\ \text{N/m})^2 \cdot (0.866)^2}{1.0 \cdot 10^3\ \text{kg/m}^3 \cdot 9.8\ \text{m/s}^2} = 5.0 \cdot 10^{-6}\ \text{J}$$

Answer. The rise of water in the capillary is about 13 cm; the work of the surface tension forces is $5.0 \cdot 10^{-6}$ J.

Dimensions and Mass of Molecules. Osmotic Pressure

6.1. An oil drop of volume 0.050 mm^3 has spread over the surface of water having formed a film of area 600 cm^2. Assuming that the film is a bimolecular layer, calculate the size across an oil molecule.

6.2. Fig. 6.2 depicts an apparatus designed to determine the size of a "long" molecule of fatty acids. In the tray M five drops of the solution of palmitic acid in benzene are applied on the surface of water. The solution spreads over the surface limited by the tray walls and two movable blocks A and B. Benzene evaporates rapidly. The acid molecules arrange in a single layer perpendicular to the water surface coupling with it by their "acid" ends (see Fig. 6.6). The

mass of 1000 drops of the solution is 0.0033 kg, 4 g of acid was taken for 996 g of solvent, the surface of the film formed

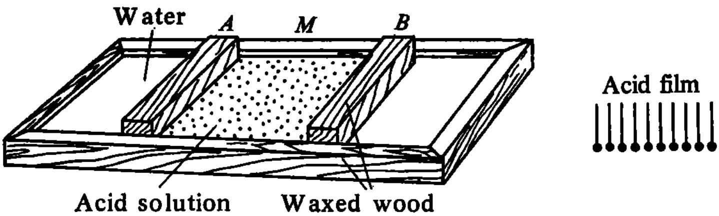


Fig. 6.2

is 0.23×0.14 m^2, the density of palmitic acid is $\rho = 850$ kg/m^3. Compute the length l of the acid molecule.

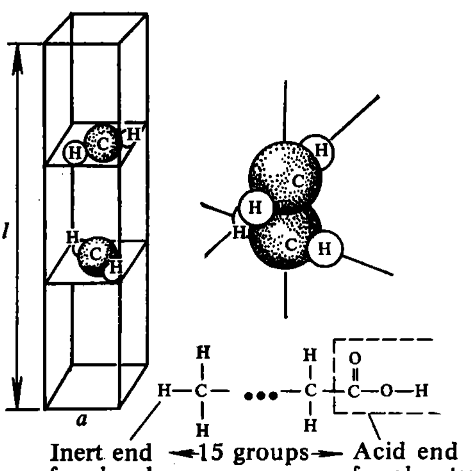


Fig. 6.6

6.3. Determine the number of molecules contained in 1 g of water; in 1 g and 1 cm^3 of sulfuric acid ($\rho = 1800$ kg/m^3); in a drop of water of 0.1 mm in diameter at 4°C.

6.4. Compute the intermolecular separation for water at 4°C.

6.5. Calculate the mass of a molecule for water, sulfuric acid, and mercury.

6.6. Using the structural formula of palmitic acid to determine its molar mass μ (Fig. 6.6) calculate the mass of a molecule of the acid. Assuming the section of the molecule to be a square with side a, find the ratio a/l (l is the length of the molecule determined in problem 6.2).

6.7. Compute the osmotic pressure caused by three lumps of sugar of total mass 20.0 g dissolved in 250 cm^3 of water at a temperature of 312 K. The formula of sugar is $C_{12}H_{22}O_{11}$.

6.8. What amount of sodium sulphate (Na_2SO_4) is to be dissolved in one litre of water at 298 K for the osmotic pressure to be $2.6 \cdot 10^5$ Pa?

6.9. An inverted funnel closed by a semipermeable membrane and having a long tube is immersed into a jar of water (see Fig. 2.8). The funnel contains a solution of 0.50 g of blue vitriol in 3.0 l of water at 288 K. What is the difference of levels in the tube and in the jar?

Surface Tension. Viscosity of Liquid

6.10. Why cannot aluminium be solded with conventional (tin) solder? The surface tension of soapy water is less almost by half than that of pure water. But soapy water produces stable bubbles and films that are not to be obtained from pure water. Why?

6.11. A spherical glass vessel three-quarters full of water is rendered weightless. What happens to the water? How will the result of the experiment change, if we substitute mercury for water?

6.12. A frame enclosing a surface of 40 cm^2 is covered with a soapy film. What is the change in the excess energy of the film if we curtail its area by half, the temperature being constant?

6.13. Compute the excess potential energy of the surface of a soap bubble of 50 mm diameter.

6.14.* To measure the surface tension of alcohol use is

* In problems 6.14-6.16 the diameter of the drop neck (d_n) is taken to be 0.9 of the diameter of canal tube (d_c).

made of a vertically positioned buret with an orifice of 1.6 mm diameter (Fig. 6.14). 100 drops are counted off with a total mass of 1.02 g. Compute the surface tension of the alcohol.

6.15. Kerosene flows out of a buret drop-by-drop through a 2.0 mm dia orifice, the drops following one after another at 1 s time intervals. How long before 25 cm^3 of kerosene leaves the buret?

6.16. A tray of capacity 6 cm^3 is being filled drop-by-drop with water from a tube with internal diameter 1.0 mm.

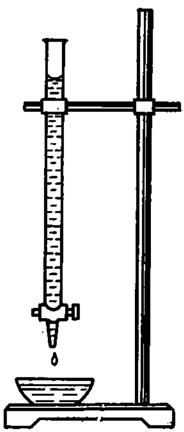


Fig. 6.14

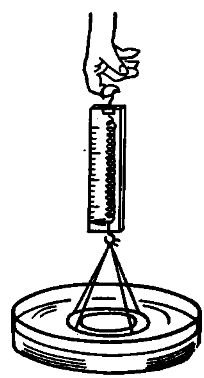


Fig. 6.18

How many drops are required to fill the tray at a water temperature of 20°C?

6.17. A wooden board placed at the bottom of a vessel and flooded with water rises to surface. A glass plate placed at the bottom of a vessel and flooded with mercury does not rise to surface, though the buoyancy of glass in mercury is much larger than that of wood in water. Explain.

6.18. To measure surface tension of water use is made of a spring balance and aluminium ring (Fig. 6.18); the ring is placed on the surface of water and then removed. The mass of the ring is 5.7 g, its average diameter is 200 mm. At the instant of separation of the ring from the water sur-

face the spring balance reads 0.15 N. Compute the surface tension of water.

6.19. What is the force needed to detach an aluminium ring of mass 5.0 g and average diameter 80 mm from the surface of glycerin (see Fig. 6.18)?

6.20. A steel ball of 1 mm diameter descends in a vessel with glycerin at an established speed of 0.25 cm/s. The resistance force in the liquid is $F = 6\pi r v \eta$, where r is the

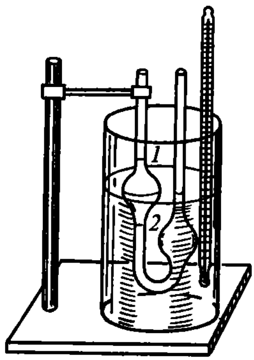


Fig. 6.21

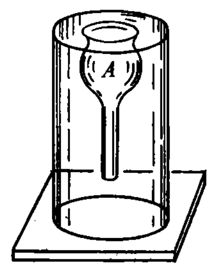


Fig. 6.22

ball radius, v is its speed, and η is the dynamic viscosity of the liquid. Find η for glycerin, perform the calculation for castor oil ($\rho = 960$ kg/m^3), if in this case the speed of the ball is 0.19 cm/s.

6.21. Figure 6.21 shows schematically the Ostwald viscometer used to determine dynamic viscosity under or just over that of water. First the duration t_1 is found experimentally of the flowing through the apparatus (between marks *1* and *2*) of a volume of water, and then the duration t of flowing of exactly the same amount of liquid. Next the dynamic viscosity η is calculated from the formula $\eta = \eta_1 \frac{\rho t}{\rho_1 t_1}$, where η_1 and ρ_1 are the viscosity and density of water at 293 K, respectively. Calculate the dynamic viscosity at 293 K of benzene, ethyl alcohol, nitrobenzene, if the flowing through the viscometer of the same volume

of the above liquids took 42, 90, and 100 s, respectively; for water $t_1 = 60$ s, $\eta_1 = 1.00 \cdot 10^{-3}$ Pa·s. How long will the flowing of the same volume of ether take ($\eta = 2.43 \times \times 10^{-4}$ Pa·s)?

6.22. Figure 6.22 shows schematically the Engler viscometer meant for the determination of dynamic viscosity of liquids which are by far more viscous than water. One calculates the ratio E of the time t taken for 200 g of a liquid being tested to flow out of a vessel A, to the time t_1 of outflow of 200 g of water from the same vessel: $E = t/t_1$. Then the viscosity η for the liquid tested is obtained by the formula

$$\eta = (7.32E - 6.31/E)\, \rho \cdot 10^{-6}$$

where ρ is the density of the given liquid at 293 K, and η is expressed in Pa·s.

Using this relationship determine the dynamic viscosity at 293 K of aniline and the saturated solution of phenol ($\rho = 1\,250$ kg/m^3) and mazout, if the flowing through the apparatus of 200 g of each of the above liquids takes 63.5, 87.5, and 400 s, respectively, and of 200 g of water—50 s. How long will 200 g of nitrobenzene take to flow through the apparatus ($\eta = 2 \cdot 10^{-3}$ Pa·s)?

Curved Liquid Surface. Capillarity

6.23. On the opposite sides of a T-tube two soap bubbles A and B of different diameters have been blown, thereafter the tube outlet has been closed (Fig. 6.23). What is to happen to the bubbles if these latter are left on their own protected from exposure?

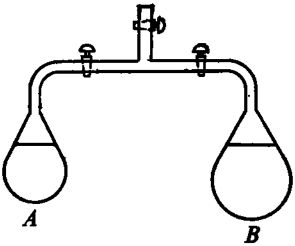


Fig. 6.23

6.24. Find the additional (Laplacian) pressure exerted by the surface of (a) a 18 mm dia air bubble under water, (b) 20.0 mm dia soap bubble.

6.25. What diameter of a soap bubble is required for the additional pressure at its surface to be 200 Pa?

6.26. What is the internal diameter of a glass tube, if

in it the curved surface of water gives rise to an additional pressure of 320 Pa? The contact angle is 30°.

6.27. The bottom of a vessel is fine net (sieve) of an unwettable material. What is the maximum height to which water can be poured into this vessel without flowing out through the bottom? Solve the problem for kerosene. The diameters of openings in both nets are taken to be 0.20 mm.

6.28. Given that the surface of contact of two soap bubbles with diameters $r_1 = 24.0$ and $r_2 = 20.0$ mm is also

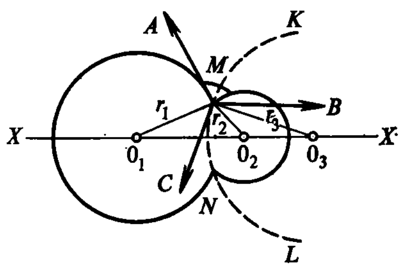


Fig. 6.28

spherical, determine the additional pressure caused by this surface and its radius of curvature r_3 (Fig. 6.28). Find the angles formed by soap films at points of their contact.

6.29. A cylinder with base diameter 24.0 mm made of a material wettable with water floats vertically on the surface of water, submerged by 16.7 mm. What is the mass of the cylinder?

6.30. Determine the maximum value of the diameter of a steel needle slightly smeared with fat, that floats horizontally over the surface of water. The depth of immersion of the needle is one half of its diameter. The needle diameter is 0.05 of its length ($d = 0.05\ l$).

6.31 Explain the action of a wick, gauze bandage, and blotting paper.

6.32. Under drought conditions a hard crust is formed on the surface of ground. Would it pay to preserve it in order to prevent the underlying horizons from drying?

6.33. In measuring the surface tension of alcohol use is made of a capillary tube with canal diameter 0.15 mm.

At 293 K the alcohol rises at a height of 7.6 cm. What is the surface tension of alcohol from these measurements?

6.34. The capillary tube diameter is 0.20 mm. Compute at room temperature the rise of water in it and the descent of mercury. Calculate for kerosene the work of surface forces and the potential energy of the thread of the liquid.

6.35. Calculate the diameters of canals in three capillary tubes, given that water at 293 K rises in them at heights 2.5 cm, 50 and 80 mm.

6.36. What is the rise of ether in a capillary tube with canal diameter 0.66 mm, if the contact angle θ at the glass-ether-air interface is 20°? How far down the same capillary will mercury descend? The contact angle for mercury is taken to be 155°.

6.37. The internal diameters of the right and left sections of an U-shaped capillary tube are 1 and 0.2 mm, respectively. What is the difference in levels in the tube for water, kerosene, and mercury? What is the surface tension of saturated solution of copper sulfate, if the level difference for it in the given tube is 105 mm?

6.38. In a capillary tube placed vertically in water down to a depth l water has risen to a height h. The tube is closed from beneath with a finger, removed from water, and opened again. Find the length x of water column that remained in the tube. The curvature of the upper (concave) and lower (convex) menisci is regarded as equal.

SEC. 7. PROPERTIES OF SOLIDS. FUSION AND CRYSTALLIZATION. DEFORMATIONS

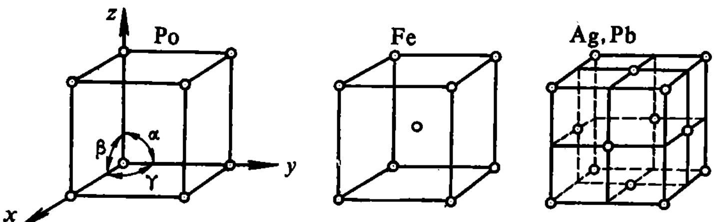
Example 18. Given that single crystals of silver have face-centered cubic lattice (Fig. 18, *c*) determine the number of immediate "neighbours" of each silver atom; the number of atoms per lattice cell; the lattice constant*; the least separation between atoms.

Given: $\mu = 107.88 \cdot 10^{-3}$ kg/mol is the molar mass of silver, $\rho = 1.05 \cdot 10^{4}$ kg/m^3 is the density of silver, $N_A = 6.023 \cdot 10^{23}$ mol^{-1} is Avogadro's number.

* The constant of a cubic space lattice is the length of the edge of a lattice cell.

Determine: a—the lattice constant,
d—the least separation between the atoms.

Solution. If we take an atom at one of the vertices of a cube, it will have 4 neighbouring atoms in centres of faces lying in each of the mutually perpendicular planes. In all, these are 12.



(a) (b) (c)

Fig. 18

A lattice cell includes one atom from those at cube vertices, and three atoms from those located at centres of its faces: 4 atoms all told.

The parameters of cubic lattice are given by

$$a = \sqrt[3]{\frac{\mu}{\rho N_A} \cdot \frac{k}{q}} \quad \text{and} \quad d = \frac{\sqrt{2}}{2} a$$

where k is the number of atoms per lattice cell, $q = 1$ is the number of atoms (ions) per molecule. For ionic lattices NaCl, ZnS $q = 2$.

Substituting the values into the expressions for the quantities we get

$$a = \sqrt[3]{\frac{107.88 \cdot 10^{-3}\ \text{kg/mol} \cdot 4}{1.05 \cdot 10^{4}\ \text{kg/m}^3 \cdot 6.023 \cdot 10^{23}\ \text{mol}^{-1}}} \approx 4.09 \cdot 10^{-10}\ \text{m}$$

$$d = 4.09 \cdot 0.707 \cdot 10^{-10}\ \text{m} \approx 2.89 \cdot 10^{-10}\ \text{m}$$

Answer. The lattice constant is about $4.09 \cdot 10^{-10}$ m, the least distance between atoms is $2.89 \cdot 10^{-10}$ m.

Example 19. The efficiency of refrigerator is 80 per cent. What amount of coolant (Freon-12) will evaporate to change into ice 150 g of water with initial temperature 289 K?

Given: $\eta = 0.80$ is the efficiency of the refrigerator, $m_W = 0.150$ kg is the mass of the water to be cooled, $T_1 = 289$ K is the initial temperature of the water, $T_2 = 273$ K is the temperature of melting of ice, $\lambda = 3.32 \cdot 10^5$ J/kg is the specific heat of ice melting, $r_F = 1.68 \cdot 10^6$ J/kg is the specific heat of Freon evaporation.

Determine: m_F —the mass of Freon evaporated.

Solution. The problem is solved using the equation of thermal balance. The amount of heat released by water in cooling and freezing is

$$Q_1 = c_W m_W (T_1 - T_2) + \lambda m_W$$

The quantity of heat required to evaporate Freon will be

$$Q = r_F m_F$$

The heats Q_1 and Q are related to each other by

$$\eta = Q_1/Q$$

By the energy conservation law we derive the equation of thermal balance

$$c_W m_W (T_1 - T_2) + \lambda m_W = r_F m_F \eta$$

Solving the thermal balance equation for m_F we obtain

$$m_F = \frac{c_W m_W (T_1 - T_2) + \lambda m_W}{r_F \eta}$$

Making the appropriate substitutions we arrive at

$$m_F = \frac{0.15\ \text{kg} \cdot [4\,187\ \text{J/(kg}\cdot\text{K)} \cdot 16\ \text{K} + 3.32 \cdot 10^5\ \text{J/kg}]}{1.7 \cdot 10^6\ \text{J/kg} \cdot 0.8} \approx 0.044\ \text{kg}$$

Answer. The mass of the Freon evaporated is about 0.044 kg.

Example 20. From a steel rod of section 2.0 cm^2 and length 0.50 m a 5.0 t weight is suspended. What is the margin of safety of the rod, if for steel the ultimate strength (breaking stress) in expansion is $1.25 \cdot 10^9$ Pa? What is the relative extension of the rod? What is the energy of elastic deformation of the rod? Ignore the mass of the rod.

Given: $m = 5.0 \cdot 10^3$ kg is the mass of the weight, $l = 0.5$ m is the length of the rod, $S = 2.0 \cdot 10^{-4}$ m^2 is the rod cross-sectional area, $g = 9.8$ m/s^2 is the free fall accele-

ration, $\sigma_l = 1.25 \cdot 10^9$ Pa is the ultimate strength of steel, $E = 2.2 \cdot 10^{11}$ Pa is Young's modulus.

Determine: n—the margin of safety;
ε—the relative extension;
Π—the energy of elastic deformation.

Solution. The margin of safety is to be found from the formula $n = \sigma_l/\sigma$, where $\sigma = F/S$, and $F = mg$, hence

$$n = \sigma_l S/mg$$

The relative extension is given by

$$\varepsilon = \frac{1}{E}\sigma = \frac{mg}{ES}$$

Knowing the force $F = mg$ responsible for the deformation, and the absolute deformation Δl, we determine the energy of elastic deformation Π:

$$\Pi = \frac{F\Delta l}{2}$$

where $\Delta l = \varepsilon l = \frac{mgl}{FS}$.

Hence

$$\Pi = \frac{(mg)^2 l}{2ES}$$

Substituting the numerical values, we arrive at the required results

$$n = \frac{1.25 \cdot 10^9 \text{ Pa} \cdot 2.0 \cdot 10^{-4} \text{ m}^2}{5.0 \cdot 10^3 \text{ kg} \cdot 9.8 \text{ m/s}^2} \approx 5.1$$

$$\varepsilon = \frac{5.0 \cdot 10^3 \text{ kg} \cdot 9.8 \text{ m/s}^2}{2.2 \cdot 10^{11} \text{ Pa} \cdot 2.0 \cdot 10^{-4} \text{ m}^2} \approx 1.1 \cdot 10^{-3}$$

$$\Pi = \frac{(5.0 \cdot 10^3 \cdot 9.8)^2 (\text{kg} \cdot \text{m/s}^2)^2 \cdot 0.5 \text{ m}}{2 \cdot 2.2 \cdot 10^{11} \text{ Pa} \cdot 2.0 \cdot 10^{-4} \text{ m}^2} \approx 14 \text{ J}$$

Answer. The margin of safety is around 5; the relative extension is about $1.1 \cdot 10^{-3}$; the energy of elastic deformation of the rod is about 14 J.

Example 21. The limit of elasticity of temperatured steel is $5.72 \cdot 10^8$ Pa, Young's modulus for steel is $E = 1.96 \cdot 10^{11}$ Pa. Will the deformation be elastic or plastic, if a steel wire of

length 3.00 m and section 1.20 mm^2 stretches by 8.00 mm under a tensile force? What is this force?

Given: $\sigma_{el} = 5.72 \cdot 10^8$ Pa is the limit of elasticity, $l = 3.00$ m is the length of the wire, $\Delta l = 8.00$ mm $= 8.00 \cdot 10^{-3}$ m is the absolute extension of the wire, $S = 1.20$ $mm^2 = 1.2 \cdot 10^{-6}$ m^2 is the cross-sectional area of the wire, $E = 1.96 \cdot 10^{11}$ Pa is Young's modulus.

Determine: Whether the deformation is elastic or plastic? What is the force that causes such an extension?

Solution. To determine the kind of deformation in the wire we obtain the stress σ in it and compare the result with the limit of elasticity σ_{el}. From the Hooke law we have $\sigma = E\varepsilon$. Considering that $\varepsilon = \Delta l/l$, we obtain

$$\sigma = E\frac{\Delta l}{l} = \frac{8.00 \cdot 10^{-3}\ \text{m} \cdot 1.96 \cdot 10^{11}\ \text{Pa}}{3.00\ \text{m}} \approx 5.23 \cdot 10^8\ \text{Pa}$$

The comparison of the obtained value of the stress σ with the limit of elasticity $\sigma_{el} = 5.72 \cdot 10^8$ Pa gives that the stress in the wire is less than the limit of elasticity ($\sigma < \sigma_{el}$). Thus the deformation of the wire is elastic.

Using the stress obtained we arrive at the force responsible for the deformation:

$$F = \sigma S = 5.23 \cdot 10^8\ \text{Pa} \cdot 1.2 \cdot 10^{-6}\ \text{m}^2 \approx 6.27 \cdot 10^2\ \text{N}$$

Answer. The deformation is elastic. It is caused by a force of about $6.27 \cdot 10^2$ N.

Space Lattice

7.1. How many molecules are there in 1 g of common salt (NaCl)? In 1 g of copper sulfate ($CuSO_4$)? In 1 cm^3 of zinc sulfide (ZnS, $\rho = 3\,980$ kg/m^3)?

7.2. How many atoms are there at room temperature in 1 g of tin? In 1 g of iron? In 1 cm^3 of iron?

7.3. Fig. 18 depicts the cells of atomic cubic lattices (*a*) primitive (polonium); (*b*) and (*c*) volume- and face-centered (α-iron*, lead). In each case determine the number of the nearest neighbours for each atom and the number of

---

* α-Fe is a form of the solid phase of iron. On heating to 910°C it changes into γ-Fe with face-centered lattice. At 1 392°C it returns to the former structure (α-Fe) and retains it all the way to melting.

atoms per cell, k. Calculate the parameters of the lattices: the lattice constant a and the least distance between atoms d. The d/a ratios are 1, $\sqrt{3}/2$, and $\sqrt{2}/2$, respectively.

7.4. What is the volume of a plate of zinc sulfide containing $5 \cdot 10^{22}$ molecules? A block of common salt containing $1.75 \cdot 10^{24}$ molecules?

7.5. Fig. 7.5 shows schematically the cell of a cubic space lattice of diamond or germanium (atoms inside the cube are represented by filled circles). Represent it as a plane net and as a combination of two face-centered lattices that are initially superposed. One of them is shifted as regards the other along their common diagonal line by 1/4 of its length. Determine the number of the nearest neighbours for each atom and the number of atoms per cell.

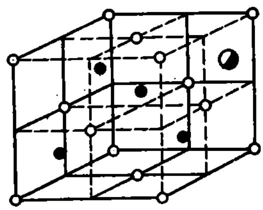


Fig. 7.5

7.6. Find the lattice constant for diamond and the least distance between atoms (see Fig. 7.5). What is the number of atoms in a germanium plate of dimension $12 \times 3 \times 0.74$ cm?

7.7. Fig. 7.7 presents the cells of ionic cubic lattices of caesium chloride (CsCl) and sylvine (KCl). For both

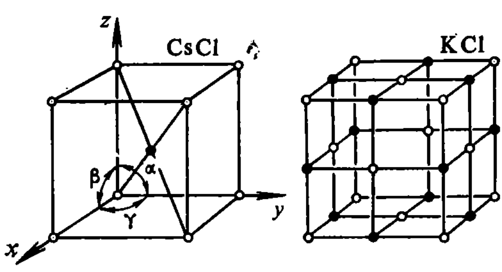


Fig. 7.7

substances find the number c of the nearest neighbours for each ion and the number k of ions per cell. Calculate the lattice constant a and the crystal density ρ, if the least

distance d between ions is $3.58\cdot10^{-10}$ m and $3.14\cdot10^{-10}$ m, respectively. Represent each of the lattices as a composite one (see problem 7.5).

Fusion and Crystallization

7.8. When water in reservoirs freezes, its surface layer turns to ice first. Explain.

7.9. An ice drift is accompanied by a cold snap. Why?

7.10. A snow fall is accompanied by a rise in temperature. Explain why.

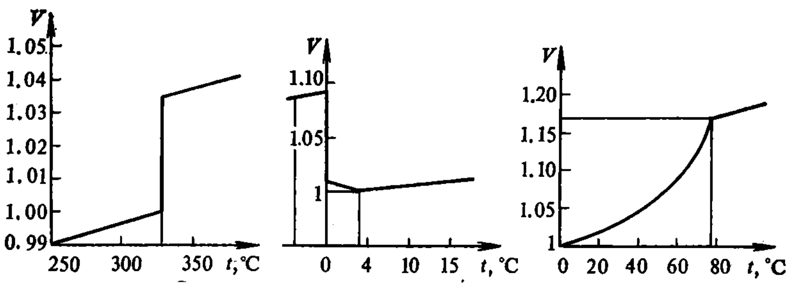
7.11. Why do fuses use a lead wire, and incandescent lamps tungsten filaments. Explain.

7.12. Which of the two materials—river ice or snow—their volumes and temperatures being equal, is the better coolant?

7.13. Hot-water heating tubing and automobile radiators are drained in a cold spell during prolonged downtime of the boiler room and the engine. Explain why.

7.14. A test tube with ice at a temperature of 0°C is placed in melting snow. Will the ice in the test tube melt?

7.15. Sketched in Fig. 7.15 are the variations in heating of the volume of (*a*) lead, (*b*) ice and water, and (*c*) wax.



(a) (b) (c)

Fig. 7.15

What are the melting points of these materials? What distinguishes the process in (*a*) from those in (*b*) and (c)?

7.16. Of what significance for spring field work is the melting heat of ice and heat of evaporation of water?

7.17. What amount of heat is it required to melt 100 g of ice at 0°C? 100 g of ice at —20°C?

7.18. What quantity of heat is it required to bring from 20°C to the melting point and melt 200 g of sulphur and 200 g of naphthalene?

7.19. What amount of heat is it required for 8.0 kg of ice at —30°C to be brought to the melting point, melted and for the water obtained to be heated to 60°C?

7.20. In an aluminium calorimeter of mass 0.20 kg, in which there is 0.34 kg of water at 23.5°C, a 81.5 g piece of ice at 0°C is placed. All the ice has melted. What is the new temperature in the calorimeter?

7.21. To determine the heat of fusion for ice the following experiment is performed: ice cubes at 0°C are placed in a copper calorimeter of mass 0.20 kg, in which there is 0.70 kg of water at 25°C. After all the ice has melted, the calorimeter contains 0.775 kg of water at a temperature of 15.2°C. From the results of the experiment determine the heat of fusion of ice.

7.22. It is necessary to cool from 80°C to 5°C 3.0 l of water in a 1.2 kg glass vessel, by placing in the water pieces of ice at 0°C. Determine the quantity of ice required.

7.23. Water can be cooled much below 0°C without changing into ice in the process, provided appropriate conditions are met. How much ice is formed, if in 1000 g of water supercooled to —10°C a small piece of ice is introduced to cause freezing? Down to what temperature is the water to be supercooled to change completely to ice under these conditions?

7.24. An iron vessel of mass 100 g contains 500 g of water and 200 g of ice at a common temperature of 0°C. How much water at 100°C is it required for the common temperature of the water in the vessel to rise to 32.0°C?

7.25. What will happen if 100 g of ice at 0°C are added to 410 g of water at 24°C contained in an aluminium vessel of mass 100 g? 150 g of ice at 0°C?

7.26.* A 100 g cast iron cylinder heated up to 100°C is placed on ice having a temperature of 0°C. What amount of ice will melt under the cylinder, when the latter cools down to 0°C?

---

* Ignore the loss coefficient, unless stated otherwise.

7.27. An aluminium cube is placed on ice at 0°C, and heated. What temperature is required for the cube to sink in ice fully?

7.28. A vessel containing a quantity of water is brought in a warm room, and in 15 minutes the water temperature rose to 4°C. Determine the time required for the same amount of ice to melt in this room. In both cases the heat exchange rates are taken to be equal.

7.29. A domestic refrigerator cools in 20 min 1.5 l of water from 16 to 4°C. If the process proceeds under the same conditions for another hour, how much food service ice will form in the refrigerator?

7.30. To remove snow from a surface of 200 m² a snow melter of efficiency 45 per cent is used. The snow density is 200 kg/m³, the thickness of snow cover is 25 cm, and the air temperature is —1°C. At the outlet of the snow melter the water temperature is 2°C. What quantity of fire wood is it required?

7.31. At —10°C each square metre of the surface of a pond loses 180 kJ of heat to air. What is the thickness of the ice cover formed in 24 hours, if at the surface of the pond the water temperature is 0°C?

7.32. In manufacture of shot, melted lead at a temperature of 327°C is poured in water. What quantity of shot is produced, if in the process 3.0 l of water heats from 25 to 47°C? The heat loss accounts for 25 per cent.

7.33. Fig. 7.33 shows diagrammatically the heat exchange process for the case where a piece of ice at $-t_1$°C is placed in a calorimeter containing water at a temperature of t_2°C. What is the meaning of individual sections of the curve. How should the curve be changed, if the initial temperature of ice were 0°C?

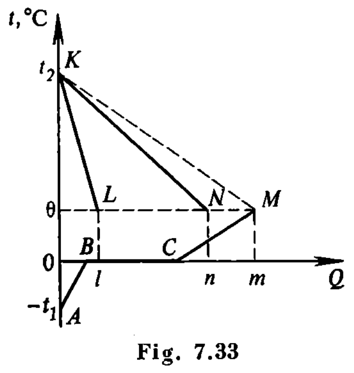


Fig. 7.33

Solutions and Melts. Triple Point

7.34. Why does the solving in water of common salt or sal ammoniac crystals lower the temperature of the solution? What is to happen with a crystal placed in an unsaturated solution of the same substance? In an oversaturated solution or supercooled melt of the same substance?

7.35. In refrigerator tubing, common salt solution, and not pure water, is made to circulate. Why?

7.36. Can you think of a way of producing fresh water from saline water at freezing temperature?

7.37. Figure 7.37 presents the variation of the melting point for a tin-zinc alloy with the zinc percentage of alloy. Discuss the curve. Indicate the zinc content at which the alloy shows the lowest melting temperature. Explain why cast iron melts at a lower temperature than iron.

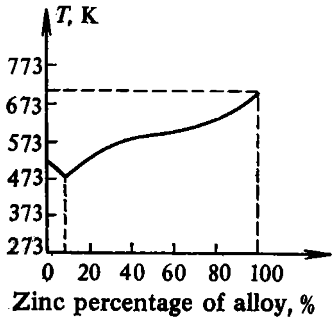
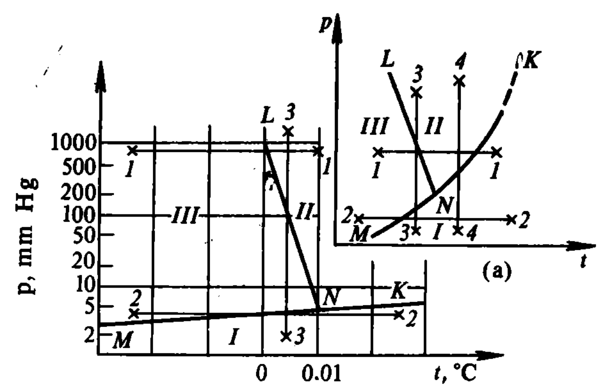


Fig. 7.37



(b)

Fig. 7.41

7.38. The melting point of tin is lower than that of olive oil. Explain why it is possible to fry in olive oil on a tin-plated frying pan.

7.39. Explain why ice in icehouses and ice-cream boxes is sprinkled with salt.

7.40. In winter, pavements are sometimes sprinkled with salt. Why? In which case will feet be colder: on a snow-covered pavement or on the same pavement sprinkled with salt?

7.41. Given in Fig. 7.41 (*a*) is the constitution diagram of a crystalline matter, and in Fig. 7.41 (*b*) a small section of the same diagram for water, ice, and water vapour near 0°C. Consider both diagrams. What is the meaning of sections *I*, *II*, *III*, of curves *KN*, *LN*, and *MN*; of point *N*? What processes are represented by curves *1-1*, *2-2*, *3-3*, and *4-4*?

Phase Transitions

7.42. What amount of heat is it required to change 6.0 kg of ice at —20°C to vapour at 100°C?

7.43. An iron reservoir of mass 5.0 kg contains 20 kg of water and 6.0 kg of ice at 0°C. What quantity of water vapour at 100°C is to be introduced into the reservoir in order that the ice might be melted and the water heated to 70°C?

7.44. Consider a vessel containing 2.0 l of water and a quantity of ice at 0°C; 0.38 kg of water vapour at a temperature of 100°C is added with the result that all the ice melts and the water in the vessel heats to 70°C. What was the mass of the ice?

7.45. Into a 6.0 kg copper container with 20.5 l of water at a temperature of 19°C some melted tin is added at 232°C. As a result, 0.10 kg of water evaporates, and the remainder reaches a temperature of 32°C. What is the mass of the tin?

7.46. To 0.80 l of water at 15°C contained in a vessel, 0.20 kg of melted lead is added at a temperature of 327°C. In the process, 1.0 g of water changes into vapour. What is the new temperature of the water in the vessel?

7.47.* Consider a test tube containing 575 g of water at 0°C. From the tube air and water vapour are pumped out thus freezing a part of the water in the test tube. Determine the mass of the ice formed.

7.48.* A test tube contains 518 g of water at 0°C. By pumping air and water vapour from the tube, the water is frozen. What amount of water evaporates during the freezing?

7.49. A quantity of 0.60 l of water at 18°C is changed into ice by evaporating 240 g of ammonia. What is the efficiency of a refrigerating installation involved?

7.50. How much ice at 0°C can be produced by evaporating 100 g of Freon, if the refrigerator efficiency is 87 per cent, the initial temperature of water is 15°C, and one-fourth of the water changes into ice?

7.51. The chamber of a domestic refrigerator in 2 hours of operation cools 2.5 l of water from 20 to 0°C, with one-fifth of the water changing into ice. Calculate the efficiency of the refrigerator and the amount of heat lost to the air in the room, if the refrigerator is rated at 75 W.

7.52. From what height must a piece of tin fall to melt as it hits the ground? The initial temperature of tin is 0°C, and 50 per cent of gravitational work goes into the heating and melting.

7.53. A lead bullet travelling at 430 m/s hits a wall. The initial temperature of the bullet is 50°C, 50 per cent of its kinetic energy is used in heating the bullet. What fraction of the mass of the bullet melts if (a) it comes through a wall to slow down to 200 m/s, (b) it sticks in the wall?

7.54. An iron meteor at a temperature near absolute zero enters the Earth's atmosphere, heats, melts, and changes into vapour. What is the minimal velocity required?

7.55. In soldering with an electric soldering iron a 5.0 g piece of tin heats in 25 s from 20°C to the melting point and melts. If the body of the iron bears an inscription 220 V, 40 W, what is the efficiency of the tool?

7.56. Consider two ice particles of equal masses at initial temperature —100°C travelling in opposite directions to melt each other. Ignoring heat losses to the environment,

---

* The heat of vaporization at 0°C (r) is taken to be $2.54 \cdot 10^6$ J/kg.

what is the velocity needed for these particles to change into vapour in collision?

7.57. How much fuel (coke) with an initial temperature of 0°C will be consumed in the foundry to obtain one ton of aluminium casting? Cast shapes of grey cast iron*? The efficiency of the casting machine (heat utilization efficiency) is taken to be 35 per cent in the first case, and 45 per cent in the second.

7.58. What amount of white cast iron with an initial temperature of 293 K can be melted in a furnace with 18 per cent efficiency, if 3.0 t of A-I coal is available?

7.59. How much petroleum is to be burned in a melting furnace with 30 per cent efficiency, in order to bring to the melting point and melt 10 t of copper? The initial temperature of the copper is 25°C.

7.60. Compute the efficiency of a melting furnace which uses 32 kg of A-I coal to melt 300 kg of copper. The initial temperature of the copper is 13°C.

7.61. The initial temperature of cast iron being 10°C, how much coal of brand A-II is needed to melt 1.2 t of gray iron in a cupola with an efficiency of 55 per cent?

7.62. Consider two Dewar flasks: one containing boiling liquid nitrogen, the other river ice melting at 0°C. In the first flask 99.65 g of nitrogen evaporated in 6 hours, in the second one 6.94 g of ice melted in 8 hours. In both cases the temperature and pressure of ambient air are 20°C and 101.3 kPa. Given that the heat of vaporization of nitrogen is $1.88 \cdot 10^5$ J/kg, determine its boiling temperature.

Stress. Deformation and Strain

7.63. What kinds of deformation are characteristic of walls of a building? Ropes of a crane? Railway rails? Shafts of machinery? Paper in cutting?

7.64. How does the cross-section of a piston change under longitudinal compression? Under longitudinal tension?

---

* White, or conversion, cast iron is used for making steel, gray foundry iron is used in casting.

7.65. A body is subjected to opposite couples of forces in two parallel planes. What is the kind of deformation involved?

7.66. What kind of strain does stone resist well: compression, bending, or torsion? What kind of strain is it subjected to in walls of buildings, columns, and arches?

7.67. Concrete is resistant to compression, but not to tension. Steel shows good tensile strength. What properties does ferrous concrete exhibit?

7.68. What longitudinal force in a rod of 0.40 cm diameter gives rise to a stress of $1.5 \cdot 10^8$ Pa?

7.69. Consider a lifting crane. What must the diameter of the hook shank be in order that in steady lifting of a 25 kN weight the stress might not be higher than $6.0 \cdot 10^7$ Pa?

7.70. An elastic rod of a mass m and cross-sectional area S travels longitudinally with acceleration a equal for all the points. Determine the stress at section through the middle of the rod due to its accelerated motion.

7.71. From a steel rod of 3.0 cm^2 cross-section a weight of mass 7.5 t is suspended, the breaking stress for this brand of steel in tension is $6.0 \cdot 10^8$ Pa. Ignoring the mass of the rod, what is the margin of safety of the rod?

7.72. A rope consists of 200 steel wires of 1.0 mm diameter and margin of safety 5.0; the ultimate strength of the steel is $5.0 \cdot 10^8$ Pa. What maximum weight can the rope support (a) in uniform lifting, and (b) in lifting under acceleration 0.50 m/s^2?

7.73. A lift having a mass of 500 kg goes up with acceleration 0.50 m/s^2 on a rope with tensile strength $5.0 \cdot 10^8$ Pa. What must the cross-sectional area be with a safety margin of 10?

7.74. A mass of 2.0 t is lifted uniformly by a steel-wire rope consisting of 1.0 mm dia wires. If the safety margin is 10 and the ultimate strength of the steel is $5.8 \cdot 10^8$ Pa, how many wires must the rope include? What would the margin of safety of the rope be in lifting the weight with an acceleration of 1.0 m/s^2?

7.75. A steel wire of a length l is suspended vertically. What is the stress developed in the wire under gravitational force? How is it related to the cross-sectional area of the wire?

7.76. A steel wire of ultimate strength $6.0 \cdot 10^8$ Pa is

suspended vertically. At what maximum length does the wire break under gravitational force (a) in air, and (b) in sea water of density $1.03 \cdot 10^3$ kg/m^3?

7.77. What is the stress at the foundation of a brick wall 20 m high? Must the strength of brickwork be similar at the foundation and at the top of the wall?

7.78. The ultimate compression strength of a brick is $6.00 \cdot 10^7$ Pa. Given that the margin of safety is 8.00, what is the maximum permissible height of a brick house?

7.79. Two rods of like material and cross-section have different length ($l_1 > l_2$). Find if their tensile strains are equal under equal forces; what rod requires a greater force to show equal tensile strain. Ignore the masses of rods.

7.80. Consider two wires. The diameter and length of a first are twice those of the second. Neglecting the masses of the wires, what is the difference in tensile strains of the wires? What is the difference in their tensile deformations?

7.81. When stretching a 4.0 mm^2 aluminium wire the residual deformation is found to begin under 120 N. What is the limit elasticity of aluminium?

7.82. Consider a brass wire of length 4.0 m and cross-sectional area 2.0 mm^2, and having a limit of elasticity $\sigma_{el} = 1.1 \cdot 10^8$ Pa. What is the minimal load required for residual deformation to set in? What is the absolute elongation under these conditions? Ignore the mass of the wire.

7.83. A copper wire having a cross-sectional area of 2.0 mm^2 breaks under the action of a 440 N weight. What is the ultimate strength of copper?

7.84. The limit of elasticity of aluminium is $\sigma_{el} = 3.10 \cdot 10^7$ Pa, and its ultimate strength is $\sigma_l = 1.3 \cdot 10^8$ Pa. Is aluminium plastic or elastic? Does it lend itself for cold stamping?

7.85. The ultimate compression strength of cast iron is close to the limit of elasticity. Is cast iron suitable for stamping? Or rolling?

7.86. Under a force of 100 N a wire of length 5.0 m and cross-section 2.5 mm^2 elongates by 1.0 mm. Determine the stress in the wire, Young's modulus and the energy of elastic deformation.

7.87. A copper rod of length 5.0 m elongates by no more than 1.0 mm under a load of 480 N. The ultimate tensile

strength of copper is $2.2 \cdot 10^8$ Pa. Ignoring the mass of the rod, would the rod bear such a stress?

7.88. Consider a wire of length 1.8 m, diameter 0.50 mm and breaking strength $1.2 \cdot 10^9$ Pa. What is the tensile deformation of the wire under a load of 15 N? Under a load of 100 N?

7.89. A steel rod has a length of 2.0 m and cross-sectional area 10.0 mm^2. What is the absolute elongation required for the potential energy to be $4.4 \cdot 10^{-2}$ J?

7.90. A copper rod is 2.0 m long, its cross-sectional area is 2.0 mm^2. The work of elastic force done in stretching is 0.24 J. Determine the tensile strain of the rod.

7.91. A 100 kg weight is suspended from a copper rod of length 1.0 m and cross-sectional area 10 mm^2. What is the potential energy of the elastic deformation of the rod?

7.92. A spring needs 100 N to stretch by 10 cm. What is its potential energy?

7.93. A 0.10 kg weight is suspended on an elastically strained spring of stiffness 1.0 N/cm. Regarding the spring to be massless, determine its potential energy.

7.94. A load is suspended from a spring of stiffness 1000.0 N/m to extend it by 4.0 cm. What is the weight of the load? What is its potential energy under the load?

7.95. To an unstrained spring of stiffness k a weight of mass m is suspended, thereafter the spring is let alone. If the spring is taken to be massless, what is the potential energy of the elastically deformed spring at the instant of its maximum extension?

7.96. Consider a massless spring of stiffness $k = 1.00 \times \times 10^3$ N/m to which a 1.00 kg weight is suspended. What is its potential energy? What is the potential energy of the system of two such springs connected in series? In parallel? The mass of the weight is the same.

7.97. Two springs—steel and copper—are subjected to an elastic strain under the action of the same force, other things being equal. Considering the springs massless, which of them has higher potential energy?

SEC. 8. THERMAL EXPANSION OF BODIES

Example 22. A copper wire with a cross-sectional area of 10 mm^2 is under strain. What is the force needed to stretch it just as it expands in heating by 20 K?

Given: $S = 10 \cdot 10^{-6}$ m^2 is the cross-sectional area of the wire, $\Delta T = 20$ K is the change in the wire temperature, $E = 1.2 \cdot 10^{11}$ Pa is Young's modulus for copper, $\alpha = 1.7 \cdot 10^{-5}$ K^{-1} is the linear thermal expansion coefficient of copper.

Determine: F—the stretching force (load).

Solution. The tensile strain $\Delta l/l_0$ of the wire or rod under the stretching force will be the larger, the larger the stress $\sigma = F/S$ under the load per unit area of cross-section, and the lower the modulus of elasticity E (Young's modulus) of the material:

$$\frac{\Delta l}{l_0} = \frac{F}{S}\frac{1}{E}$$

The absolute extension of the wire is thus

$$\Delta l = \frac{Fl_0}{ES}$$

As stated, the wire must have the same elongation if heating by ΔT:

$$\Delta l = l_0 \alpha \Delta T$$

Equating the right sides of the equations and solving for F gives

$$F = ES\alpha\, \Delta T$$

Making substitutions into the last relationship we obtain

$$F = 1.2 \cdot 10^{11}\ \text{Pa} \cdot 10 \cdot 10^{-6}\ \text{m}^2 \cdot 17 \cdot 10^{-6}\ \text{K}^{-1} \cdot 20\ \text{K} \approx 410\ \text{N}$$

Answer. The stretching force is 410 N.

Example 23. A solid iron cube absorbs 296.4 kJ of heat. What is the change in its volume?

Given: $\Delta Q = 296{,}400$ J is the amount of heat absorbed by the body, $c = 460$ J/(kg·K) is the specific heat of iron, $\rho = 7\,800$ kg/m^3 is the density of iron, $\alpha = 12 \cdot 10^{-6}$ K^{-1} is the linear thermal expansion coefficient for iron.

Determine: ΔV—the change in volume of the iron cube.

Solution. The required change in the volume of the iron cube is

$$\Delta V = V_0 \beta \Delta T$$

The change in temperature ΔT is obtainable from the formula for the amount of heat absorbed by the body

$$\Delta Q = cm\, \Delta T = c\rho V_0\, \Delta T$$

Hence

$$\Delta T = \frac{\Delta Q}{c\rho V_0}$$

Substituting for ΔT into the relation for ΔV and considering that $\beta \approx 3\alpha$, we get

$$\Delta V = \frac{\beta}{c\rho} \Delta Q = \frac{3\alpha}{c\rho} \Delta Q$$

Substituting the numerical values we arrive at

$$\Delta V = \frac{36 \cdot 10^{-6}\ \mathrm{K^{-1}} \cdot 2.96 \cdot 10^{5}\ \mathrm{J}}{460\ \mathrm{J/(kg \cdot K)} \cdot 7\,800\ \mathrm{kg/m^3}} = 3.0 \cdot 10^{-6}\ \mathrm{m^3}$$

Answer. The cube volume will increase by 3.0 $\mathrm{cm^3}$.

Note. It is seen from the equation for ΔV that the increase in the volume is dependent neither on initial volume, nor on temperature. If the initial volume V_0 were given, the change in the body temperature could be determined, e.g., at $V_0 = 1\ \mathrm{dm^3}$ $\Delta T = 100$ K.

Example 24. A glass flask contains 680 g of mercury at 0°C and 670 g of mercury at 100°C. Determine the linear thermal expansion coefficient of glass.

Given: $m_0 = 680\ \mathrm{g} = 0.68$ kg is the mass of mercury filling the flask at 273 K, $T = 373$ K is the final temperature of the flask and mercury that remained in it, $m = 670\ \mathrm{g} = 0.67$ kg is the mass of mercury that remained in the flask at 373 K, $\beta_{mer} = 1.8 \cdot 10^{-4}\ \mathrm{K^{-1}}$ is the volume thermal expansion coefficient of mercury.

Determine: α_{gl}—the linear thermal expansion coefficient of glass.

Solution. The linear thermal expansion coefficient of glass is $\alpha_{gl} = {}^1/_3 \beta_{gl}$, where β_{gl} can be obtained from the relationship

$$V = V_0 (1 + \beta_{gl}\, \Delta T)$$

where V and V_0 are the volumes of the flask and the mercury contained in it at appropriate temperatures.

The mass of mercury occupying the volume V is $m = \rho V$, where

$$\rho = \frac{\rho_0}{1+\beta_{mer}\Delta T}$$

The mass of mercury occupying the volume V_0 is $m_0 = \rho_0 V_0$. We take the ratio of masses m and m_0, substitute in them the relationships obtained for ρ and V, and find the magnitude of α_{gl}.

Dividing m by m_0 gives

$$\frac{m}{m_0} = \frac{\rho V}{\rho_0 V_0} = \frac{\rho_0 V_0 (1+\beta_{gl}\Delta T)}{\rho_0 V_0 (1+\beta_{mer}\Delta T)} = \frac{1+\beta_{gl}\Delta T}{1+\beta_{mer}\Delta T}$$

Hence we find α_{gl}:

$$\alpha_{gl} = \frac{1}{3}\beta_{gl} = \frac{m(1+\beta_{mer}\Delta T) - m_0}{3m_0\Delta T}$$

Substitution gives

$$\alpha_{gl} = \frac{1}{3} \cdot \frac{0.67\ \text{kg}\,(1+0.00018\ \text{K}^{-1}\cdot 100\ \text{K}) - 0.68\ \text{kg}}{0.68\ \text{kg}\cdot 100\ \text{K}} \approx 1\cdot 10^{-5}\ \text{K}^{-1}$$

Answer. The linear thermal expansion coefficient of glass is about $1\cdot 10^{-5}\ \text{K}^{-1}$.

Linear and Superficial Thermal Expansion

8.1. In connecting railway rails, gaps are often let at joints, whereas tramway rails are often welded without gaps? Why?

8.2. Will the equilibrium of a precision balance be disturbed if one of the arms of the balance beam is heated?

8.3. Describe the construction and action of the compensated pendulum (Fig. 8.3), in which the distance between the centre of gravity and the suspension point is invariable at any temperature.

8.4. A glass tube is 2000.0 mm long at 0°C. Find its length at 100°C.

8.5. Consider a 500.0 m length of aluminium wire and the same length of steel wire at 0°C. What is the difference in lengths of the wires at 100°C?

8.6. A steam piping has a length of 10.00 m at 0°C. Find the length at 110°C.

8.7. Fig. 8.7 shows Lermantov's laboratory device designed to measure the linear thermal expansion coefficient of solid bodies. At room temperature 293 K the length of the brass rod is 405 mm, and the distance between the lugs *a* and *b* is 18.73 mm. In passing steam through the bushing the distance increases to 19.35 mm. What value will be obtained for the linear expansion of brass?

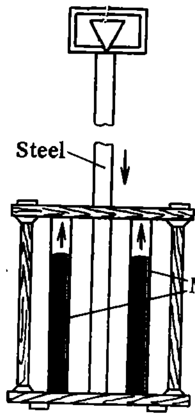


Fig. 8.3

8.8. The diameter of a glass plug that stuck in the bottleneck is 60.0 mm. To remove the plug the bottleneck is heated by 120°C, the very plug being heated by 20°C only. Determine the gap between the plug and the bottleneck.

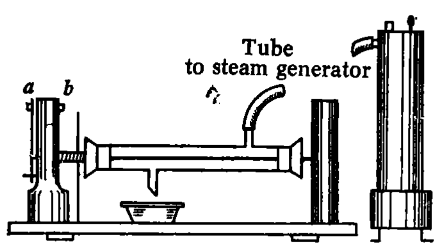


Fig. 8.7

8.9. What is the change in length of a 60 m piece of copper telegraph wire on raising the temperature from 10 to 40°C? What is the change in length of the wire on lowering the temperature from 10 to —30°C?

8.10. A steel truss of a railway bridge has a length of 75 m at 10°C. Determine the displacement of a trolley, on which the free end of the truss rests, in changing the temperature from —35 to 40°C.

8.11. The length of a railway rail at 30°C is 12.015 m. Determine the length of the rail at 0°C? At —35°C?

8.12. Prior to be slided on the wheel of a carriage an iron ring is heated by 700 K. The diameter of the wheel is 1310 mm, the initial diameter of the ring is 1300 mm. Will the ring fit the wheel?

8.13. A cast iron wheel of a tram car has a diameter of 1150 mm at 20°C, and a steel band designed for the wheel has a diameter of 1145 mm. At what temperature will the gap between the band and the wheel be 1.00 mm?

8.14. In machining a cast iron pulley on a lathe the temperature of the pulley rises up to 200°C. What diameter of the pulley at this temperature is needed to become 400.0 mm in cooling to 0°C?

8.15. Through a 6.00 m iron wire an electric current is flowing. The wire heats to red heat and stretches by 37.0 mm. What is the change in temperature of the wire?

8.16. Rolled stock is cut into strips immediately at the outlet of the mill at a temperature of 900°C. Determine the length of strips when hot, if their length at 20°C is 15.0 m.

8.17. Consider an aluminium wire of cross-section 6.0 mm^2. By how many degrees must the wire be heated to attain a length obtainable under a stretching force of 508 N?

8.18. The ends of a steel beam are rigidly fixed in the opposite walls of a room. What pressure will the beam exert on the walls with the temperature raised by 30 K?

8.19. The ends of a 150 cm^2 steel beam are rigidly fastened to two supports hindering the elongation of the beam. By how many degrees must the beam temperature increase for the pressure force acting on the support to be 1400 kN?

8.20. Why does the heating and cooling of ferrous concrete structures not result in separation of reinforcement from concrete?

8.21. What requirements are placed on material of electrodes soldered into a glass bulb of an incandescent lamp?

8.22. A zinc sheet measures 120×70 cm^2 at 0°C. What is the increase in its area on heating up to 100°C?

8.23. An aluminium plate measures 150×80.0 mm^2 at 0°C. Compute the area of the plate at a temperature of 600°C.

8.24. An iron sheet measuring 2×1 m^2 at 0°C is subjected to heating with the result that its area increases by 6000 mm^2. What is the rise in its temperature?

Volume Thermal Expansion

8.25. A steel cube at 0°C has a volume of 800.0 cm^3. Calculate its volume at a temperature of 200°C.

8.26. A copper sphere at 0°C has a diameter of 200 mm. What is the change in its volume in heating to 100°C?

8.27. A glass jar at 50°C has a volume of 3500 cm^3. What is the change in its volume in cooling down to 10°C?

8.28. A brass reservoir at 0°C has a volume of 12.00 l. What will the volume of the reservoir be at temperatures 30 and −25°C?

8.29. Find the density of iron at temperatures 200 and −70°C.

8.30. A heat of 1.62 MJ is used to heat an iron part. What is the rise in the volume of the part? By how many degrees will it heat, if its initial volume is 3000 cm^3?

8.31. A heat of $1.63 \cdot 10^8$ J is used to heat a cast iron block of 80 cm^2 cross-section. What is its thermal expansion?

8.32. A heat of 13.53 kJ is used to heat a piece of copper wire with the result that the wire elongates by 34 mm. Determine the cross-sectional area of the wire. Find the initial length of the wire, when heated by 250 K.

8.33. A 5.0 mm^2 aluminium rod is heated by 150 K. The heating requires 11.178 kJ of heat. What is the elongation of the rod? What was its initial length?

8.34. A vessel of capacity 250 cm^3 at 0°C is filled with water at the same temperature and then heated to 100°C. As a result, 3.5 cm^3 of water left the vessel. Ignoring the expansion of the vessel, determine the average volume thermal expansion coefficient of water.

8.35. A bottle having at 0°C a capacity of 20 l is filled to the brim with kerosene at the same temperature. What

is the rise in temperature required for 0.50 l of kerosene to pour out of the bottle?

8.36. A cylindrical, vertical tank at —10°C is filled with petroleum up to a level of 6.0 m. What will the level of petroleum in the tank be, if the initial temperature is raised to 20°C? At what temperature will the petroleum overflow, if at —10°C the petroleum level is by 24 cm below the brim?

8.37. The level of electrolyte in a cell jar at 5°C is 4.0 mm below an opening in the lid. At what temperature will

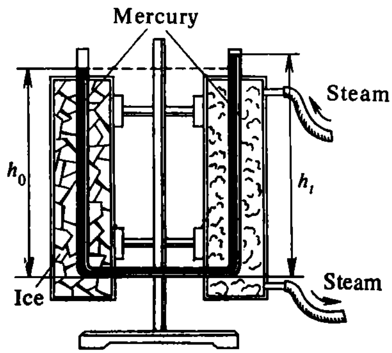


Fig. 8.44

the electrolyte spill? The jar is 300 mm high, the volume thermal expansion coefficient of the electrolyte is 0.00043 K^{-1}.

8.38. A petroleum tank ship is filled at a temperature of 30°C. Into one of its compartments 1600 m^3 of petroleum is loaded. What is the change in the volume of the petroleum at —5°C?

8.39. A reservoir of capacity 20 l at 0°C is filled to the brim with transformer oil at the same temperature. When heated to 80°C the reservoir lost 0.85 l of oil through overflowing. What is the apparent volume expansion coefficient of the oil? What is the material of the reservoir?

8.40. A glass flask with mercury at 0°C has a capacity of 250 cm^3. If the flask is heated to 100°C, how much mercury will overflow?

8.41. A glass vessel with mercury at 0°C has a capacity of 600 cm^3. Up to what temperature is the vessel with mercury heated, if it lost 13.8 cm^3 of mercury?

8.42. A tank confines at 0°C 12.3 kg of glycerin, and at 20°C 12.2 kg. Compute the coefficient of linear thermal expansion for the material of which the tank is made.

8.43. A glass flask contains at 0°C 1000 g of a liquid, and at 150°C, 978 g. Determine the coefficient of volume expansion for the liquid.

8.44. Fig. 8.44 shows a device designed to measure the volume expansion coefficient of liquids. One end of a U-tube containing the liquid to be tested is cooled down to 0°C with ice, and the other is heated to 100°C by water steam. In one experiment the heights of liquid columns were 250 and 254.5 mm, respectively, and in an experiment with another liquid, 30 and 33 cm. What were the results of the experiments?

8.45. Compute the density of mercury at 150 and −30°C.

8.46. Compute the density of alcohol at 50 and −100°C.

8.47. What quantity of heat is to be used to heat mercury so that its initial volume increased by 45 cm^3? Compute the same quantity for transformer oil.

8.48. A 850 g brass cube is descended on a string in kerosene first at 40°C, and then at 80°C. In both cases calculate the weight of the liquid displaced by the cube.

8.49. A glass cube of mass 205 g is submerged in a liquid at 20°C and is acted upon by an upward force of 1.00 N. When repeated with the same substances at a liquid temperature of 70°C, the measurement yielded 0.977 N. Calculate the volume expansion coefficient and density of the liquid.

CHAPTER II

ELECTRICITY

SEC. 9. INTERACTION BETWEEN CHARGES. CHARGE CONSERVATION LAW. COULOMB'S LAW

Example 25. Two identical conducting balls of mass 1.5 g each are suspended at one point from silk strings. After one of the balls has been charged negatively and brought to contact with the other, the balls separated by 10.0 cm with the strings making an angle of 36° (Fig. 25). Determine the charge of the ball before its coming into contact with the other, and the number of excess electrons on each ball after their contact.

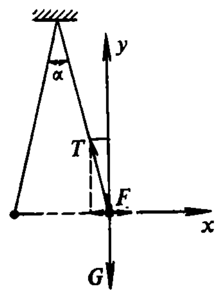


Fig. 25

Given: $m_1 = m_2 = m = 1.5 \cdot 10^{-3}$ kg is the mass of the balls, $r = 0.10$ m is the separation between the balls, $\alpha = 36°$ is the angle between the strings, $\varepsilon_0 = 1/36\pi \cdot 10^9\ C^2/N \cdot m^2$ is the electric constant, $e = 1.6 \cdot 10^{-19}$ C is the electron charge.

Determine: q—the initial charge of the ball, n—the number of excess electrons on each ball.

Solution. As the problem statement carries no indication as to the medium, we will consider that the charges are interacting in a vacuum. According to the charge conservation law, the contact of the balls results in each of them having the charge $q/2$. The balls are acted upon by three forces: (1) gravitational force $G = mg$, (2) reaction force of tension in the string, T, (3) electric interaction force $F = (q/2)^2/4\pi\varepsilon_0 r^2$. In equilibrium, vertical and horizontal components of these forces add, respectively, to zero (see Fig. 25):

$$T \cos(\alpha/2) - mg = 0$$

$$F - T \sin(\alpha/2) = 0$$

Solving the set of equations for F gives

$$F=\frac{mg\sin(\alpha/2)}{\cos(\alpha/2)}=mg\tan\frac{\alpha}{2}$$

Substituting for F from the Coulomb formula we arrive at an expression for the charge q

$$q=4r\sqrt{\pi\varepsilon_0 mg\tan(\alpha/2)}$$

Considering that on each ball there is the charge $q/2$, we obtain

$$n=\frac{q/2}{e}$$

Substitution gives

$$q=4\cdot 0.1\ \mathrm{m}\sqrt{\frac{\pi\cdot\mathrm{C}^2\cdot 1.5\cdot 10^{-3}\ \mathrm{kg}\cdot 9.8\ \mathrm{m/s^2}\cdot 0.32}{36\pi\cdot 10^9\ \mathrm{N\cdot m^2}}}\approx 14.6\cdot 10^{-8}\ \mathrm{C}$$

Determine the number of excess electrons on each ball

$$n=\frac{7.3\cdot 10^{-8}\ \mathrm{C}}{1.6\cdot 10^{-19}\ \mathrm{C}}\approx 4.6\cdot 10^{11}$$

Answer. The initial charge of the balls was about $7.3\cdot 10^{-8}$ C, The number of excess electrons on each ball is around $2.3\cdot 10^{11}$.

9.1. Is it possible to charge a brass stick by rubbing?

9.2. How will you proceed to identify the charge sign of an electroscope with an ebonite rod and woollen cloth available?

9.3. Is it possible to transfer all the charge from a conductor to another isolated conductor?

9.4. In which case will the force of electrostatic interaction of two metal balls be the larger: with like, or unlike charges? The diameters of the balls are compatible with their separation, their charges are q, the distance between them being constant in both cases.

9.5. Consider two isolated metal balls of equal diameters. Can you think of a way of obtaining on them like charges equal in magnitude and sign? Equal in magnitude, but of opposite signs?

9.6. Two small balls with like charges are suspended from isolating strings of a similar length l at one point. What will happen to the balls under weightlessness?

9.7. To improve the contact of belts and pulleys in factories use is made of colophony, but in places with explosion hazard it is strictly forbidden. Why? Driving belts are smeared with a conducting paste, and pulleys are earthed. Why?

9.8. Explain why in filling a tank truck with petrol the truck and source reservoir are connected with a wire and earthed?

9.9. In fabrication of gun powder the latter is covered with graphite dust. Explain.

9.10. A metal ball has $5.0 \cdot 10^5$ excess electrons. What is its charge in coulombs? How many excess electrons will remain on the ball after a contact with another identical ball with a charge of $+3.2 \cdot 10^{-14}$ C?

9.11. A small conducting ball having a charge of $-4.8 \times \times 10^{-11}$ C is brought into contact with an identical uncharged ball. How many excess electrons will remain on the ball? What charge will the other ball get? What will the force of electrostatic interaction be, if the balls are placed in a vacuum 2.4 cm apart?

9.12. Determine the charge that a 1.0 cm radius aluminium ball would acquire should it lose all the conduction electrons, if there is one conduction electron per aluminium atom.

9.13. What is the change in the interacton force between two point charges, if each charge is increased four-fold and the separation between the charges is decreased by half?

9.14. To get some idea of the magnitude of charge of one coulomb, determine the force of interaction of two 1 C point charges separated by 1 m in vacuum, and in water with the same separation.

9.15. A metal ball of 20.0 cm diameter has a charge of $3.14 \cdot 10^{-7}$ C. What is the surface-charge density on the ball?

9.16. A conducting sphere of 4.0 cm radius receives a charge, such that the surface-charge density is $0.50 \cdot 10^{-4}$ C/m^2. What is the charge?

9.17. In your opinion, how will the surface-charge density of a sheet conductor change if it is rolled into a cylinder surface?

9.18. Two point charges equal in magnitude and of like

signs are placed 3.0 m apart in a vacuum. The repulsive force is 0.40 N. Determine each charge.

9.19. Two charges of $0.66 \cdot 10^{-7}$ C and $1.1 \cdot 10^{-5}$ C are separated by 3.3 cm in water. What is the force of electrostatic interaction between the charges? What separation is needed in vacuum for the force to remain the same?

9.20. Two charges, one of them three times as large as the other, are placed in a vacuum 0.30 m apart; the interaction force is 30 N. Determine the charges. In water, at what distance will the charges interact with the force three times as large?

9.21. Two balls with equal charges are in a vessel with ice at —18°C at a distance of 20.0 cm from each other. What is the dielectric constant of ice, if on forming in the vessel of some water at 0°C the balls are brought nearer to 3.8 cm for the interaction to remain the same? The dielectric constant of water is taken to be 88 at 0°C.

9.22. Two identical charges in a vacuum at 20.0 cm interact with the same force as in transformer oil when 0.140 m apart. If the force of electrostatic interaction in vacuum is 90.0 N, what is the permittivity of the oil, and what are the charges?

9.23. Two small conducting balls of equal radius carrying charges of opposite signs are attracted to each other with a force of $4.00 \cdot 10^{-3}$ N when placed 30.0 cm apart. After the balls have been brought into contact for a short while and restored to their initial place, the electrostatic force becomes $2.25 \cdot 10^{-3}$ N. Determine the charges of the balls before the contact.

9.24. Two small identical conducting balls are positioned in air so that the centre distance is 60.0 cm and the charges are $4.0 \cdot 10^{-7}$ and $0.80 \cdot 10^{-7}$ C. The balls are brought into contact and then restored to their initial position. Determine the force of their interaction before and after the contact.

•9.25. Prove that at any magnitudes of like charges q_1 and q_2 and arbitrary r (see problem 9.24) the inequality holds $F_2/F_1 > 1$ (where F_1 and F_2 are the forces before and after the contact, respectively).

9.26. A ball of mass 150 mg suspended from an unconducting string has a charge of $—10.0 \cdot 10^{-9}$ C. At a distance of 32 cm from it there is a second small ball. What must the

charge be in magnitude and sign for the tension in the string to increase twofold?

9.27. A thin silk string withstands a load of up to $9.8 \times \times 10^{-3}$ N. A 0.67 g ball suspended from the string carries a charge $q_1 = 1.1 \cdot 10^{-9}$ C. A ball with a charge q_2 of unlike sign is brought from below at a distance of 1.8 cm on the line of the string. What is the magnitude of charge q_2 needed for the string to break?

9.28. Two point electric charges $60.0 \cdot 10^{-9}$ and $2.4 \cdot 10^{-7}$ C are in transformer oil 16 cm apart. A third charge $30.0 \times \times 10^{-5}$ C is positioned in-between so that under the action of the electrostatic forces it stays in equilibrium. Where is it to be placed? Will the equilibrium be stable? Will the equilibrium be violated if the magnitude of the third charge is changed?

9.29. An electric field is formed by two charges $5 \cdot 10^{-4}$ C and $-5 \cdot 10^{-4}$ C spaced 10 cm apart at points A and B. There is a droplet lying on the axis of symmetry at a distance of 5 cm from the centre of the segment AB. The droplet carries a charge equal to that of 10 electrons, its mass is $0.4 \cdot 10^{-7}$ kg. What is the force acting on the droplet? What is the initial acceleration of the droplet?

9.30. Two small balls of identical mass and radius are suspended in the air from strings of equal length at one point. After the balls have received charges $40.0 \cdot 10^{-8}$ C each, the strings deflect to form an angle of 60°. Find the mass of each ball, if the distance from the point of suspension to the centre of the ball is 20.0 cm.

9.31. The gravitational force between two electrified balls of mass 1 g each is balanced out by the electrostatic repulsive force. Regarding the charges of the balls to be equal, determine their magnitude. Why is it possible to ignore the gravitational force when dealing with small electrified bodies interacting? Illustrate by an example of two interacting electrons.

9.32. Two water droplets of radius 0.30 mm carry charges equal in magnitude and of like signs. Determine these charges if the electrostatic repulsive force is balanced out by the mutual gravitational force of the droplets.

9.33. A small negatively charged ball moves uniformly in a circle round a point charge of $q_1 = 3.0 \cdot 10^{-9}$ C under a grav-

itational force. What is the ratio of the charge of the ball to its mass, if the radius of the circle is 2.0 cm, and the angular velocity of rotation is 3.0 rad/s?

9.34. A charge Q is uniformly distributed over a thin ring of radius r. Determine the force acting on a point charge q at a distance h from the ring centre (Fig. 9.34).

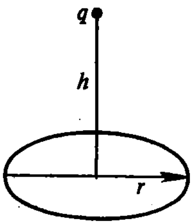


Fig. 9.34

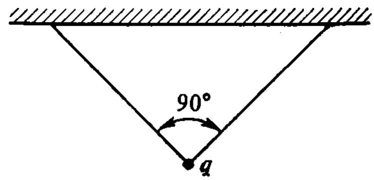


Fig. 9.35

9.35. A charged ball of mass $5.88 \cdot 10^{-4}$ kg is suspended from silk strings at an angle of 90° (Fig. 9.35). Another ball with a charge equal in magnitude but opposite in sign is placed below the first ball at a distance of $4.2 \cdot 10^{-2}$ m on the vertical with the result that the tension in strings doubles. Determine the charge of the ball and the tension in the string with electrostatic interaction.

SEC. 10. ELECTRIC FIELD

Example 26. Two point charges equal in magnitude $q_1 = q_2 = 2 \cdot 10^{-8}$ C are placed at vertices of a right-angled isosceles triangle (Fig. 26). The charges are separated by a distance of 0.6 m. Determine the electric field strength and the potential at the right angle vertex and at the crossing of the altitude with the base of the triangle. Take the cases of like and unlike charges.

Given: $q_1 = q_2 = 2 \cdot 10^{-8}$ C is the charge, $l = AB = 0.6$ m is the distance between the charges, $\varepsilon_0 = 1\ C^2/36\pi \cdot 10^9\ N \cdot m^2$ is the permittivity of free space.

Determine: E_C and E_D—the field strength at points C and D;
φ_C and φ_D—the potentials at the same points.

Solution. The electric field strength at specified points is to be obtained from the relation $E = q/4\pi\varepsilon_0\varepsilon r^2$. If the field is formed by several charges, then the resultant strength is the vectorial sum of strengths due to individual charges. The potential of the field caused by each charge is determined

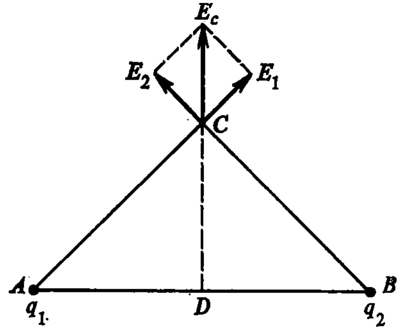


Fig. 26

from the formula $\varphi = q/4\pi\varepsilon_0\varepsilon r$, and the resultant potential is the algebraic sum of potentials φ_1 and φ_2 created at a given point by charges q_1 and q_2.

The distances from the charges to point C are equal to $r = l/\sqrt{2}$, therefore

$$E_1 = E_2 = \frac{2q_1}{4\pi\varepsilon_0\varepsilon l^2} \qquad E_C = \sqrt{2}\,E_1$$

Substituting for the quantities, we have

$$E_C = \sqrt{2}\,\frac{2\cdot 2\cdot 10^{-8}\ \mathrm{C}\cdot 36\pi\cdot 10^9\ \mathrm{N\cdot m^2}}{4\pi\ \mathrm{C^2}\cdot 0.36\ \mathrm{m^2}} \approx 1.4\cdot 10^3\ \mathrm{N/C}$$

The resultant vector of the electric intensity at C is directed upward for like charges and horizontally for unlike charges, the magnitude of the field strength in both cases being equal.

The potential at point C caused by each charge is

$$\varphi_1 = \varphi_2 = \frac{2\cdot 10^{-8}\ \mathrm{C}\cdot 36\pi\cdot 10^9\ \mathrm{N\cdot m^2}\cdot\sqrt{2}}{4\pi\ \mathrm{C^2}\cdot 0.6\ \mathrm{m}} \approx 420\ \mathrm{V}$$

The resultant potential of the field formed by the two charges at point C is to be found as an algebraic sum of potentials φ_1 and φ_2, i.e.

$$\varphi_C = \varphi_1 + \varphi_2 \approx 840 \text{ V}$$

With unlike charges $\varphi_C = 0$.

The point D being at the middle of the segment connecting the charges, the vectors of strength $\mathbf{E}_1$ and $\mathbf{E}_2$ are equal in magnitude

$$E_1 = E_2 = \frac{q_1}{4\pi\varepsilon_0\varepsilon (l/2)^2} = \frac{2\cdot 10^{-8}\ \text{C}\cdot 36\pi\cdot 10^9\ \text{N}\cdot\text{m}^2\cdot 4}{4\pi\ \text{C}^2\cdot 0.36\ \text{m}^2} = 2\cdot 10^3\ \text{N/C}$$

With like charges, the required strength is $E_D = 0$, as the vectors $\mathbf{E}_1$ and $\mathbf{E}_2$ are opposite in direction, and for like charges the field strength is $E_D = 4\cdot 10^3$ N/C and directed to the negative charge. The potential at point D is

$$\varphi_D = 2\varphi_1 = 2\frac{2\cdot 10^{-8}\ \text{C}\cdot 36\pi\cdot 10^9\ \text{N}\cdot\text{m}^2}{4\pi\ \text{C}^2\cdot 0.3\ \text{m}} = 1\,200\ \text{V}$$

For unlike charges $\varphi_D = 0$.

Answer. The field strength and potential at point C for the like charges are about 1400 N/C and 840 V; for the unlike charges the direction of the strength vector changes and the potential is zero. At point D the values of the strength are 0 and $4\cdot 10^3$ N/C, and of the potential 1200 V and 0 V.

Example 27. A single conducting sphere is charged so that in a vacuum at points at distances 5 and 10 cm from its surface the potentials are 300 and 210 V, respectively. What is the potential of the sphere?

Given: $r_1 = 5.00\cdot 10^{-2}$ m, $r_2 = 0.100$ m are the distances from the sphere's surface to the points with known potentials, $\varphi_1 = 300\ \text{V} = 3.00\cdot 10^2$ V, $\varphi_2 = 210\ \text{V} = 2.10\cdot 10^2$ V are the potentials at the two points in the electric field of the charged sphere.

Determine: φ_{sh} —the potential of the charged sphere.

Solution. The potential of the charged sphere in vacuum is obtained by the formula

$$\varphi_{\text{sh}} = q/4\pi\varepsilon_0 R$$

where R is the radius of the sphere.

In order to determine the charge we assume that it is concentrated at the centre of the sphere (the electric field of a charged sphere starts at its surface and coincides with the field of a point charge positioned at the centre of the sphere). Therefore

$$\varphi_1 = q/4\pi\varepsilon_0 (R + r_1) \qquad \varphi_2 = q/4\pi\varepsilon_0 (R + r_2)$$

Next we obtain the charge of the sphere

$$q = \varphi_1 \cdot 4\pi\varepsilon_0 (R + r_1)$$
$$q = \varphi_2 \cdot 4\pi\varepsilon_0 (R + r_2)$$

Solving the above set of equations for R we arrive at the relationship for the sphere radius

$$R = \frac{\varphi_2 r_2 - \varphi_1 r_1}{\varphi_1 - \varphi_2}$$

In the formula for the electrified sphere we substitute for q and R after transformations we obtain

$$\varphi_{sh} = \varphi_1 \varphi_2 \frac{r_2 - r_1}{\varphi_2 r_2 - \varphi_1 r_1}$$

And after the substitution of numerical values and some algebra we get

$$\varphi_{sh} = 3.00 \cdot 10^2 \text{ V} \cdot 2.10 \cdot 10^2 \text{ V} \times$$
$$\times \frac{(0.100 - 0.05) \text{ m}}{2.10 \cdot 10^2 \text{ V} \cdot 0.100 \text{ m} - 3.00 \cdot 10^2 \text{ V} \cdot 5.00 \cdot 10^{-2} \text{ m}} = 525 \text{ V}$$

Answer. The sphere is electrified to a potential of 525 V.

Example 28. An electric field is formed by a charge of $5.0 \cdot 10^{-7}$ C in a medium with dielectric constant 2.0 (Fig. 28). Determine the potential difference at points B and C spaced from the charge 5.0 cm and 0.20 m apart. What is the work done by the field in transporting a charge of $0.30 \cdot 10^{-7}$ C between points B and C?

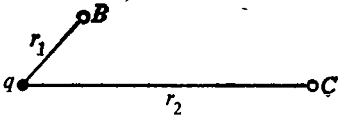


Fig. 28

Given: $q = 5.0 \cdot 10^{-7}$ C is the charge producing the field, $\varepsilon = 2.0$ is the dielectric constant of the medium, $r_1 = r_B = 5.0$ cm $= 5.0 \cdot 10^{-2}$ m is the separation between

the charge q and point B, $r_2 = r_C = 0.20$ m is the separation between the charge q and point C, $q_1 = 0.30 \cdot 10^{-7}$ C is the test charge, $\varepsilon_0 = 1$ C^2/$36\pi \cdot 10^9$ N$\cdot$m^2 is the electric constant.

Determine: U—the potential difference between points B and C;
A—the work performed by the field to shift the test charge.

Solution. Using the formula $\varphi = q/4\pi\varepsilon_0\varepsilon r$, we obtain the potential difference between the points B and C in the electric field

$$U = \varphi_B - \varphi_C; \quad U = \frac{q}{4\pi\varepsilon_0\varepsilon r_B} - \frac{q}{4\pi\varepsilon_0\varepsilon r_C} = \frac{q}{4\pi\varepsilon_0\varepsilon}\left(\frac{1}{r_B} - \frac{1}{r_C}\right)$$

The work performed in carrying the charge in the electric field is given by

$$A = q_1 U$$

Using the data of the problem we perform some manipulations

$$U = \frac{5.0 \cdot 10^{-7}\ \text{C} \cdot 36\pi \cdot 10^9\ \text{N} \cdot \text{m}^2}{4\pi \cdot 2\ \text{C}^2}\left(\frac{1}{5.0 \cdot 10^{-2}\ \text{m}} - \frac{1}{0.20\ \text{m}}\right) \approx 34{,}000\ \text{V}$$

$$A \approx 0.30 \cdot 10^{-7}\ \text{C} \cdot 34{,}000\ \text{V} \approx 0.001\ \text{J}$$

Answer. The potential difference between the two points is about 34,000 V; in carrying between these points of the charge q_1 the field performs a work of around 0.001 J.

Example 29. An alpha-particle produced in a radioactive decay travels at $1.6 \cdot 10^7$ m/s in the direction of a stationary uranium nucleus. How close to the uranium nucleus can it approach? The charges of the particles are to be regarded as point ones, and the masses of a proton and neutron are considered equal.

Given: $v = 1.6 \cdot 10^7$ m/s is the velocity of the alpha-particle, $q_\alpha = 2e$ is the charge of alpha-particle, $m_\alpha = 4m_p$ is the mass of alpha-particle, $q_U = 92e$ is the charge of uranium nucleus, $e = 1.6 \cdot 10^{-19}$ C is the electron charge, $m_p = 1.67 \cdot 10^{-27}$ kg is the mass of proton, $\varepsilon = 1$ is the dielectric constant of the medium, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ C^2/N$\cdot$m^2 is the electric constant.

Determine: r_0—the least distance of approach of the particles.

Solution. Assume that the potential energy of an alpha-particle infinitely distant from the uranium nucleus creating a field, is zero. As the alpha-particle approaches the uranium nucleus, its kinetic energy will decrease, and the potential energy of interaction of the particles will grow. To the least distance of approach of the particles r_0 there will correspond the transition of all the kinetic energy of the alpha-particle to potential energy

$$\frac{m_\alpha v^2}{2} = q_\alpha \varphi$$

where

$$\varphi = \frac{q_U}{4\pi\varepsilon_0 \varepsilon r_0}$$

Using the data of the problem we get

$$\frac{m_\alpha v^2}{2} = \frac{2e \cdot 92e}{4\pi\varepsilon_0 \varepsilon r_0}$$

Hence

$$r_0 = \frac{92e^2}{\pi\varepsilon_0 \varepsilon m_\alpha v^2}$$

Substituting and transforming, we arrive at

$$r_0 = \frac{92 \cdot 1.6^2 \cdot 10^{-38}\ \mathrm{C^2 \cdot N \cdot m^2}}{3.14 \cdot 8.85 \cdot 10^{-12}\ \mathrm{C^2} \cdot 4 \cdot 1.67 \cdot 10^{-27}\ \mathrm{kg} \cdot 1.6^2 \cdot 10^{14}\ \mathrm{m^2/s^2}} \approx 5 \cdot 10^{-14}\ \mathrm{m}$$

Answer. The least distance possible between the particles is about $5.0 \cdot 10^{-14}$ m.

Example 30. Two parallel thin identical rings of radius 5.0 cm have in a vacuum a common axis O_1O_2 (Fig. 30). Their centre distance is 12 cm. On the first ring a charge of $8.2 \cdot 10^{-7}$ C is uniformly distributed, and on the second $6.0 \cdot 10^{-7}$ C. What work is done in carrying a charge of 3.0×10^{-9} C from the centre of one ring to the centre of the other?

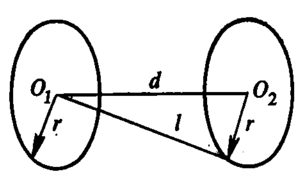


Fig. 30

Given: $r_1 = r_2 = r = 5.0 \cdot 10^{-2}$ m are the radii of the rings, $d = 1.2 \cdot 10^{-1}$ m is the distance between the ring centres, $q_1 = 8.2 \cdot 10^{-7}$ C is the charge on the first ring, $q_2 =$

$= 6.0 \cdot 10^{-7}$ C is the charge on the second ring, $q = 3.0 \times 10^{-9}$ C is the charge transported in the electric field, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ C^2/N$\cdot$m^2 is the electric constant.

Determine: A—the work performed in carrying the charge between the centres of the charged rings.

Solution. The charges on the rings cannot be regarded as point ones, therefore the formula $\varphi = q/4\pi\varepsilon_0 r$ cannot be utilized directly to compute the potential in vacuum, as the formula is only valid for point charges. As the work in carrying the charge depends on the difference of potentials of the beginning and end of the transportation (in this case, ring centres), the solution of the problem requires the calculation of potentials of these points φ_{O_1} and φ_{O_2}.

We break down each of the rings into n equal segments so that the charge of each segment might be regarded as a point one

$$q_1' = q_1/n \quad \text{and} \quad q_2' = q_2/n$$

The potential caused by the point charge q_1' at the centre of the first ring is

$$\varphi_1' = \frac{q_1'}{4\pi\varepsilon_0 r}$$

All the charge q_1 distributed over the first ring produces at its centre the potential φ_1 equal to the algebraic sum of potentials from n point charges, or

$$\varphi_1 = \varphi_1' n = \frac{q_1'}{4\pi\varepsilon_0 r} n = \frac{q_1}{4\pi\varepsilon_0 r}$$

Reasoning along these lines we find the potential at the point O_1 due to the charge q_2, recalling that $l_2 = \sqrt{d^2 + r^2}$.

$$\varphi_2 = \frac{q_2' n}{4\pi\varepsilon_0 \sqrt{d^2 + r^2}} = \frac{q_2}{4\pi\varepsilon_0 \sqrt{d^2 + r^2}}$$

The potential of electric field at the centre of the first ring φ_{O_1} produced by the charges q_1 and q_2 will be

$$\varphi_{O_1} = \varphi_1 + \varphi_2$$

or

$$\varphi_{O_1} = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 \sqrt{d^2 + r^2}}$$

Using the same argument we obtain the expression for the potential at the centre of the second ring φ_{0_2}

$$\varphi_{0_2} = \frac{q_2}{4\pi\varepsilon_0 r} + \frac{q_1}{4\pi\varepsilon_0 \sqrt{d^2+r^2}}$$

The work done in carrying the charge q from point O_1 to point O_2 is to be determined from the relationship

$$A = q\,(\varphi_{0_1} - \varphi_{0_2})$$

After substituting for φ_{0_1} and φ_{0_2} and rearranging we get

$$A = q\left(\frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0\sqrt{d^2+r^2}} - \frac{q_2}{4\pi\varepsilon_0 r} - \frac{q_1}{4\pi\varepsilon_0\sqrt{d^2+r^2}}\right)$$

$$= q\frac{q_1-q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{d^2+r^2}}\right)$$

$$A = 3\cdot 10^{-9}\ \text{C}\ \frac{2.2\cdot 10^{-7}\ \text{C}\cdot\text{N}\cdot\text{m}^2}{4\cdot 3.14\cdot 8.85\cdot 10^{-12}\ \text{C}^2}$$

$$\times\left(\frac{1}{5\cdot 10^{-2}\ \text{m}} - \frac{1}{\sqrt{1.44\cdot 10^{-2} + 0.25\cdot 10^{-2}}\ \text{m}}\right) \approx 7.3\cdot 10^{-5}$$

Answer. The work performed in carrying the charge q in the electric field of the charges q_1 and q_2 is around $7.3\cdot 10^{-5}$ J.

Example 31. Protons travel at $9.5\cdot 10^4$ m/s between and parallel to two horizontal plates so that the distance to the

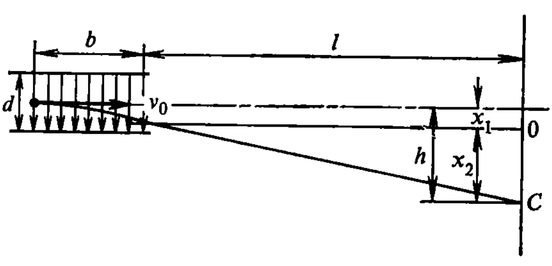


Fig. 31

plates is equal (Fig. 31). The voltage across the plates being 14 V, the protons are deflected and hit the point C on a screen. Determine the displacement OC of the protons, if the separation between the plates is 2.4 cm, the length of the plates

is 6.2 cm and the distance from the plates to the screen is 45 cm. The protons move in vacuum. Ignore the gravity force.

Given: $U = 14$ V is the voltage across the plates, $d = 2.4\cdot10^{-2}$ m is the distance between the plates, $b = 6.2\cdot10^{-2}$ m is the length of the plates, $l = 0.45$ m is the distance from the plates to the screen, $v_0 = 9.5\cdot10^4$ m/s is the initial velocity of protons, $m_p = 1.672\cdot10^{-27}$ kg is the mass of a proton, $q = 1.6\cdot10^{-19}$ C is the charge of proton.

Determine: h—the displacement of protons on the screen under the action of the electric field.

Solution. The displacement $h = OC$ is obtainable as the sum of displacements x_1 and x_2, where x_1 is the displacement within the confines of the plates under the action of the electric force $F = qE$, and x_2 is the displacement as a proton travels in free flight outside the field (we ignore the field irregularities at the edges of the plates). As the force F is directed perpendicular to the vector of velocity v_0, we assume that the horizontal component of the velocity is a constant equal to v_0. Hence the time required for the proton to pass inside the plates is given by

$$t_1 = b/v_0$$

The uniformly accelerated vertical movement of a proton in an electric field occurs during the time t_1, i.e.

$$x_1 = \frac{1}{2}at_1^2$$

Using Newton's second law we get

$$a = F/m_p = qE/m_p$$

To solve the problem we need the strength of the electric field which we find from the formula $E = U/d$.

The vertical component of the proton velocity v_V at the instant it leaves the confines of the plates will be $v_V = at_1$ and later is to remain constant. Therefore

$$x_2 = v_V/t_2$$

where t_2 is the time of travelling of the proton outside the plates that is defined by the relation $t_2 = l/v_0$.

Now we determine the displacement of proton within the plates

$$x_1 = \frac{at_1^2}{2} = \frac{Fb^2}{2m_p v_0^2} = \frac{Uqb^2}{2dm_p v_0^2}$$

$$x_1 = \frac{14\,V\cdot 1.6\cdot 10^{-19}\,\mathrm{C}\cdot(6.2)^2\cdot 10^{-4}\mathrm{m}^2}{2\cdot 2.4\cdot 10^{-2}\,\mathrm{m}\cdot 1.672\cdot 10^{-27}\,\mathrm{kg}\cdot(9.5)^2\cdot 10^8\,\mathrm{m}^2/\mathrm{s}^2} = 0.0119\,\mathrm{m}$$

Next we obtain the displacement outside the plates

$$x_2 = v_v t_2 = at_1\frac{l}{v_0} = \frac{Fbl}{m_p v_0 v_0} = \frac{Eqbl}{m_p v_0^2} = \frac{Uqbl}{dm_p v_0^2}$$

$$x_2 = \frac{14\,\mathrm{V}\cdot 1.6\cdot 10^{-19}\,\mathrm{C}\cdot 6.2\cdot 10^{-2}\,\mathrm{m}\cdot 0.45\,\mathrm{m}}{2.4\cdot 10^{-2}\,\mathrm{m}\cdot 1.672\cdot 10^{-27}\,\mathrm{kg}\cdot(9.5)^2\cdot 10^8\,\mathrm{m}^2/\mathrm{s}^2} = 0.172\,\mathrm{m}$$

And now we obtain h

$$h = x_1 + x_2 \qquad h = 1.2\ \mathrm{cm} + 17.2\ \mathrm{cm} = 18.4\ \mathrm{cm}$$

Answer. The vertical displacement of proton by the field between the plates is 18.4 cm.

Electric Field Strength

10.1. In which case will the electric field strength at a point and the force acting on a test charge at the same point have opposite signs?

10.2. Electrostatic filters used in thermal power stations and other plants to catch solid particles in smoke are essentially metal tubes with a wire stretched along the axis of the tube. What is the principle of the filter?

10.3. In electrostatic painting used in automobile works the jobs to be painted are passed under a metal net electrode connected to one of the poles of the high-voltage source. Sprayed paint is fed through the net electrode. Under what condition will paint particles travel to the jobs only?

10.4. Draw the pattern of electric field lines for two point charges unequal in magnitude and opposite in sign.

10.5. Using spherical conductors as an example, show the variation of the field strength with the surface charge density.

10.6. When a conductor having sharp points is electrified, "electric wind" is formed near them, that may be detected

through a deflection of flame of a candle. Explain the phenomenon.

10.7. Inside a hollow metal uncharged sphere a ball with a positive charge is placed. Are there electric fields inside and outside the sphere? Where and what charges develop? What will happen if (*a*) the charged ball is moved about within the sphere, and (b) the ball is left alone but an electrified body is brought near the sphere?

10.8. What is the principle underlying the electrostatic screening? Why are bodies of some of electronic tubes covered with metal cups?

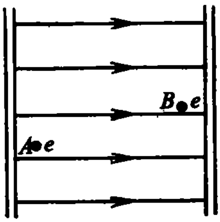


Fig. 10.9

10.9. Two electrons are at points *A* and *B* in a uniform electric field (Fig. 10.9). At which of these points does an electron "feel" the greater force?

10.10. A force of 0.015 N acts upon a charge of $2.0 \cdot 10^{-7}$ C at a point in an electric field. What is the field strength at that point?

10.11. The electric field strength at a point is $0.40 \cdot 10^{3}$ N/C. Determine the force acting at the point on a charge of $4.5 \cdot 10^{-6}$ C.

10.12. Determine the electric field strength produced by a point charge of $8 \cdot 10^{-6}$ C in air at a point located at a distance of 30 cm from the charge.

10.13. An electric field is formed by a point charge producing at 12 cm a strength of $3.45 \cdot 10^{5}$ N/C. What is the strength of the field 3.0 cm from the charge? What is the locus of the points with a field strength of equal magnitude?

10.14. A field is produced by a charge of $1.6 \cdot 10^{-8}$ C. Determine the field strength at a point 6.0 cm from the charge. What is the force with which the field at this point acts on a charge of $1.8 \cdot 10^{-9}$ C?

10.15. A conducting sphere of 24.0 cm radius is given a charge of $6.26 \cdot 10^{-9}$ C. Determine the electric field strength (*a*) at the sphere centre, (*b*) at a half radius distance from the centre, (*c*) at a distance of 24.0 cm from the sphere surface.

10.16. For a point charge draw a field strength vs. dis-

tance curve $E = f(r)$, and the curve for a field produced by a charged sphere of radius R.

10.17. According to Bohr's theory the electron in the hydrogen atom goes around the nucleus in a circular orbit. Determine the strength of the field created by the nucleus charge at a distance equal to the radius of the first electron orbit, i.e. $5.3 \cdot 10^{-11}$ m and the velocity of travelling of the electron around the nucleus in this orbit.

10.18. An electric field in glycerin is produced by a point charge of $7.0 \cdot 10^{-8}$ C. What is the field strength at a point at 7.0 cm from the charge?

10.19. A point charge produces in a vacuum at a distance of 9.0 cm a field strength of $4.0 \cdot 10^{5}$ N/C. What is the magnitude of the charge? How closer to the charge is a point where the strength is the same, if the charge is placed in a medium with $\varepsilon = 2.0$?

10.20. A point electric charge of $4.5 \cdot 10^{-7}$ C causes a strength of $2.0 \cdot 10^{4}$ N/C at 5.0 cm. What medium is it? What is the dielectric constant of the medium?

10.21. What is the difference between polarized charges of a dielectric and induced charges of a conductor?

10.22. The surface charge density of a conducting sphere is $\sigma = 3.2 \cdot 10^{-7}$ C/m^2. Determine the electric field strength at a point separated from the sphere surface by a distance equal to three radii.

10.23. Three like and equal charges q are separated from each other by an equal distance a. What is the field strength at a point equidistant from the charges and lying with them in one plane? What will the magnitude of the strength be, if one of the charges has an opposite sign?

10.24. A field is created by two equal, like charges separated by a distance. What is the field strength at a point located midway between the charges? Would the strength change with unlike charges?

10.25. Two charges $q_1 = 2.0 \cdot 10^{-8}$ C and $q_2 = 1.6 \cdot 10^{-7}$ C are separated by a distance of 5.0 cm. What is the field strength at a point separated by a distance of 3.0 cm from the first charge and by a distance of 4.0 cm from the second.

10.26. Charges $2.0 \cdot 10^{-7}$ C each are situated at two opposite corners of a square 30.0 cm on a side. Find the electric field strengths at two other square corners.

10.27. A pith ball of mass 0.40 g charged with $4.9 \cdot 10^{-9}$ C is suspended from a silk string in a horizontal uniform field of $1.0 \cdot 10^5$ N/C in vacuum. At what angle will the ball deflect?

10.28. A dust particle of mass $40.0 \cdot 10^{-8}$ g charged with $-1.60 \cdot 10^{-11}$ C is in a uniform electric field in vacuum. What must the magnitude and sign of the field strength be for the particle to remain stationary?

10.29. A droplet of mass $1.0 \cdot 10^{-4}$ g is in equilibrium in a uniform electric field of strength 98 N/C. Determine the charge of the droplet.

10.30. A 0.016 mg droplet that has lost 100 electrons is located 3 cm from a charge of $2.0 \cdot 10^{-6}$ C. What is the initial acceleration of the droplet?

10.31. A 5 g body falls on the ground. The body is given a charge of $+4 \cdot 10^{-8}$ C, the field strength at the surface of the Earth is 100 N/C. How will the acceleration of the body change?

10.32. It is common knowledge that with the charges on a conductor being in equilibrium there is no field inside the conductor. But if the conductor moves with an acceleration, an electric field sets up in it. What acceleration is needed for a metal rod so that the strength of a uniform field in it be $1.0 \cdot 10^{-6}$ N/C?

10.33. An electron travelling at $1.8 \cdot 10^4$ m/s enters a uniform electric field in a vacuum with strength 0.0030 N/C and moves in the direction opposite to field lines. What is the acceleration of the electron and its velocity after a distance of 7.1 cm? How long will it take the electron to attain this velocity?

10.34. An electron in a uniform electric field in a vacuum travels along the field lines. How long will it take the electron to come to rest, if the electric field strength is 90 N/C, and the initial velocity of the electron is $1.8 \cdot 10^3$ km/s?

10.35. Consider an electron at rest in a uniform electric field in vacuum. What is the field strength needed for the electron to gain an acceleration of $2.0 \cdot 10^{12}$ m/s^2? How long will it take the electron to reach $5.0 \cdot 10^6$ m/s?

10.36. A uniformly charged infinite plane creates a uniform field with the strength defined by the relationship $E = \sigma/2\varepsilon_0\varepsilon$. A charge of 0.15 nC is placed in the field of

the plane, the surface charge density of the plane being $2.0 \cdot 10^{-5}$ C/m^2, $\varepsilon = 1$. What is the force on the charge?

10.37. A field of a uniformly charged plane acts in vacuum on charge 0.2 nC with a force of $2.26 \cdot 10^{-5}$ N. Compute the electric field strength and the surface charge density on the plane.

10.38. Using the relation for the field strength produced by a uniformly charged plane (see problem 10.36) derive the expression for the field strength between two parallel, unlikely charged planes with equal surface charge density.

10.39. Two infinite parallel plates are uniformly charged with surface charge densities of $\sigma_1 = 0.4 \cdot 10^{-7}$ C/m^2 and $\sigma_2 = -0.1 \cdot 10^{-7}$ C/m^2. Determine the electric field strength between the plates and outside them.

10.40. A conducting surface of area 200 cm^2 carries a uniformly distributed charge of $2.0 \cdot 10^{-7}$ C. What is the force of attraction between two such plates arranged parallel to each other, if they have unlike charges?

Work Done in Moving a Charge in Electric Field. Potential. Potential Difference

10.41. In moving a charge of 120 μC from beyond a field to a point in the field a work of $6.0 \cdot 10^{-4}$ J is required. Find the potential of the field at that point.

10.42. A separate conducting sphere of diameter 6 cm in vacuum receives a charge of 20 nC. What is its potential?

10.43. How many electrons are required for the potential of a separate metal ball of 7.2 cm radius in vacuum to be 6 000 V?

10.44. A separate conducting sphere of 30.0 cm diameter is given a charge of $9.0 \cdot 10^{-8}$ C. What is the potential of the sphere? What is the potential at the centre of the sphere and at a distance of 15 cm from its surface in air?

10.45. Plot the curves for the dependence of potential on distance, $\varphi(r)$, for the field of a point charge, and for the field produced by a conducting sphere of radius R.

10.46. An electric field is formed by a point charge of $4.0 \cdot 10^{-7}$ C positioned in transformer oil. If the dielectric constant of the medium is taken to be 2.5, what is the strength

and potential at a point separated by a distance of 20.0 cm from the charge?

10.47. The potential of a conducting ball of 4.0 cm radius submerged in kerosene is 180 V. Determine the charge given to the ball. Compute the work done by the field in moving a charge of $0.5 \cdot 10^{-10}$ C to a point separated by a distance of 8.0 cm from the ball surface along the field line.

10.48. The electric field strength at the ground surface is around 130 V/m. Considering that the Earth is a sphere of radius 6 400 km, estimate the charge of the Earth.

10.49. A point charge of $2.0 \cdot 10^{-8}$ C is brought from infinity to a point separated by a distance of 28 cm from the surface of a conducting sphere of 2.0 cm radius, the sphere potential being 300 V. What is the work done?

10.50. A sphere of radius 10.0 cm is in vacuum. At 1.00 m from its surface the potential is 20.0 V. What is the potential of the sphere? What charge is given to the sphere?

10.51. What is the potential difference between any point on the surface of a conducting charged sphere and any point inside?

10.52. The body of an electrometer is made of metal and earthed. Explain why. Is it possible to measure the potential of a conductor by connecting it with the body of an electrometer installed on an insulating support, and the shaft and pointer of the electrometer, with earth? What will the electrometer read, if a test ball connected by a long conductor with the electrometer is moved about the surface of an electrified conductor of any configuration?

10.53. A conducting sphere with a positive charge is disposed under a metal sheet. Sketch an approximate pattern of equipotential surfaces and field lines.

10.54. An electric field is produced by a point charge of $1.5 \cdot 10^{-9}$ C. How far are two equipotential surfaces with potentials of 45 and 30.0 V in vacuum spaced apart?

10.55. Consider a conducting sphere of radius 5.0 cm. The potential difference of two points separated from its surface by a distance of 10.0 and 15 cm is 3.0 V. Determine the charge on the sphere.

10.56. An electric field in glycerin is formed by a point charge of $0.90 \cdot 10^{-8}$ C. What is the potential difference between two points separated from the charge by distances of

3.0 cm and 12 cm? What work will the field perform in moving between these points a like charge of 5.0 nC?

10.57. At a distance of 5.00 cm from the surface of an electrified conducting sphere the potential is 1 200 V, and at 10.0 cm, 900 V. Determine the radius, charge, and potential of the sphere.

10.58. Two metal concentric spheres with radii 15 and 30 cm are located in the air. The internal sphere carries a charge of $-2.0 \cdot 10^{-8}$ C, and the potential of the external sphere is 450 V. Compute the field strength and potential at points separated from the centre of the spheres by a distance of 10, 20, and 36 cm.

10.59. Two parallel thin rings of equal radii r are located in a vacuum so that they have a common axis. The centre distance is d. Over a first ring a charge q_1 is distributed uniformly, and over the second $(-q_2)$. Calculate the potential difference between the ring centres.

10.60. One hundred of identical droplets with a potential of 3.0 V each merge to form one drop. What is its potential?

10.61. Two charges of $3.0 \cdot 10^{-6}$ C each in air are brought together from a separation of 0.60 m to 0.20 m. What is the work done?

10.62. Two electrified plates cause a uniform electric field of 250 V/cm. What is the voltage of the plates? What is the force on a charge of $6.0 \cdot 10^{-6}$ C and what is the work performed in moving the charge from one plate to the other, if the plates are 4.0 cm apart?

10.63. A dust particle of mass $1.0 \cdot 10^{-11}$ g has a charge equal to 20 elementary charges, and is in equilibrium between two horizontal parallel plates with a potential difference of 153 V. What is the separation of the plates? In what direction and with what acceleration will the dust particle travel, if the potential difference between the plates is increased by 2 V?

10.64. A dust particle of mass $4.9 \cdot 10^{-12}$ g carrying an excess negative charge is suspended in equilibrium in a field of two parallel conducting plates spaced 1.0 cm apart, their potential difference being $1.0 \cdot 10^{2}$ V. When irradiated by ultraviolet rays the particle loses some of its charge and its equilibrium is disturbed. How many electrons has the particle lost, if to restore the equilibrium it is required to

increase the potential difference between the plates by 50 V?

10.65. An electron moves in a vacuum along a field line and loses its velocity completely between points with a potential difference of 400.0 V. Determine the electron velocity at the entry into the electric field. At what potential difference does the electron velocity decrease by half?

10.66. In a radioactive decay of uranium salts alpha-particles are produced with a velocity of about $2.0 \cdot 10^4$ km/s. What must the potential difference between two points in an electric field be so that alpha-particle could gain the same velocity in travelling through the distance?

10.67. A conducting ball of mass 1.6 g carrying a charge of $0.4 \cdot 10^{-7}$ C is moved from point M to point N, the potential of the latter being zero. What is the velocity of the ball at point M, if at point N it travelled at 0.4 m/s? The potential $\varphi_M = 700$ V.

10.68. An electric field is created in a vacuum by two point charges $q_1 = 4.0 \cdot 10^{-8}$ C and $q_2 = -0.50 \cdot 10^{-8}$ C

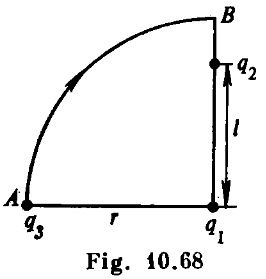


Fig. 10.68

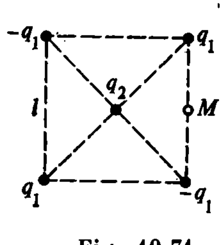


Fig. 10.71

(Fig. 10.68). The charges are placed $l = 30$ cm apart. What is the work done in carrying a charge $q_3 = 5.0 \cdot 10^{-9}$ C from point A to point B in a segment of a circle of radius $r = 0.40$ m?

10.69. An electron is travelling at 85,000 km/s within capacitor plates in a vacuum, parallel to the plates. When the capacitor is under a voltage, the electrons within it are displaced toward one of the plates by 1.8 mm. Determine the electric field strength within the capacitor.

10.70. A proton beam in a vacuum is ejected in a flat capacitor 5.5 cm long perpendicular to the electric field

lines. If the field strength in the capacitor is 30,000 V/m, then the protons having passed through the capacitor are displaced in the direction of the field by 1.5 mm. Determine the kinetic energy of protons entering the capacitor. Ignore the gravity force.

10.71. At the corners of a square of side l there are two positive and two negative charges of equal magnitude q_1 (Fig. 10.71). What is the work required to move a charge q_2 from the centre of the square to point M in the middle of any of the sides?

10.72. An electron is ejected into a flat air capacitor parallel to the plates, the velocity of the electron is $3 \cdot 10^7$ m/s, at the outlet of the capacitor it is displaced by $1.76 \cdot 10^{-3}$ m from its initial direction. Determine the ratio of the electron charge to its mass, if the capacitor is 3 cm long, its plates are $2 \cdot 10^{-2}$ m apart and the potential difference between them is 400 V.

SEC. 11. CAPACITANCE AND CAPACITORS *

Example 32. A flat capacitor in which the area of each plate is $6.20 \cdot 10^{-3}$ m^2 is filled with mica with dielectric constant 6. The separation between the plates is 2.00 mm and each carries a charge of $4.00 \cdot 10^{-8}$ C.

Given: $S = 6.20 \cdot 10^{-3}$ m^2 is the area of a plate, $d = 2.00 \cdot 10^{-3}$ m is the separation between the plates, $\varepsilon = 6$ is the dielectric constant of mica, $q = 4.00 \cdot 10^{-8}$ C is the charge on one plate, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the electric constant.

Determine: (1) C—the capacitance of the capacitor,
(2) U—the potential difference across the plates,
(3) E—the electric field strength in the capacitor,
(4) F—the force of mutual attraction of the plates.

Solution. The capacitance of the capacitor is obtained from the relationship for the flat capacitor

$$C = \varepsilon_0 \varepsilon S/d$$

* In this section the dielectric constant ε is taken to be unity, unless stated otherwise.

Given the charge q and the capacitance C, we determine the potential difference across the capacitor plates

$$U = q/C = qd/\varepsilon\varepsilon_0 S$$

The electric field strength inside the capacitor is related to the potential difference across its plates by

$$E = U/d = q/\varepsilon\varepsilon_0 S$$

As the field between the plates is caused by the superposition of two fields (from the two plates), the field strength due to one plate is $E_1 = E/2$. The force of attraction of one plate by the other is

$$F = E_1 q = Eq/2$$

Using the data of the problem and the above relationships we arrive at the quantities required

$$C = \frac{8.85\cdot10^{-12}\ \text{F/m}\cdot6\cdot6.2\cdot10^{-3}\ \text{m}^2}{2.0\cdot10^{-3}\ \text{m}} = 1.65\cdot10^{-10}\ \text{F}$$

$$U = \frac{4.0\cdot10^{-8}\ \text{C}}{1.65\cdot10^{-10}\ \text{F}} = 242\ \text{V}$$

$$E = \frac{242\ \text{V}}{2.00\cdot10^{-3}\ \text{m}} = 1.21\cdot10^{5}\ \text{V/m}$$

$$F = \frac{1.21\cdot10^{5}\ \text{V/m}}{2}\cdot4.00\cdot10^{-8}\ \text{C} = 2.42\cdot10^{-3}\ \text{N}$$

Answer. The capacitance of the capacitor is $1.65\cdot10^{-10}$ F, the potential difference across the plates is 242 V, the electric field strength in the capacitor is $1.21\cdot10^{5}$ V/m, the force of attraction of the plates is $2.42\cdot10^{-3}$ N.

Example 33. Consider a capacitor of an aluminium foil belt 157 cm long and 90.0 mm wide separated by a waxed paper 0.10 mm thick. What is the capacitance of the capacitor? What is the energy stored in the capacitor, if its working voltage is $4.0\cdot10^{2}$ V?

Given: $l = 157$ cm $= 1.57$ m is the length of the aluminium foil, $h = 90.0$ mm $= 9.0\cdot10^{-2}$ m is the width of the foil, $d = 0.10$ mm $= 0.10\cdot10^{-3}$ m is the thickness of the waxed paper, $U = 4.0\cdot10^{2}$ V is the voltage across the capacitor plates, $\varepsilon_0 = 1/36\pi\cdot10^{9}$ F/m is the electric constant, $\varepsilon = 2.0$ is the dielectric constant of the waxed paper.

Determine: C—the capacitance of the capacitor,
W—the energy stored in the capacitor.

Solution. To find the capacitance we use the formula

$$C = \varepsilon_0 \varepsilon S/d$$

As $S = hl$, we have

$$C = \varepsilon_0 \varepsilon hl/d$$

The energy of the capacitor is given by

$$W = CU^2/2$$

The substitution of the data gives

$$C = \frac{2\cdot 0.09\ \text{m}\cdot 1.57\ \text{m}\cdot \text{C}^2}{36\pi\cdot 10^9\ \text{N}\cdot\text{m}^2\cdot 0.1\cdot 10^{-3}\ \text{m}} = 25\cdot 10^{-9}\ \text{F} = 25\cdot 10^{-3}\ \mu\text{F}$$

$$W = \frac{25\cdot 10^{-9}\ \text{F}\cdot 16\cdot 10^4\ \text{V}^2}{2} = 0.002\ \text{J}$$

Answer. The capacitance of the capacitor is $25\cdot 10^{-3}$ μF, the energy of the capacitor is 0.002 J.

Example 34. Three capacitors of capacitance $C_1 = 0.2$ μF, $C_2 = C_3 = 0.4$ μF are arranged as shown in Fig. 34 and connected to a constant voltage source $U_{AB} = 250$ V. Find the total charge, the charges and potential differences of individual capacitors. Determine the electric energy stored by the capacity bank.

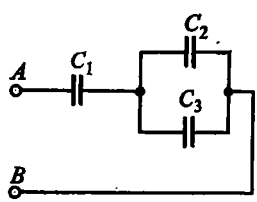


Fig. 34

Given: $C_1 = 0.2\cdot 10^{-6}$ F, $C_2 = C_3 = 0.4\cdot 10^{-6}$ F are the capacitances of the capacitors, $U_{AB} = 250$ V is the voltage applied to the capacitors.

Determine: q, q_1, q_2, and q_3—the total charge and the charges of individual capacitors,
U_1, U_2, and U_3—the potential differences across the capacitors,
W—the energy stored by all the capacitors.

Solution. The total charge is obtainable from the relationship

$$q = CU_{AB}$$

where C is the capacitance of all the capacitors (bank) and it is obtained using the relationship for a mixed (parallel and series) connection

$$C = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} = \frac{2C_1C_2}{C_1+2C_2}$$

The charge of the first capacitor is equal to the total charge

$$q_1 = q$$

and the charges of the two other capacitors are

$$q_2 = q_3 = q/2$$

Knowing the capacity and the charge of each capacitor, we may obtain the potential difference across the capacitors.

To find the energy stored by the capacitor bank we use the relationship

$$W = \frac{C(U_{AB})^2}{2} = \frac{C_1C_2}{C_1+2C_2} U_{AB}^2$$

Next we find the charge stored by the bank

$$q = \frac{2C_1C_2}{C_1+2C_2} U_{AB}$$

$$q = \frac{2\cdot 0.2\cdot 10^{-6}\,\mathrm{F}\cdot 0.4\cdot 10^{-6}\,\mathrm{F}\cdot 250\,\mathrm{V}}{(0.2\cdot 10^{-6}+2\cdot 0.4\cdot 10^{-6})\,\mathrm{F}} = 4\cdot 10^{-5}\,\mathrm{C}$$

and the charges of individual capacitors

$$q_1 = 4\cdot 10^{-5}\,\mathrm{C} \qquad q_2 = q_3 = 2\cdot 10^{-5}\,\mathrm{C}$$

Now we would like to compute the potential difference and the total energy

$$U_1 = \frac{4\cdot 10^{-5}\,\mathrm{C}}{0.2\cdot 10^{-6}\,\mathrm{F}} = 200\,\mathrm{V} \qquad U_2 = U_3 = \frac{2\cdot 10^{-5}\,\mathrm{C}}{0.4\cdot 10^{-6}\,\mathrm{F}} = 50\,\mathrm{V}$$

$$W = \frac{0.2\cdot 10^{-6}\,\mathrm{F}\cdot 0.4\cdot 10^{-6}\,\mathrm{F}\cdot (250)^2\,\mathrm{V}^2}{(0.2\cdot 10^{-6}+0.8\cdot 10^{-6})\,\mathrm{F}} = 5\cdot 10^{-3}\,\mathrm{J}$$

Answer. The charge of the bank and the first capacitor is $4\cdot 10^{-5}$ C, of the second and third capacitors $2\cdot 10^{-5}$ C each; the potential differences are 200, 50, and 50 V, respectively; the total energy is $5\cdot 10^{-3}$ J.

11.1. Compare the potentials of two conducting balls of different size in a vacuum with equal charges; of two identical detached spheres in a vacuum with different charges.

11.2. On the surface of molded plastic parts there develops an electrostatic charge owing to contacts with the walls of metal dies. When the part is removed from the mold the potential of the charged surface relative to the Earth increases. Why?

11.3. Are the capacitances of two conductors of similar configuration and dimensions always equal?

11.4. To get some idea of the magnitude of unit capacitance—farad—compute the capacitance of the Earth.

11.5. Consider a conducting sphere in a vacuum. What is the radius of the sphere required for its capacitance to be equal to one farad?

11.6. As a conductor is given a charge of 0.008 C, its potential becomes 1000 V. Determine the capacitance of the conductor.

11.7. Determine the capacitance of a separate conducting ball of 3.0 cm diameter in the air. Express the result in farads, microfarads, and picofarads.

11.8. As a conducting sphere is given a charge of $3 \cdot 10^{-8}$ C, its potential becomes $6.0 \cdot 10^{3}$ V. Determine the radius and capacitance of the sphere in the air.

11.9. A metal ball of capacitance $0.45 \cdot 10^{-11}$ F is charged with $1.8 \cdot 10^{-7}$ C in the air. Determine the potential and radius of the ball.

11.10. Two conducting balls of radii 1.5 and 6.0 cm are given charges $0.50 \cdot 10^{-9}$ and $6.0 \cdot 10^{-9}$ C, respectively. What will happen after the balls are connected with a thin wire? Determine the potentials of the balls before and after the connecting, if these are in the air. Find the final charges of both balls.

11.11. The potentials of balls with capacitances 6.0 and 9.0 pF are $2.0 \cdot 10^{2}$ and $8.0 \cdot 10^{2}$ V, respectively. Determine the total charge and the potential of the balls after their coming into contact with each other.

11.12. Consider a capacitor with metal foil plates of area $4.7 \cdot 10$ cm^{2} separated by 15 sheets of waxed paper with a thickness of 0.03 mm. What is the capacitance of the capacitor?

11.13. What hazard do deenergized circuits present if they contain capacitors? What is to be done after the circuit is deenergized?

11.14. A capacitor is an arrangement of 21 brass sheets interlarded by glass layers of 2 mm thickness. Areas of the brass sheets and glass layers are equal to 200 cm^2 each. The sheets are connected so that they form a bank of parallel capacitors. The dielectric constant of glass is 7. What is the capacitance of the bank?

11.15. Consider a variable capacitor used for school demonstrations. It consists of an array of plates shaped as semicircles with radius 10 cm, and as the dielectric 7 mm thick glass is used ($\varepsilon_g = 5$). What is the maximum capacitance of the capacitor? Can you think of a way to increase the energy of the capacitor without changing its charge?

11.16. A flat capacitor of capacitance 0.020 μF has an electric field strength of 320 V/cm, the plates being spaced 0.50 cm apart. What is the charge of the capacitor? What will the voltage across the plates be, if the gap between them is doubled? What is the energy of the capacitor in both cases?

11.17. A mica capacitor has a plate area of 36 cm^2, the thickness of the dielectric layer being 0.14 cm. The potential difference across the plates is $3.0 \cdot 10^2$ V, the dielectric constant of mica is 7. What are the capacitance, charge, and energy of the capacitor?

11.18. A flat capacitor of capacitance 0.3 μF is made of sheets of foil interlarded with mica. The area of each of the mica plates is 50 cm^2, the thickness 0.177 mm. The dielectric constant of mica is taken to be 6. How many mica plates are required?

11.19. A flat air capacitor is charged with $2.00 \cdot 10^{-7}$ C. The area of each plate is $2\pi \cdot 10^4$ mm^2. What is the work required to increase the separation between the plates by 0.400 mm?

11.20. A conducting ball of 2.0 cm diameter and $9.0 \cdot 10^4$ V potential is connected with earth by a conductor. What amount of energy will be released in the conductor?

11.21. Six capacitors with capacitance $5.0 \cdot 10^{-3}$ μF each are connected in parallel to make a bank, and charged up to 4,000 V. What is the total charge stored by all the capaci-

tors? How much heat will evolve in discharging such a bank?

11.22. The area of each plate of a flat mica capacitor is 300 cm^2, the thickness of mica is 1.0 mm. Given that in discharging the capacitor there evolved 0.21 J of heat, what was the potential difference across the plates?

11.23. A flat air capacitor of capacitance $1.6 \cdot 10^3$ pF is charged up to a potential difference of 500 V and disconnected from the voltage source, then the separation between the plates is increased threefold. Determine the potential difference across the capacitor plates after they have been forced apart, and the work done by external forces to do so.

11.24. In a flat capacitor a glass plate of 15 mm thickness is used as dielectric. The capacitor is charged to 200 V, disconnected from the voltage source, and thereafter the glass plate is removed. What is the direction and amount of the potential difference across the capacitor plates? The dielectric constant of glass is taken to be 7.5.

11.25. A capacitor of unknown capacitance with a voltage of 1000 V across the plates is connected in parallel with another capacitor of 2.0 μF with a voltage of 400 V. If after the connection the voltage is 570 V, what is the capacitance of the first capacitor? What is the total charge?

11.26. Think of a way to produce with two identical capacitors a capacitance twice lower and twice higher than that of one of them? If any, how?

11.27. Two identical variable capacitors are connected to form a bank. Within what range is it possible to vary the capacitance of this bank? The capacitance of each capacitor can vary from 15 to 250 pF.

11.28. A capacitor of 6.0 μF with a voltage of $4.0 \cdot 10^2$ V across the plates is connected in parallel with an uncharged capacitor of 10.0 μF. What is the new potential difference across the plates of both capacitors? How is the charge distributed?

11.29. A flat capacitor with a plate area of 20 cm^2 uses for a dielectric a layer of mica $3.0 \cdot 10^{-3}$ mm thick and a layer of waxed paper $1.0 \cdot 10^{-3}$ mm. Assuming that for mica $\varepsilon = 6$, what is the capacitance of the capacitor?

11.30. Why do electrolytic capacitors possess higher capacitance?

11.31. After charging, a flat air capacitor is disconnected and placed in kerosene. What is the change in the energy accumulated by the capacitor?

11.32. A flat air capacitor has a capacitance C. Determine the capacitance of the capacitor if it is half immersed in transformer oil so that its plates are perpendicular to the surface of oil. The dielectric constant of the oil is 2.2.

11.33. Two isolated conducting spheres of radii R_1 and R_2 are given charges q_1 and q_2, respectively. Prove that as a result of a redistribution of charges after they are connected, the potential $\varphi = (q_1 + q_2)/4\pi\varepsilon_0 (R_1 + R_2)$ is established.

11.34. One million of spherical conducting droplets merge to form one drop. The radius of each droplet is $5.0 \cdot 10^{-4}$ cm, and its charge is $1.6 \cdot 10^{-14}$ C. What is the work performed against the repulsive forces in connecting the droplets?

11.35. Three capacitors are arranged as shown in Fig. 11.35. The voltage at points A and B is 250 V, $C_1 = 1.5$ μF, $C_2 = 3.0$ μF, and $C_3 = 4.0$ μF. What is the charge stored by all the capacitors? What is the energy of all capacitors?

11.36. The capacitance of the capacitor bank of Fig. 11.36 is 5.8 μF. What are the capacitance and charge of the first capacitor, if $C_2 = 1.0$ μF, $C_3 = 4.0$ μF, and the voltage applied to points A and B is 220 V?

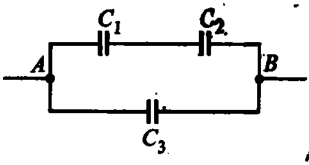


Fig. 11.35

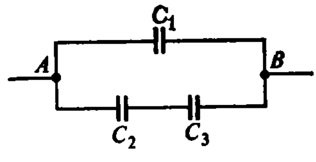


Fig. 11.36

11.37. Two capacitors of capacitances 4.0 and 1.0 μF are connected in series to a constant voltage source of 220 V. Determine the total capacitance. How will the voltage distribute between the capacitors?

11.38. Three capacitors with capacitances $C_1 = 1.0$, $C_2 = 1.0$, and $C_3 = 2.0$ μF are arranged as shown in Fig. 11.38 and connected to a constant voltage source of

120 V. What is their total capacitance? Determine the charge and voltage at each capacitor.

11.39. Consider a flat capacitor of capacitance 0.015 μF with the plates 2.5 mm apart. What charge must be given to it for a dust particle that has lost 20 electrons to be in equilibrium in the field of the capacitor?

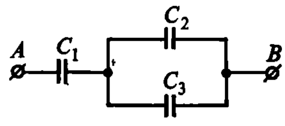


Fig. 11.38

11.40. A capacitor of capacitance 0.6 μF is charged to a potential difference of 200 V and connected in parallel so that to make a bank with a capacitor of 0.4 μF and 300 V. Determine the capacitance of the bank, the potential difference across the plates, and the energy stored in it.

11.41. A flat air capacitor with a plate area of 80 cm^2 and a separation between the plates of 1.5 mm is charged by a 100 V source, then disconnected from it and immersed in a liquid dielectric with a dielectric constant of 2.5. What is the change in the energy of the capacitor?

11.42. Metal balls of radii 0.20 and 0.60 cm are brought to contact and given a total charge of $28 \cdot 10^{-8}$ C, then they are separated from each other at a distance of 10.0 cm (centre distance). How are the charges distributed? Find the ratio of charge distribution densities, considering them uniform. What is the force of repulsion between the balls?

11.43. A uniform electric field of $1.0 \cdot 10^4$ V/m is produced by two electrified plates spaced 2.0 cm apart in air. What is the potential difference between the plates? What will the potential difference be if between the plates parallel to them a metal sheet 0.50 cm is placed?

11.44. Two metal plates are spaced parallel to each other at 0.60 cm in air. What is the potential difference needed for the field strength to be $7.0 \cdot 10^2$ V/cm? What is the energy stored by the capacitor, if the plates are charged with $8.0 \cdot 10^{-4}$ C? Is the field uniform at the edges of the plates?

11.45. Two parallel vertical plates A and B are separated by a distance of 2.50 mm (Fig. 11.45), their potentials are 50.0 and −50.0 V, respectively. Midway between them a metal sheet 0.40 mm thick is placed. Determine the strength of the new uniform field and the potential difference between

the plates. What is the work done by the field in carrying a charge of $0.10 \cdot 10^{-2}$ C between plate A and the sheet?

11.46. A flat capacitor of capacitance C is in a vacuum. The area of a plate is S, and the field strength inside the capacitor is E. Determine the velocity to be gained by the electron travelling from one plate to the other. The initial velocity of the electron is $v_0 = 0$.

11.47. A flat air capacitor is fully immersed in kerosene, the field strength between the plates is $5 \cdot 10^6$ N/C. What is

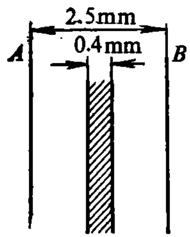


Fig. 11.45

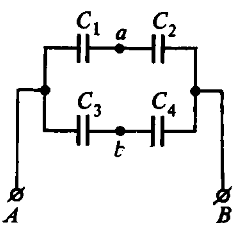


Fig. 11.49

the volume density of the energy within the capacitor?

11.48. Two identical flat air capacitors are placed in series and connected to a constant voltage source. By how many times will the potential difference on one of the capacitors change, if the other is completely immersed in a liquid with a dielectric constant ε?

11.49. Four capacitors with capacitances $C_1 = 1$ μF, $C_2 = 1.5$ μF, $C_3 = 2.5$ μF, and $C_4 = 0.5$ μF are connected so that to form a bank (Fig. 11.49) and are supplied by a constant voltage source $U_{AB} = 15$ V. What is the potential difference between points a and b?

SEC. 12. ELECTRIC CURRENT IN METALS. DIRECT CURRENT LAWS

Example 35. An electric boiler is rated at a voltage of 120 V and a current of 4.0 A, it uses a Nichrome heating element with a permissible current density of 10.2 A/mm² and

resistivity of $1.3 \cdot 10^{-6}$ Ω·m. Ignoring the change in the length and cross-sectional area of the wire in heating, what is the length and cross-sectional area required for the heating element?

Given: $U = 120$ V is the voltage of the boiler, $I = 4.0$ A is the current, $j = 10.2 \cdot 10^6$ A/m² is the permissible current density, $\rho = 1.3 \ 10^{-6}$ Ω·m is the resistivity of Nichrome.

Determine: S—the cross-sectional area,
l—the length of the wire.

Solution. Using Ohm's law and the relation between current and density, we determine the resistance and cross-sectional area of the wire

$$R = U/I \qquad S = I/j$$

Given the formula relating resistance to material and dimensions of the conductor we obtain its length

$$l = \frac{RS}{\rho} = \frac{US}{I\rho}$$

And using the data of the problem, we find

$$S = \frac{4 \text{ A}}{10.2 \cdot 10^6 \text{ A/m}^2} = 0.39 \cdot 10^{-6} \text{ m}^2$$

$$l = \frac{120 \text{ V} \cdot 0.39 \cdot 10^{-6} \text{ m}^2}{4 \text{ A} \cdot 1.3 \cdot 10^{-6} \, \Omega \cdot \text{m}} = 9 \text{ m}$$

Answer. The cross-sectional area of the wire is 0.39 mm², the length of the wire is 9 m.

Example 36. A battery of 2.8 V is connected as shown in Fig. 36, where $R_1 = 1.8$ Ω, $R_2 = 2$ Ω, $R_3 = 3$ Ω. The ammeter reads 0.48 A. Determine the internal resistance of the battery. Ignore the ammeter resistance.

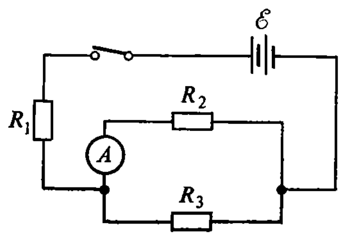


Fig. 36

Given: $\mathscr{E} = 2.8$ V is the e.m.f. of the battery, $R_1 = 1.8$ Ω is the resistance of the first conductor, $R_2 = 2$ Ω is the resistance of the second conductor, $R_3 = 3$ Ω is the resistance of the third conductor, $I_2 = 0.48$ A is the reading of the ammeter.

Determine: r—the internal resistance of the battery.

Solution. The internal resistance of the battery is obtainable from Ohm's relationship for all network

$$I=\frac{\mathscr{E}}{R_{tot}+r} \quad r=\frac{\mathscr{E}-IR_{tot}}{I}$$

As $I = I_1 = I_2 + I_3$, we first determine the current in the third conductor, and then the total current

$$R_2/R_3 = I_3/I_2 \quad I_3 = I_2R_2/R_3$$

Thus,

$$I = I_2(R_3 + R_2)/R_3$$

As R_1 is connected in parallel with the branch, we have

$$R_{tot}=R_1+R_{eq}$$

where

$$R_{eq}=\frac{R_2R_3}{R_2+R_3}$$

Hence

$$R_{tot}=\frac{R_1(R_2+R_3)+R_2R_3}{R_2+R_3}$$

As $r = \mathscr{E}/I - R_{tot}$, we get

$$r=\frac{\mathscr{E}R_3}{I_2(R_3+R_2)}-\frac{R_1(R_2+R_3)+R_2R_3}{R_2+R_3}$$

Substituting gives

$$r=\frac{2.8\text{ V}\cdot 3\ \Omega}{0.48\text{ A}\cdot 5\ \Omega}-\frac{1.8\ \Omega\cdot 5\ \Omega+6\ \Omega^2}{5\ \Omega}=0.5\ \Omega$$

Answer. The internal resistance of the battery is 0.5 Ω.

Example 37. In a room separated from a generator by a distance of 100 m, 44 incandescent lamps with resistance 440 Ω each are connected in parallel. The voltage across the lamps is 220 V. The wiring is made of a copper wire with a cross-sectional area of 17.0 mm^2. Determine the voltage drop across the wiring and the voltage across the terminals of the generator.

Given: $l = 1.00\cdot 10^2$ m is the distance from the generator to the load, $n = 44$ is the number of lamps, $R_1 = 4.40\cdot 10^2\ \Omega$ is the resistance of each lamp, $U_1 = 2.20\cdot 10^2$ V is the volt-

age rating of the lamps, $S = 17\ \text{mm}^2 = 17.0 \cdot 10^{-6}\ \text{m}^2$ is the cross-sectional area of the wiring, $\rho = 0.017 \cdot 10^{-6}\ \Omega \cdot \text{m}$ is the resistivity of copper.

Determine: U_w—the voltage drop across the wiring,
U—the voltage across the generator terminals.

Solution. The voltage at the generator terminals is higher than that of lamps by the voltage drop across the wiring:

$$U = U_1 + U_w$$

where

$$U_w = IR_w$$

The current in wiring is equal to the sum of currents through the lamps

$$I = \frac{U_1}{R_1} n$$

The resistance of the wiring is

$$R_w = \rho \frac{2l}{S}$$

Substituting for current and wiring resistance into the relation $U_w = IR_w$, we arrive at

$$U_w = \frac{U_1}{R_1} n\rho \frac{2l}{S}$$

$$U_w = \frac{220\ \text{V} \cdot 44 \cdot 0.017 \cdot 10^{-6}\ \Omega \cdot \text{m} \cdot 200\ \text{m}}{440\ \Omega \cdot 17.0 \cdot 10^{-6}\ \text{m}^2} = 4.4\ \text{V}$$

We now calculate U

$$U = 220\ \text{V} + 4.4\ \text{V} = 224.4\ \text{V}$$

Answer. The voltage drop in the wiring is 4.4 V, the voltage across the generator terminals is 224.4 V.

Example 38. Two cells of 1.6 V and 1.3 V with internal resistances of 1.0 Ω and 0.50 Ω, respectively, are arranged as shown in Fig. 38. Determine the currents in all the branches. Ignore the resistance of conducting wires.

Given: $\mathscr{E}_1 = 1.6$ V is the e.m.f. of the first cell, $\mathscr{E}_2 = 1.3$ V is the e.m.f. of the second cell, $r_1 = 1.0\ \Omega$ is the internal resistance of the first cell, $r_2 = 0.50\ \Omega$ is the internal resistance of the second cell, $R = 0.60\ \Omega$ is the resistance of segment AB.

Determine: I_1—the current through the first cell,
I_2—the current through the second cell,
I_3—the current through the segment with resistance R.

First approach.

Solution. According to Kirchhoff's rules, and considering

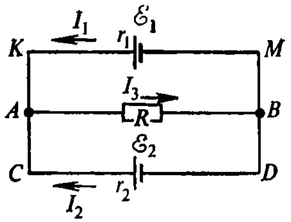


Fig. 38

the arbitrarily chosen direction of currents, we derive the equations for various segments of the network.

For node A

$$I_1 + I_2 = I_3$$

For a closed contour $KCDM$

$$\mathscr{E}_1 - \mathscr{E}_2 = I_1 r_1 - I_2 r_2$$

For a closed contour $KABM$

$$\mathscr{E}_1 = I_1 r_1 + I_3 R$$

Eliminating I_3 from the last equation and solving the set of equations for I_1 and I_2, we get

$$I_1 = \frac{\mathscr{E}_1 r_2 + (\mathscr{E}_1 - \mathscr{E}_2) R}{r_1 R + r_1 r_2 + R r_2}$$

$$I_2 = \frac{I_1 r_1 + \mathscr{E}_2 - \mathscr{E}_1}{r_2}$$

Substitution gives

$$I_1 = \frac{1.6\ \text{V}\cdot 0.5\ \Omega + (1.6\ \text{V} - 1.3\ \text{V})\cdot 0.6\ \Omega}{1\ \Omega\cdot 0.6\ \Omega + 1\ \Omega\cdot 0.5\ \Omega + 0.6\ \Omega\cdot 0.5\ \Omega} = 0.7\ \text{A}$$

$$I_2 = \frac{0.7\ \text{A}\cdot 1\ \Omega - 0.3\ \text{V}}{0.5\ \Omega} = 0.8\ \text{A}$$

$$I_3 = 0.7\ \text{A} + 0.8\ \text{A} = 1.5\ \text{A}$$

Second approach.

Solution. Now, to solve the problem we will use the method of node potentials. We denote the potential of node A as φ_A, and the potential of node B we take to be zero. Then $\varphi_A - \varphi_B = U_{AB}$. We write the relationships for currents (their selected directions are indicated in Fig. 38) by Ohm's law for a network segment with and without an e.m.f.

$$I_1 = \frac{\mathscr{E}_1 - U_{AB}}{r_1} \qquad I_2 = \frac{\mathscr{E}_2 - U_{AB}}{r_2} \qquad I_3 = \frac{U_{AB}}{R}$$

As $I_1 + I_2 = I_3$, we have

$$\frac{\mathscr{E}_1 - U_{AB}}{r_1} + \frac{\mathscr{E}_2 - U_{AB}}{r_2} = \frac{U_{AB}}{R}$$

Substituting the data we determine U_{AB}

$$\frac{1.6\ \text{V} - U_{AB}}{1.0\ \Omega} + \frac{1.3\ \text{V} - U_{AB}}{0.50\ \Omega} = \frac{U_{AB}}{0.60\ \Omega}$$

$$U_{AB} = 0.9\ \text{V}$$

Next we find the currents

$$I_1 = \frac{1.6\ \text{V} - 0.9\ \text{V}}{1.0\ \Omega} = 0.7\ \text{A}$$

$$I_2 = \frac{1.3\ \text{V} - 0.9\ \text{V}}{0.50\ \Omega} = 0.8\ \text{A}$$

$$I_3 = 1.5\ \text{A}$$

Answer. The current through the first cell is 0.7 A, through the second 0.8 A, and through the conductor with resistance R, 1.5 A.

Example 39. An electric circuit shown in Fig. 39 consists of a source of an e.m.f. of 12 V with internal resistance 1.0 Ω, two resistors $R_1 = 3.0\ \Omega$ and $R_2 = 6.0\ \Omega$ and two capacitors with capacitances $C_1 = 1.0\ \mu\text{F}$ and $C_2 = 2.0\ \mu\text{F}$. Determine the potential difference between points a and b and the charge stored by each capacitor.

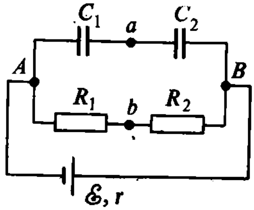


Fig. 39

Given: $\mathscr{E} = 12$ V, $r = 1.0\ \Omega$ are the e.m.f. and internal resistance of the source of electric energy, $R_1 = 3.0\ \Omega$,

$R_2 = 6.0\ \Omega$ are the resistances of the network segment AB, $C_1 = 1.0 \cdot 10^{-6}$ F, $C_2 = 2.0 \cdot 10^{-6}$ F are the capacitances of the capacitors.

Determine: $\Delta\varphi$—the potential difference between points a and b,

q—the charge stored by each capacitor.

Solution. We take the potential of point A to be zero, and the potentials of points a and b we denote by φ_a and φ_b, then $\Delta\varphi = \varphi_a - \varphi_b$. The solution of the problem is reduced to the finding of φ_a and φ_b.

We determine the current in the network

$$I = \mathscr{E}/(R_1 + R_2 + r)$$

The potential of point b is above zero by the amount of the voltage drop across the resistor R_1

$$\varphi_b = IR_1 = \mathscr{E}R_1/(R_1 + R_2 + r)$$

The voltage drop across the segment AB is

$$U_{AB} = \mathscr{E} - Ir = \frac{\mathscr{E}(R_1 + R_2)}{R_1 + R_2 + r}$$

The potential difference across the two capacitors connected in series is also U_{AB}. Considering the way of connection of the capacitors we note that their charge will be identical and equal to

$$q = U_{AB}\frac{C_1C_2}{C_1 + C_2} = \frac{\mathscr{E}(R_1 + R_2)C_1C_2}{(R_1 + R_2 + r)(C_1 + C_2)}$$

Given the charge and capacitance of the first capacitor we may determine the potential difference across its plates, and hence φ_a.

Now we write $\Delta\varphi$ and determine its numerical value

$$\Delta\varphi = \frac{\mathscr{E}(R_1 + R_2)C_2}{(R_1 + R_2 + r)(C_1 + C_2)} - \frac{\mathscr{E}R_1}{R_1 + R_2 + r}$$

$$= \mathscr{E}\frac{R_2C_2 - R_1C_1}{(R_1 + R_2 + r)(C_1 + C_2)}$$

$$\Delta\varphi = 12\ \text{V}\ \frac{(6.0 \cdot 2.0 \cdot 10^{-6} - 3.0 \cdot 1.0 \cdot 10^{-6})\ \Omega \cdot \text{F}}{(3.0 + 6.0 + 1.0)\ \Omega \cdot 3.0 \cdot 10^{-6}\ \text{F}} = 3.6\ \text{V}$$

We obtain the charge of the capacitor

$$q=\frac{12\ \mathrm{V}\cdot 9.0\ \Omega\cdot 1.0\cdot 10^{-6}\ \mathrm{F}\cdot 2.0\cdot 10^{-6}\ \mathrm{F}}{10.0\ \Omega\cdot 3.0\cdot 10^{-6}\ \mathrm{F}}=7.2\cdot 10^{-6}\ \mathrm{C}$$

Answer. The potential difference between points a and b is 3.6 V, the charge carried by each capacitor is $7.2\cdot 10^{-6}$ C.

12.1. What will a galvanometer read, if it has passed 18 C of electricity in 10 min? How many electrons must pass through the cross-section of a conductor for a galvanometer to read 1 mA?

12.2. An electric stove has passed a charge of 9,720 C in 1 hour. Determine the energy liberated, if the line voltage is 220 V. Determine the current in the circuit.

12.3. A conductor of cross-sectional area 1.5 mm^2 carries a current of 0.3 A. Regarding the concentration of free electrons in the substance to be equal to 10^{28} m^{-3}, determine the average velocity of ordered motion of free electrons.

12.4. A boiler is rated at 125 V. What is the energy dissipated in the boiler in 10 min, if during this time it passes a charge of 4,800 C? Determine the current through the heating element of the boiler and the resistance of the boiler.

12.5. In a segment of a network the current grows steadily from 0 to 1.5 A in 6 s. Draw the current vs time curve and from it determine the charge which flew through the cross-section of the conductor during this time.

12.6. It took 0.0004 s to charge a bank of 4 identical capacitors connected in parallel with a current of 0.2 A up to 1 000 V. Given that the current during the charging is constant, determine the capacitance of one capacitor.

12.7. An electric iron for 5 min draws 2.0 A of current from a 220 V line voltage. What is the charge passed through the iron and what is the energy given up by the iron? Compute the resistance of the heating element of the iron under operating conditions.

12.8. The average velocity of ordered motion of electrons in a copper wire of 1 mm^2 cross-sectional area is $7.4\cdot 10^{-3}$ cm/s. What is the current in the wire, given that each copper atom yields two free electrons?

12.9. At power stations special-purpose buses, and not conventional wires of circular section, are used between generators and step-up transformers. Why?

12.10. A power line of length 500 m carries a current of 15 A. If the wire cross-sectional area is 14 mm^2, what is the voltage drop across it?

12.11. From the generator to the user is 250 m. The conducting wire of a cross-sectional area of 25 mm^2 carries a current of 50 A. What is the voltage drop across the line, if the wire is of copper; aluminium?

12.12. In the manufacturing of electrolytic copper for cathode a plate with a working surface area of 80 dm^2 is used. If the circuit carries a current of 160 A, what is the current density through the cathode?

12.13. A wire of cross-sectional area 1.2 mm^2 passes $6 \cdot 10^{18}$ electrons in 0.4 s. What is the current density?

12.14. There is a rheostat made of a nickeline wire of length 7.5 m that carries a current of 1.5 A/mm^2 when engaged in full. What is the voltage drop across the rheostat?

12.15. The voltage across a 20 Ω circuit steadily increases from 0 to 30 V. Plot the curve of the current variation.

12.16. The heating element of an electric furnace is made of constantan wire of 0.80 mm diameter and 24.2 m length. Determine the resistance of the element.

12.17. A rheostat of 6.0 Ω capacity is made of nickeline wire of 0.80 mm diameter. What is the length of the wire? If the current is 1.5 A, what is the voltage drop across the rheostat engaged in full?

12.18. A Nichrome wire of a resistance of 24 Ω has a length of 4.8 m. Compute the diameter of the wire.

12.19. Determine the resistance and length of a nickeline wire of mass 88 g and cross-sectional area 0.50 mm^2.

12.20. Calculate the weight amount of copper needed to produce a piece of wire with a resistance of 1.72 Ω and cross-sectional area of 0.500 mm^2.

12.21. What quantity of copper is required for a wire 5.0 km long so that its resistance be 5.0 Ω?

12.22. A quantity of 4.45 kg of copper is used to manufacture a bundle of wire of resistance 16.8 Ω. How many metres of wire are there in the bundle? What is its cross-sectional area?

12.23. An electric wiring is made of a copper wire of a length of 200.0 m and cross-section of 10 mm^2. What is its resistance? What cross-section is needed for an aluminium wire to have the same resistance?

12.24. Find the ratio of resistances of (a) two iron wires of the same weight such that the diameter of one is twice that of the other, (b) two conductors—copper and aluminium—of the same weight and diameter.

12.25. How many turns of manganin wire of cross-sectional area 0.70 mm^2 are needed to be wound on a cylindrical form of 2.0 cm diameter to obtain a resistance of 1.0 Ω?

12.26. Consider a wire of Fechral (cheap substitute of Nichrome) with a cross-sectional area of 0.50 mm^2, length of 2.5 m, and resistance of 5.47 Ω. What is the resistivity of Fechral? How many metres of the wire is needed to produce an electric heater drawing 3.0 A at 220 V?

12.27. Draw a circuit diagram for the lighting of a long through corridor such that a man entering at any end could turn on the lighting, and going out through another door turn off the light.

12.28. A rheostat has 80 turns of nickeline wire of 0.8 mm diameter. The diameter of a turn is 3 cm. What is the resistance of the rheostat? What is the length of the wire?

12.29. The starting current through an incandescent lamp is higher than the operating current. Explain why.

12.30. Explain why rail joints of electrified railways are connected by thick connectors of copper wire.

12.31. A Nichrome insulated wire is wound on a reel. The wire cross-sectional area is 0.55 mm^2. If the coil draws a current of 1.2 A when connected to a 120 V line voltage, compute the length of the wire without unwinding it from the reel.

12.32. When a conductor of 0.50 mm diameter and 4.5 m length is connected to a mains, the potential difference across it is 1.2 V at a current of 1.0 A. What is the resistivity of the material of the conductor?

12.33. A rheostat is made of a nickeline wire of length 15 m and cross-sectional area 1.0 mm^2. What is the resistance capacity of the rheostat? What current will the rheostat draw if a voltage of 12 V is applied to it when used to capacity?

12.34. A tungsten filament of an incandescent lamp has a resistance of 484 Ω at 2 100°C. Determine the resistance of the filament at 20°C.

12.35. A Fechral heating element has a resistance of 15 Ω

at 18°C. At what temperature will its resistance be 15.3 Ω?

12.36. The resistance of tungsten filament of a lamp is 20 Ω at 20°C. The resistance of the lamp under operating conditions is 188 Ω. What is the temperature of the filament?

12.37. The resistance of a rheostat at 20°C is 15 Ω. The rheostat winding is made of rheotan (zinc-copper-manganese alloy). What is the rise in the rheostat resistance if the latter is heated to a temperature of 100°C?

12.38. A lamp takes 0.4 A at 120 V. If the resistance of the tungsten filament at 0°C is 30 Ω, what is its temperature under operating conditions?

12.39. A flash-light lamp is marked: 3.5 V, 0.28 A. The filament temperature is 425°C, the filament resistance when cold is 4 Ω. What is the temperature coefficient of resistance of the material of the filament?

12.40. The resistance of a coal rod drops from 5.0 to 4.5 Ω as the temperature increases from 50 to 545°C. What is the temperature coefficient of resistance for coal? What is the meaning of the negative sign in the answer?

12.41. As an electric current is passed through an iron wire, its temperature increases by 250°C, and the resistance doubles. What is the temperature coefficient of resistance for iron?

12.42. The resistance of a lamp filament at 0°C is ten times lower than at 1 900°C. What is the temperature coefficient of resistance for the material of the filament?

12.43. A constantan wire designed for thermocouples has a mass of 89 g and cross-sectional area of 0.10 mm². What is the resistance of the wire at 100°C?

12.44. Three conductors with resistances 10, 20, and 30 Ω are connected in series, and a d.c. voltage of 120 V is applied to them. Determine the total resistance and the voltage drop across each conductor.

12.45. Three resistors (4, 6, and 10 Ω) connected in series draw a current of 5 A. What is the voltage in the mains and what is the voltage drop across each resistor?

12.46. There is a circuit as shown in Fig. 12.46. What is the resistance of the rheostat and the lamp, if the maximum and minimum currents in the circuit are 2.5 and 1.5 A? The voltage across the terminals *A* and *B* is constant and equal to 12 V.

12.47. A constant voltage source supplies three conductors with resistances 10, 16, and 18 Ω connected in series. If the voltage drop across the second conductor is 80 V, determine the equivalent resistance, current, voltage drop across the first and third conductors, voltage across the terminals of the source.

12.48. A conductor carries a current of 1.5 A at a voltage of 120 V. When an additional resistance is hooked into the circuit, the current becomes 1.2 A at the same voltage. Determine the additional resistance.

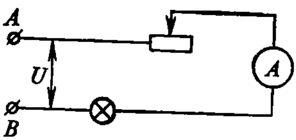


Fig. 12.46

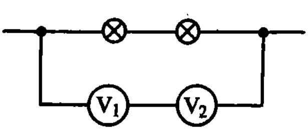


Fig. 12.53

12.49. An arc lamp of resistance 5.0 Ω is connected in series with a 7.5 Ω rheostat engaged in full. The voltage at the generator terminals is 127 V, the wiring is made of a copper wire of length 20 m and cross-sectional area 18 mm^2. What is the current through the lamp?

12.50. A projection lamp rated at 110 V and current 3.0 A is supplied by a 127 V line voltage through a rheostat. The voltage drop in the conducting copper wires accounts for 2 per cent of the voltage in the mains. The wire has a cross-sectional area of 1.8 mm^2. What is the resistance of the rheostat that is fully on? What is the length of the double copper conducting wire?

12.51. A 220 V mains supplies 10 lamps with resistance 24 Ω each and rated at 12 V. The excessive voltage is absorbed by a rheostat. If the rheostat is thrown in fully, what is the current in the circuit and the resistance of the rheostat?

12.52. A Christmas-tree is decorated with a string of 20 series-connected lamps of 19 Ω resistance each. The resistance of conducting wires is 1 Ω. Determine the current through the lamps and the voltage drop across the conducting wires, if the mains voltage is 127 V.

12.53. Two series-connected lamps are placed in circuit with voltmeters as shown in Fig. 12.53. The first reads 6.0 V,

the second 20 V. The resistance of the first voltmeter is 4 kΩ. What is the resistance of the second voltmeter?

12.54. What additional resistance is needed to be applied to a voltmeter with a resistance of 1 500 Ω for the value of a division to increase fivefold?

12.55. An additional resistance of 9 kΩ is placed in series with a voltmeter of resistance 1 000 Ω. By how many times will the voltmeter range increase?

12.56. A voltmeter measuring up to 20 V is to be connected to a 120 V line voltage. If the current through the voltmeter must not exceed 5 mA, what is the additional resistance required for this purpose?

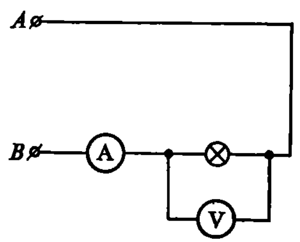


Fig. 12.58

12.57. A voltmeter designed to measure up to 30 V and equipped with a scale having 150 divisions is used to measure the potential difference up to 75 V. To this end a 3 kΩ resistor is placed in series with the instrument. Determine the scale division value in both cases. What is the internal resistance of the voltmeter?

12.58. A school voltmeter connected in parallel with a flash-light lamp (Fig. 12.58) with resistance 12 Ω reads 3.6 V. The voltmeter resistance is 60 Ω. What will the ammeter read?

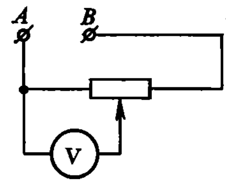


Fig. 12.59

12.59. To a potentiometer of resistance 6 000 Ω a voltage of 120 V is applied. Between the contact arm and one end of the potentiometer a voltmeter is included (Fig. 12.59) with a resistance of 10 kΩ. What will the voltmeter read when the contact arm is midway between the ends of the potentiometer?

12.60. To measure voltage of the mains a voltmeter is connected to the latter with a resistance of 450 Ω. If an additional resistance is inserted in series with the voltmeter, it will read 100 V; if another additional resistance is included that is by 60 Ω higher than the first, the voltmeter will read

90 V. Compute the mains voltage and additional resistances.

12.61. An electric motor is separated from a generator by a distance of 1 570 m and draws 15 A at 220 V. What is the weight of copper wire of 5.0 mm diameter needed for conducting wiring, and what must the voltage across the generator terminals be? Would such a wiring do, considering that the voltage drop in power lines should not be higher than 10 per cent of the generator output voltage?

12.62. There is a conductor with resistance of 24 Ω. Determine the additional resistance and way of connection required to obtain a resultant resistance of 20 Ω.

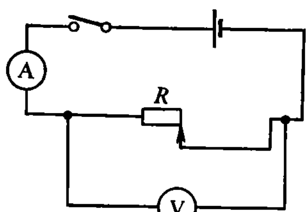


Fig. 12.63

12.63. Fig. 12.63 shows a circuit with the ammeter reading 0.30 A, and a voltmeter reading 4.0 V. The voltmeter resistance is 80 Ω. What is the resistance of the rheostat?

12.64. Into how many equal parts is it necessary to cut a 64 Ω conductor so that to obtain a resistance of 1 Ω by connecting these parts in parallel?

12.65. Can you think of a way of connecting four conductors of 4 Ω each so that the equivalent resistance remained the same as with the one conductor?

12.66. Four conductors with resistances 1.5 Ω each are to be connected so that to obtain a resistance of 2 Ω. How will you proceed?

12.67.* Consider the circuit of Fig. 12.67. If $U = 110$ V, what are the equivalent resistance of the circuit, and the ammeter reading?

12.68. There are two conductors. Their connection in series yields 20 Ω, and in parallel 5 Ω. What are the resistances of each conductor?

12.69. Consider the circuit of Fig. 12.69. The ammeter indicates 4 A. Determine the total resistance of the circuit and the voltage between points A and B.

---

* Throughout the section resistances in the figures are in ohms.

12.70. The equivalent resistance of three parallel users is 30 Ω. Their resistances are in the ratio 1 : 3 : 5. Determine the resistances of the users.

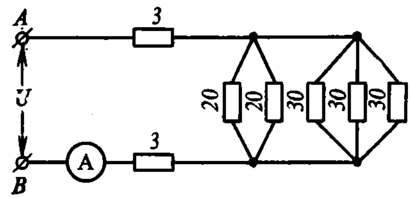


Fig. 12.67

12.71. There are six conductors with resistances 1, 2, 2, 4, 5, and 6 Ω. Connect them so that their total resistance be 1 Ω.

12.72. Two conductors when connected in series give 27 Ω, and in parallel 6 Ω. Determine their resistances.

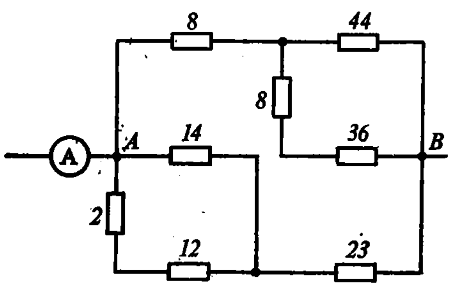


Fig. 12.69

12.73. Four identical resistors 10 Ω each are connected as shown in Fig. 12.73. What will the total resistance be, if the current is supplied to points A and C? To points A and D?

12.74. Determine the resistance R measured with a Wheatstone bridge (Fig. 12.74), if at $R_1 = 1.5$ Ω, $l_1 = 20$ cm, $l_2 = 80$ cm there is no current through the galvanometer.

12.75. Consider the circuit of Fig. 12.75. If the voltage across terminals A and B is 12 V, find the total resistance and currents in the resistors.

12.76. Four resistors are connected as shown in Fig. 12.76. The voltage between points A and B is 18 V. Determine the total resistance and currents in individual conductors.

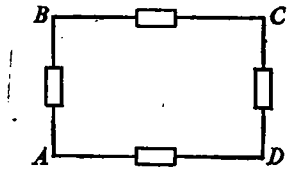


Fig. 12.73

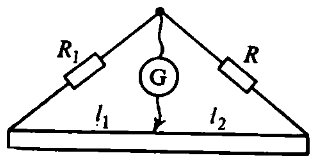


Fig. 12.74

12.77. Consider 50 lamps of resistance $2.4 \cdot 10^2$ Ω each connected in parallel to a $1.2 \cdot 10^2$ V line voltage. Determine the total current through the lamps and voltage of the main line, if the wiring from the main line to the user has a resistance of 0.280 Ω.

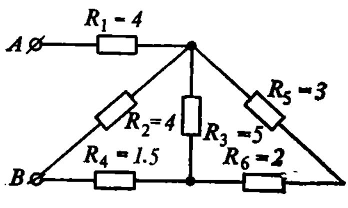


Fig. 12.75

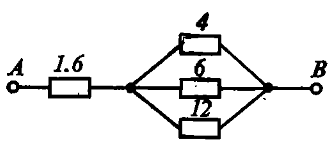


Fig. 12.76

12.78. A 127 V mains feeds 60 lamps of resistance 220 Ω each. The resistance of conducting wires is 0.20 Ω. Determine the total current through the lamps and the voltage drop across the wires.

12.79. Two groups of lamps are connected in parallel to a 220 V mains. One of the groups includes 8 lamps with resistance $1.6 \cdot 10^2$ Ω each, and the other has 10 lamps of resistance $2.0 \cdot 10^2$ Ω each. Compute the total resistance and the total current.

12.80. A generator supplying 140 V is rated at a current of 50 A. It feeds a group of normally glowing lamps with resistance 140 Ω each. The conducting wires have a resistance

of 0.30 Ω. How many lamps connected in parallel can the generator supply? What is the operating voltage across a lamp?

12.81. Referring to the circuit of Fig. 12.81, the voltage between the points A and B is 110 V. Calculate the currents

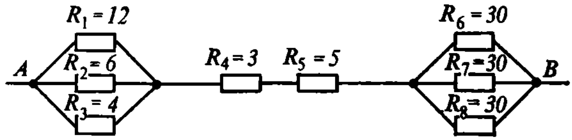


Fig. 12.81

across the third, fifth, and sixth conductors and the distribution of voltages.

12.82. Consider a circuit of Fig. 12.82 that is composed of 9 identical resistors r. The resistance of the whole circuit is 1.5 Ω. Determine the resistance r.

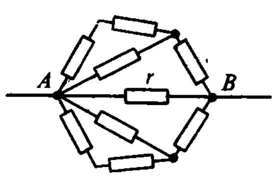


Fig. 12.82

12.83. An ammeter with a 0.04 Ω shunt is connected into a main line. It reads 5 A. The ammeter resistance is 0.12 Ω. What is the current through the main line?

12.84. The sensitivity of a galvanometer of resistance 260 Ω is to be reduced tenfold. What shunt is needed for the purpose?

12.85. Currents up to 100 A are to be measured using an ammeter with resistance 0.9 Ω rated at a maximum current of 10 A. What is the length of an iron conductor with a cross-sectional area of 0.28 mm² to be used as a shunt?

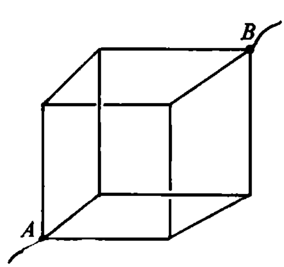


Fig. 12.86

12.86. A frame shown in Fig. 12.86 has a side with resistance 6 Ω. The conducting wire carries a current of 2 A. What is the voltage across points A and B? What is the resistance of the cube?

12.87. A generator whose e.m.f. is 120 V and internal resistance 3.0 Ω supplies a 21 Ω heater. Calculate the current through the circuit and the voltage drop at the generator.

12.88. To measure the e.m.f. and the internal resistance of a Leclanché cell a student has assembled a circuit shown in Fig. 12.88. At a current of 0.20 A the voltmeter reads 1.45 V, and at 0.60 A, 1.25 V. What are the e.m.f. and the internal resistance of the cell?

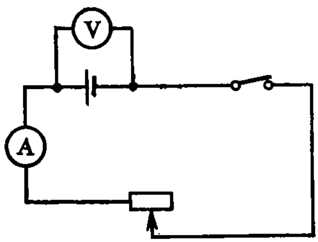


Fig. 12.88

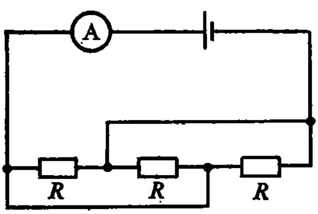


Fig. 12.93

12.89. A lead-acid battery with an e.m.f. of 2.0 V, when connected across an external resistance of 4.8 Ω, gives a current of 0.40 A. Determine the internal resistance of the battery and the voltage across its terminals.

12.90. An electric energy source of e.m.f. 1.5 V and internal resistance 0.50 Ω is connected across a resistance. If the current through the circuit is 0.60 A, what is the resistance and the voltage drop across it?

12.91. An alkaline battery yields a current of 0.8 A, if connected across a resistance of 1.5 Ω. When connected across a resistance of 3.25 Ω the battery gives a current of 0.4 A. Determine the electromotive force and internal resistance of the battery.

12.92. The internal resistance of a generator is 0.6 Ω. When connected across an external resistance of 6.0 Ω, the voltage across its terminals becomes 120 V. What is the current through the circuit and the e.m.f. of the generator?

12.93. Fig. 12.93 presents a load consisting of three identical resistances connected to an electric energy source of e.m.f. 12 V, and internal resistance 0.6 Ω. The ammeter reads 2 A. Calculate the magnitude of each resistance.

12.94. An automobile battery has an e.m.f. of 12 V and an internal resistance of 0.0050 Ω. If the resistance of the starter and conducting wires is 0.07 Ω, what is the initial current through the starter, and the voltage across the battery terminals?

12.95. How will the voltage across the terminals of a source of electric energy vary with increasing current through the circuit?

12.96. There is a source of electric energy of e.m.f. 1.45 V and internal resistance 0.40 Ω at a current of 2.0 A. What is the efficiency of the source?

12.97. Consider a galvanic cell with internal resistance 0.60 Ω to which an external circuit with a resistance of 4.0 Ω is connected. What is the efficiency of the cell?

12.98. A source of electric energy with an internal resistance of 0.250 Ω is short-circuited with an iron conductor of length 5.00 m and cross-sectional area 0.200 mm^2 causing a current of 0.500 A to flow through the circuit. What is the electromotive force of the source?

12.99. A source of electric energy has an electromotive force of 2 V and an internal resistance of 1.5 Ω. The current through the circuit being 0.5 A, what is the external resistance of the circuit, and what is the voltage drop inside the source?

12.100. A source of electric energy of internal resistance 0.50 Ω is short-circuited with a nickeline wire of length 12.5 m and cross-sectional area 0.5 mm^2. Determine the current through the circuit and the e.m.f. of the source, if the voltage across its terminals is 5.25 V.

12.101. A cell of e.m.f. 1.5 V and internal resistance 0.20 Ω is short-circuited with a 5.0 m iron conductor. If the current is 0.60 A, what must the diameter of the iron conductor be?

12.102. The electromotive force of a flash-light battery is 4.5 V. At an external resistance of 12 Ω the current through the circuit is 0.3 A. Calculate the internal resistance of the battery and the voltage drop inside it?

12.103. A generator of e.m.f. 132 V and internal resistance 0.4 Ω supplies 50 parallel-connected lamps with a resistance of 180 Ω each. Ignoring the resistance of conducting wires, what is the current through the circuit? By how many times

will the current change, if the load is increased twofold?

12.104. A voltmeter connected across the terminals of a 24 V source reads 18 V. If the resistance of the external circuit is 9 Ω, what is the current through the circuit and the resistance of the source?

12.105. Under what conditions does the voltage across the terminals of an electric energy source account for 50 per cent of its electromotive force? What is the voltage across the terminals of the source when short-circuited?

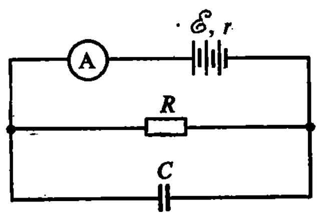


Fig. 12.106

12.106. Fig. 12.106 depicts a circuit including a battery with an electromotive force of 6.0 V and internal resistance of 1.2 Ω. The battery causes a current of 1.0 A to flow through the circuit. If the separation between the plates of the flat capacitor is 0.16 cm, what is the resistance R and the electric field strength within the capacitor?

12.107. A capacitor of 0.3 μF and a resistor of 5 Ω are placed in parallel and connected to an electric energy source with an e.m.f. of 12 V and an internal resistance of 1 Ω (see Fig. 12.106). What is the charge stored by the capacitor?

12.108. A rheostat made of 0.6 mm² constantan wire is connected across the terminals of a source of 1.80 V and internal resistance 0.250 Ω. The voltage drop across the fully engaged rheostat is 1.65 V. Calculate the resistance of the rheostat and the current through it. Determine the length of the wire used to make the rheostat.

12.109. A 4.5 V flash-light battery gives a current of 0.5 A, if connected across a resistance of 7.5 Ω. What is the short-circuit current?

12.110. A galvanic cell provides a current of 0.30 A when connected across a resistance of 6.0 Ω, and 0.15 A across a resistance of 14 Ω. What is the short-circuit current?

12.111. In determining the electromotive force of a battery use can be made of a standard cell. If the battery is connected in series with the standard cell with an e.m.f. of 2 V, the circuit takes a current of 0.3 A. When the battery

is connected into the same circuit in opposition to the standard cell the current through the external circuit becomes 0.1 A, its direction corresponding to that from the positive pole of the battery. What is the electromotive force of the battery?

12.112. The internal resistance of a generator is 0.20 Ω, the voltage across its terminals is 110 V. The circuit includes 100 parallel-connected lamps with a resistance of 400 Ω each. Ignoring the resistance of the conducting wires, compute the electromotive force of the generator.

12.113. A battery with internal resistance 0.20 Ω and e.m.f. 2 V is short-circuited with a 5.0 m wire having a resistivity of $0.10 \cdot 10^{-6}$ Ω·m. If the current is 5 A, what is the cross-sectional area of the wire?

12.114. A battery with an internal resistance of 0.20 Ω supplies ten parallel-connected lamps of resistance 250 Ω each. The conducting wires are made of copper, they have a length of 2.2 m and a cross-sectional area of 0.40 mm². The current through each lamp is 0.50 A. What is the electromotive force of the battery?

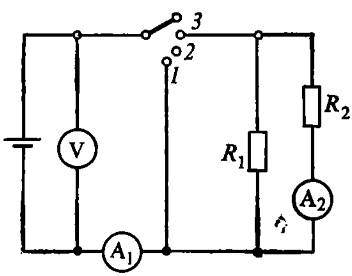


Fig. 12.115

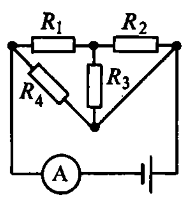


Fig. 12.116

12.115. Consider the circuit of Fig. 12.115. The e.m.f. of the source is 6.0 V, its internal resistance is 1.2 Ω, $R_1 = 8.0$ Ω and $R_2 = 4.8$ Ω. Ignoring the resistance of the wires and ammeters and considering the resistance of the voltmeter very large, what will the ammeters and the voltmeter read at positions *1*, *2*, *3* of the switch?

12.116. A circuit is arranged as shown in Fig. 12.116, where $R_1 = 2.5$ Ω, $R_2 = 2.0$ Ω, $R_3 = 6.0$ Ω, $R_4 = 14$ Ω,

$\mathscr{E} = 5.6$ V. What will the ammeter read? Ignore the internal resistance of the current source.

12.117. A number of parallel-connected 110 V lamps are fed by a 130 V battery having an internal resistance of 2.6 Ω. If the resistance of each lamp is 200 Ω and the resistance of conducting wires is 0.40 Ω, how many lamps can the battery supply?

12.118. When a voltmeter with a resistance of 90 Ω is connected across the terminals of an energy source, the former reads 36 V. If the voltmeter is substituted by another one with resistance 190 Ω, it reads 38 V. Find the electromotive force and the internal resistance of the energy source.

12.119. If two lamps of 8.0 Ω resistance are connected in series to a battery, a voltmeter connected to the poles will read 4.0 V. But if the same lamps are placed in parallel, the voltmeter will indicate 3.0 V. What are the electromotive force and the internal resistance of the battery?

12.120. A 12 V source with internal resistance 1.0 Ω gives a current of 0.80 A when short-circuited with a nickeline wire of 0.50 mm diameter. Determine the length of the wire and the voltage across the terminals of the source.

12.121. A voltmeter connected to the terminals of a galvanic cell reads 1.2 B at 0.40 A, and 1.0 V at 0.80 A. Calculate the e.m.f., internal resistance of the cell, and the maximum current that can be drawn from it.

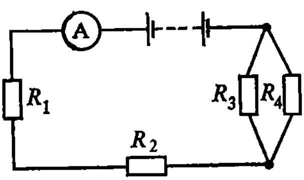


Fig. 12.122

12.122. A battery of electromotive force 12.4 V and internal resistance 0.2 Ω is connected across the circuit of Fig. 12.122. The ammeter reads 2.0 A at $R_1 = 2.9$ Ω, $R_2 = 1.6$ Ω and $R_3 = 6.0$ Ω. Compute the resistance R_4, the current through it, and the voltage across the terminals of the battery.

12.123. A 130 V generator with internal resistance 1.8 Ω supplies several parallel-connected lamps with a total resistance of 24 Ω. The conducting wires have a resistance of 0.2 Ω. Determine the current through the circuit, voltage

across the lamps, voltage drop across the conducting wires, and voltage across the terminals of the generator.

12.124. A voltmeter with a resistance of $1.0 \cdot 10^2$ Ω is connected across a 150 V source with internal resistance 4.00 Ω. What will the voltmeter read?

12.125. If a 1.44 V cell with an internal resistance of 0.20 Ω is short-circuited through an ammeter, the latter will read 4.8 A. What will it read, if supplied with a 0.15 Ω shunt?

12.126. The electromotive force of a battery is 28 V, its internal resistance is 0.80 Ω. When a lamp is connected across the battery the voltage across its terminals becomes 24 V. Determine the current through the lamp and its resistance?

12.127. A flat air capacitor with 225 cm² square plates is being immersed in a liquid dielectric at a rate of 0.6 m/s so that the plates are perpendicular to the liquid level. The capacitor is connected across a current source with an electromotive force of 200 V. The plates are 1.5 mm part, the dielectric constant of the medium is 39. Ignoring the internal resistance of the source, what is the current flowing through the conducting wires?

12.128. A flash-light battery consists of three series-connected elements, each of which has an e.m.f. of 1.5 V and internal resistance of 0.20 Ω. It supplies a lamp of resistance 11.4 Ω. What is the current flowing in the circuit and the voltage across the lamp?

12.129. The circuit of Fig. 12.129 has a battery consisting of three parallel-connected 1.44 V cells with an internal resistance of 0.60 Ω each. If $R_1 = R_2 = 1.2$ Ω, $R_3 = 2.0$ Ω, $R_4 = 3.0$ Ω, what is the current through the circuit and through the resistance R_3?

12.130. A nickeline conductor is connected across the terminals of a battery including three series-connected 2.0 V cells with an internal resistance of 0.040 Ω each. If the current through the circuit is 1.5 A and the cross-sectional area of the conductor is 0.20 mm², what is the resistance and length of the conductor?

12.131. The circuit of Fig. 12.131 is supplied by four alkaline batteries with 1.4 V e.m.f. each, and internal resistance 0.20 Ω each, which are series-connected into a battery,

$R_1 = 0.90\ \Omega$, $R_2 = R_3 = 0.60\ \Omega$. Find the distribution of currents and voltages in the external circuit.

12.132. There are three 2 V batteries with internal resistance 0.20 Ω each. Can you think of a way of connecting the batteries so that when connected across a resistance of 0.60 Ω they give a maximum current?

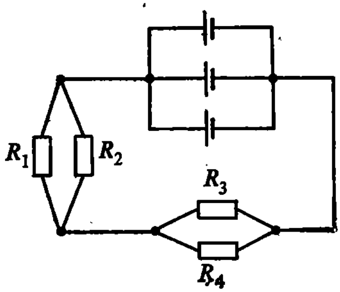


Fig. 12.129

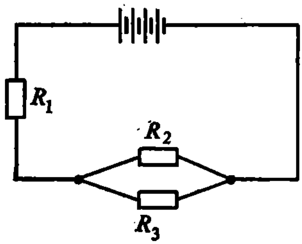


Fig. 12.131

12.133. Two 1.45 V cells with internal resistance 0.30 Ω each supply a resistance of 13.9 Ω. What connection of the cells results in a maximum current?

12.134. Three identical series-connected cells supply a resistance of 1.5 Ω with a current of 2 A. At parallel connection of the cells a current of 0.9 A flows through the same resistance. Compute the electromotive force and internal resistance of each cell.

12.135. How many 2.1 V cells with internal resistance 0.20 Ω each are to be connected in series to obtain a battery so that a conductor of resistance 6.0 Ω would carry a current of 1.5 A?

12.136. Six 1.1 V cells with internal resistance 3.0 Ω each are connected in three parallel branches with two cells apiece. If the resistance of external circuit is 2.0 Ω, determine the current.

12.137. A battery of three series-connected 2 V cells with 0.25 Ω internal resistance each supplies an external circuit consisting of two parallel-connected conductors with resistances of 3.0 and 9.0 Ω. Determine the potential difference across the battery terminal and currents carried by the conductors.

12.138. A battery with internal resistance 0.3 Ω and residual e.m.f. 11.1 V is being charged by a 15 V constant voltage source. If the charging requires an additional resistance of 1 Ω, what is the charging current?

12.139. A 11.2 V battery with an internal resistance of 0.3 Ω is being charged with a current of 4 A. What does a voltmeter connected across the terminals read?

12.140. By the end of the charging of a battery with a current of 4 A, a voltmeter connected across its terminals indicates 2.16 V. At the beginning of the discharging of the battery with a current of 5 A the voltmeter reads 1.8 V. What are the e.m.f. and internal resistance of the battery?

12.141. A battery of 50 cells connected in series is being charged by a 120 V constant voltage source. What additional resistance is to be introduced into the circuit, if the e.m.f. of the battery is 1.85 V, the internal resistance is 0.02 Ω, and the charging is done at 11 A?

12.142. A battery of 12 cells of 44.4 V placed in series is being charged by a constant voltage source. At the beginning of the charging the current is 6 A, and at the end 4 A; the e.m.f. at the end of the charging is 48 V. Determine the internal resistance of the battery and one cell, and the voltage of charging if the voltage is taken to be constant.

12.143. Two 1.5 V cells with internal resistances 3 and 2 Ω are in parallel and connected across an external resistance. What would the external resistance be for the potential difference across the terminals of the first cell to be zero?

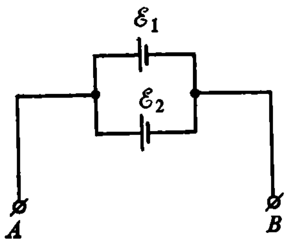


Fig. 12.144

12.144. The circuit of Fig. 12.144 consists of two parallel-connected cells whose electromotive forces are 1.44 and 1.1 V and the internal resistances 0.48 and 0.20 Ω. What is the voltage across the terminals of the battery?

12.145. Two cells with electromotive forces 1.7 and 1.4 V and internal resistances 0.80 Ω and 0.40 Ω are placed in series and connected across a resistance of 5.0 Ω. Calculate the current flowing through the circuit, the voltage across the

external section of the circuit, and across the terminals of each cell.

12.146. Two sources (6.5 V and 3.9 V) with equal internal resistance (2 Ω) are in parallel and connected across an external circuit with resistance 9.0 Ω. Determine the currents flowing through the cells and external circuit.

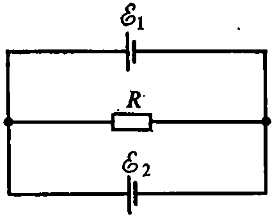


Fig. 12.147

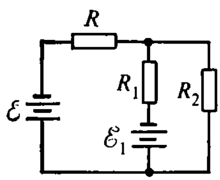


Fig. 12.148

12.147. Two sources with electromotive forces of 2.0 and 1.8 V and internal resistances of 0.50 and 0.30 Ω are connected as shown in Fig. 12.147 to supply a resistance of $R = 3.0$ Ω. Find the currents through the sources and through resistance R.

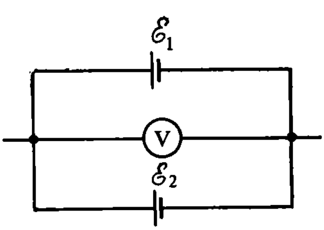


Fig. 12.149

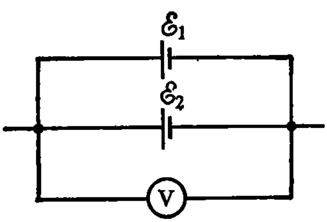


Fig. 12.150

12.148. Fig. 12.148 shows a circuit with known resistances R_1 and R_2. Neglecting the resistance of conducting wires and internal resistances of the current sources, determine the magnitude of the electromotive force $\mathscr{E}_1$ such that the current I through the resistance R be zero.

12.149. Two cells with electromotive forces 1.3 and 1.5 V are arranged as shown in Fig. 12.149. The voltmeter reads 1.45 V. What cell has the higher internal resistance

and by how many times? The voltmeter resistance is taken to be very large.

12.150. Two cells with e.m.f. 1.8 and 2 V and internal resistances 0.6 and 0.4 Ω are placed as shown in Fig. 12.150. What will the reading of the voltmeter be, if its resistance is many times higher than the internal resistances of the cells?

12.151. Several identical cells are connected as shown in Fig. 12.151. What is the potential difference between points A and B? A and C?

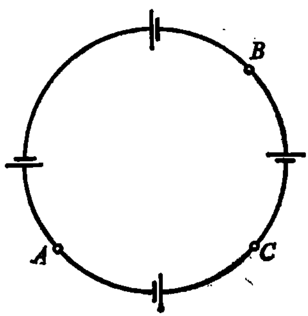


Fig. 12.151

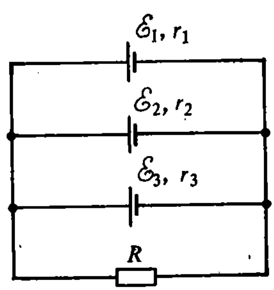


Fig. 12.152

12.152. In the circuit of Fig. 12.152 $\mathscr{E}_1 = 1.8$ V, $\mathscr{E}_2 = 1.7$ V, $\mathscr{E}_3 = 1.5$ V, $r_1 = 0.2\ \Omega$, $r_2 = r_3 = 0.1\ \Omega$. Compute the resistance R and currents flowing through all the segments, given that the current through the third cell is zero.

SEC. 13. WORK, POWER, HEATING EFFECT OF CURRENT

Example 40. A battery with residual e.m.f. 10.2 V and internal resistance 0.90 Ω is being charged with a 14 V source as given in Fig. 40. What additional resistance is to be connected in series with the battery so that the charging current be not higher than 2.0 A? What is the amount of heat liberated within the battery in 20 min, and the amount of chemical energy stored?

Given: $\mathscr{E} = 10.2$ V is the residual e.m.f. of the battery, $r = 0.90\ \Omega$ is the internal resistance of the battery, $U = 14$ V is the voltage of the source of electric energy, $I = 2.0$ A is the charging current, $t = 1\,200$ s is the time period.

Determine: R—the additional resistance, Q—the amount of heat liberated within the battery, W_{ch}—the chemical energy stored.

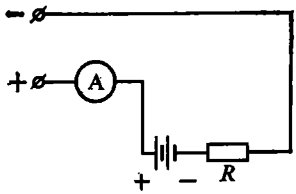


Fig. 40

Solution. The additional resistance may be found from Ohm's law for a subcircuit

$$I = \frac{U - \mathscr{E}}{R + r}$$

Hence

$$R = \frac{U - \mathscr{E} - Ir}{I}$$

The chemical energy stored is obtained from the difference between the energy used up in the battery $A = IU_{bat}\,t$, and the energy that went in heating the battery $Q = I^2rt$

$$W_{ch} = A - Q = IU_{bat}t - I^2rt = I\,(U - IR)t - Q$$

Substituting the data of the problem we get

$$R = \frac{14\ \text{V} - 10.2\ \text{V} - 2.0\ \text{A} \cdot 0.90\ \Omega}{2.0\ \text{A}} = 1\ \Omega$$

$$Q = 4\ \text{A}^2 \cdot 0.90\ \Omega \cdot 1\,200\ \text{s} = 4\,300\ \text{J}$$

$$W_{ch} = 2.0\ \text{A} \cdot (14\ \text{V} - 2.0\ \text{A} \cdot 1\ \Omega) \cdot 1\,200\ \text{s} - 4\,300\ \text{J} = 24{,}500\ \text{J}$$

Answer. The additional resistance must be 1 Ω. During the charging 4.3 kJ of heat has been liberated in the battery, and 24.5 kJ of chemical energy has been stored.

Example 41. A room is lighted by 200 W, 127 V incandescent lamps fed by a generator whose output voltage is 133 V.

The conducting wires from the generator to the user are made of aluminium wire of total length 150 m and cross-sectional area 15 mm². How many such lamps can be installed? What is the total power consumed by the user?

Given: $P_1 = 200$ W is the power of one lamp, $U_1 = 127$ V is the voltage across a lamp, $U = 133$ V is the voltage at the terminal of the generator, $l = 150$ m is the length of conducting wiring, $S = 15\ \text{mm}^2 = 15\cdot10^{-6}\ \text{m}^2$ is the cross-sectional area of wires, $\rho = 2.9\cdot10^{-8}\ \Omega\cdot\text{m}$ is the resistivity of aluminium.

Determine: n—the number of lamps,
P—the power consumed by the user.

Solution. The number of lamps that may be included into this circuit is obtained by dividing the current I flowing through the main line, by the current I_1 through one lamp

$$n = I/I_1$$

Having obtained from the relation $R = \rho l/S$ the resistance of conducting wires, we compute the current through the main line

$$I = (U - U_1)/R$$

where $(U - U_1)$ is the voltage drop across the wires. The current flowing through the lamp may be found from the relationship $I_1 = P_1/U_1$, then

$$n = \frac{I}{I_1} = \frac{(U-U_1)/R}{P_1/U_1} = \frac{(U-U_1)\,SU_1}{\rho l P_1}$$

Next we find the power consumed by the user

$$P = P_1 n$$

Substitution gives

$$n = \frac{(133\ \text{V} - 127\ \text{V})\cdot15\cdot10^{-6}\ \text{m}^2\cdot127\ \text{V}}{2.9\cdot10^{-8}\ \Omega\cdot\text{m}\cdot150\ \text{m}\cdot200\ \text{W}} = 13$$

$$P = 200\ \text{W}\cdot13 = 2\,600\ \text{W} = 2.6\ \text{kW}$$

Answer. The circuit can include 13 lamps of total power 2.6 kW.

Example 42. A 400.0 W electric stove is connected in parallel with a 100.0 W lamp. The mains voltage is 127 V. The above rated powers correspond to the voltage of 127 V. If

the resistance of conducting wires is 3.0 Ω, what is the voltage across the lamp before and after the connection of the stove?

Given: $P_1 = 100.0$ W is the power of the lamp, $P_2 = 400.0$ W is the power of the electric stove, $U = 127$ V is the mains voltage, $R_0 = 3.0$ Ω is the resistance of conducting wires.

Determine: U_1'—the voltage across the lamp before the stove is connected,
U_1''—the voltage across the lamp and stove after the latter is connected.

Solution. To solve the problem, it is required to compute the currents flowing through the circuit before and after the stove is connected. To this end, the total resistance of the circuit in both cases is required. Knowing the currents and resistances of the loads, we find the quantities required. From the relationship for power we determine the resistances of both loads:

$$R_1 = U^2/P_1 \qquad R_2 = U^2/P_2$$

We write the total resistance of the circuit at the different loads and the currents in both cases

$$R'_{\text{tot}} = R_0 + R_1 \qquad R''_{\text{tot}} = R_0 + \frac{R_1R_2}{R_1+R_2}$$

$$I' = U/R'_{\text{tot}} \qquad I'' = U/R''_{\text{tot}}$$

Substituting the data of the problem we arrive at the circuit resistances

$$R_1 = \frac{(127\ \text{V})^2}{100.0\ \text{W}} = 161\ \Omega$$

$$R'_{\text{tot}} = 3.0\ \Omega + 161\ \Omega = 164\ \Omega$$

$$R_2 = \frac{(127\ \text{V})^2}{400.0\ \text{W}} = 40.3\ \Omega$$

$$R''_{\text{tot}} = 3.0\ \Omega + \frac{161\ \Omega \cdot 40.3\ \Omega}{161\ \Omega + 40.3\ \Omega} \approx 35\ \Omega$$

Now let us calculate the currents

$$I' = \frac{127\ \text{V}}{164\ \Omega} = 0.7\ \text{A} \qquad I'' = \frac{127\ \text{V}}{35\ \Omega} = 3.6\ \text{A}$$

As in both cases the loads and wires are in series, the voltage of 127 V is distributed proportionately with the resistances of the loads and wires

$$U = U_1' + I'R_0 \qquad U = U_1'' + I''R_0$$

$$U_1' = 127\ \text{V} - 3.0\ \Omega \cdot 0.7\ \text{A} = 125\ \text{V}$$

$$U_1'' = 127\ \text{V} - 3.0\ \Omega \cdot 3.6\ \text{A} = 116\ \text{V}$$

Answer. After the electric stove has been connected in parallel to the lamp, the voltage across the lamp terminals has dropped from 125 to 116 V.

Example 43. A 60 W and 250 W lamps rated at 110 V each are connected in series to a 220 V line voltage. Find the distribution of voltage for the lamps. What is the power developed in each lamp? How much heat is liberated in 30 min by each lamp?

Given: U = 220 V is the voltage of supply line, $n = 2$ is the number of lamps, P_1 = 60 W is the power of the first lamp, P_2 = 250 W is the power of the second lamp, $U_1 = U_2 =$ = 110 V is the voltage (rated) across the lamps, $t =$ = 1 800 s is the time period.

Determine: U_1'—the voltage across the first lamp,
U_2'—the voltage across the second lamp,
P_1' and P_2'—the powers developed by each lamp when connected in series,
Q_1' and Q_2'—the amounts of heat liberated by the lamps.

Solution. The voltages across series-connected lamps are proportional to their resistances. Therefore, we first compute the resistances of the lamps

$$R_1 = U_1^2/P_1 \quad R_2 = U_1^2/P_2 \quad U_1'/U_2' = R_1/R_2$$

As $U_1' + U_2' = U$, then

$$\frac{U_1'}{U - U_1'} = \frac{R_1}{R_2}$$

By Ohm's law we obtain the current flowing through the lamps

$$I = \frac{U}{R_1 + R_2}$$

Substituting the numerical values we find the resistances of each lamp and voltages across each of them

$$R_1 = \frac{(110\ \text{V})^2}{60\ \text{W}} \approx 200\ \Omega \quad R_2 = \frac{(110\ \text{V})^2}{250\ \text{W}} \approx 48\ \Omega$$

$$\frac{U_1'}{220\ \text{V} - U_1'} \approx \frac{200\ \Omega}{48\ \Omega} \quad U_1' \approx 177\ \text{V} \quad U_2' \approx 43\ \text{V}$$

We now calculate the current flowing through the lamps

$$I \approx \frac{220\ \text{V}}{248\ \Omega} \approx 0.9\ \text{A}$$

Given the current through the lamps, their resistances, and the time of functioning, we calculate the powers developed by the lamps with series connection, and the amounts of heat liberated in them

$$P_1' = IU_1' \quad P_1' = 0.9\ \text{A} \cdot 177\ \text{V} \approx 159\ \text{W}$$

$$P_2' = IU_2' \quad P_2' = 0.9\ \text{A} \cdot 43\ \text{V} \approx 39\ \text{W}$$

$$Q_1' = IU_1't \quad Q_1' \approx 0.9\ \text{A} \cdot 177\ \text{V} \cdot 1\,800\ \text{s} \approx 2.9 \cdot 10^5\ \text{J}$$

$$Q_2' = IU_2't \quad Q_2' \approx 0.9\ \text{A} \cdot 43\ \text{V} \cdot 1\,800\ \text{s} \approx 0.7 \cdot 10^5\ \text{J}$$

Answer. The voltages across the lamps are $U_1' \approx 177$ V, $U_2' \approx 43$ V, respectively; the powers are $P_1' \approx 159$ W, $P_2' \approx 39$ W; the amounts of heat are $Q_1' \approx 29 \cdot 10^5$ J, $Q_2' \approx$ $\approx 0.7 \cdot 10^5$ J.

Example 44. A lead fuse in a circuit consisting of a copper wire with cross-sectional area 5 mm^2 melts, if the wire heats by 25 K. The initial temperature of the lead fuse is 293 K. What is the cross-sectional area of the fuse lead wire?

Given: $S_1 = 5\ \text{mm}^2 = 5.0 \cdot 10^{-6}\ \text{m}^2$ is the cross-sectional area of the copper wire, $\rho_1 = 1.68 \cdot 10^{-8}\ \Omega \cdot \text{m}$, $\rho_2 = 2.07 \times$ $\times\ 10^{-7}\ \Omega \cdot \text{m}$ are the resistivities of copper and lead, respectively, $c_1 = 3.8 \cdot 10^2$ J/(kg·C), $c_2 = 1.2 \cdot 10^2$ J/(kg·C) are specific heats of copper and lead, respectively, $D_1 = 8.9 \times$ $\times\ 10^3$ kg/m^3, $D_2 = 1.14 \cdot 10^4$ kg/m^3 are the densities of copper and lead, respectively, $T_1 = 293$ K is the initial temperature of the lead, $T_m = 600$ K is the melting point of lead, $\lambda = 2.5 \cdot 10^4$ J/kg is the specific heat of fusion for lead, $\Delta T = 25$ K is the rise in temperature of the copper wire.

Determine: S_2—the cross-sectional area of the lead wire of the fuse.

Solution. The amount of heat Q_1 given up by the copper wire can be written as

$$Q_1 = c_1 S_1 l_1 D_1 \Delta T$$

This amount of heat produced by the current flowing is

$$Q_1 = I^2 R_1 t = I^2 t \rho_1 l_1 / S_1$$

Equating the right-hand sides of the above relationships for Q_1, we obtain

$$I^2 t \rho_1 l_1 / S_1 = c_1 S_1 l_1 D_1 \Delta T$$

Reasoning along the same lines we write an expression for Q_2, i.e. the amount of heat evolved by the current flowing through the lead conductor:

$$Q_2 = I^2 R_2 t = I^2 t \rho_2 l_2 / S_2$$

This amount of heat is used to heat the lead conductor to the melting point, and to melt it

$$Q_2 = S_2 l_2 D_2 [c_2 (T_m - T_1) + \lambda]$$

Equating the right-hand sides of the relations gives

$$I^2 t \rho_2 l_2 / S_2 = S_2 l_2 D_2 [c_2 (T_m - T_1) + \lambda]$$

Considering that the current and the duration of the process are the same in both cases, we eliminate these quantities by dividing Q_1 by Q_2

$$\frac{\rho_1 S_2}{\rho_2 S_1} = \frac{c S_1 D_1 \Delta T}{S_2 D_2 [c_2 (T_m - T_1) + \lambda]}$$

The expression obtained allows the determination of the cross-sectional area S_2 of the lead wire

$$S_2 = S_1 \sqrt{\frac{\rho_2 c_1 D_1 \Delta T}{\rho_1 D_2 [c_2 (T_m - T_1) + \lambda]}}$$

Substituting the data we get

$$S_2 = 5 \cdot 10^{-6}\ \text{m}^2 \times \sqrt{\frac{2.07 \cdot 10^{-7}\ \Omega \cdot \text{m} \cdot 3.8 \cdot 10^2\ \text{J/(kg} \cdot \text{K)} \cdot 8.9 \cdot 10^3\ \text{kg/m}^3 \cdot 25\ \text{K}}{1.68 \cdot 10^{-8}\ \Omega \cdot \text{m} \cdot 1.14 \cdot 10^4\ \text{kg/m}^3\ [1.2 \cdot 10^2\ \text{J/(kg} \cdot \text{K)} \cdot 307\ \text{K} + 2.5 \cdot 10^4\ \text{J/kg}]}} = 2.5 \cdot 10^{-6}\ \text{m}^2$$

Answer. The cross-sectional area of the lead wire of the fuse is $2.5 \cdot 10^{-6}$ m^2, or 2.5 mm^2.

13.1. A 660 W electric stove is plugged into a 220 V line voltage. What is the electric energy liberated in 30 min? What is the current flowing through the circuit?

13.2. An automobile battery is rated at 54 A·h, 12 V. Calculate the energy stored.

13.3. Under working conditions the resistance of a lamp filament is 144 Ω, the nominal voltage is 120 V. Determine the current through the lamp, the power developed, and the energy used up in 10 hours.

13.4. The resistances of two lamps connected in parallel to a 120 V supply line are in the ratio 3 : 2. The current flowing through the first lamp is 0.40 A. Determine the powers developed by the lamps and their resistances under working conditions.

13.5. An arc welding uses a current of 500 A at a voltage of 40 V. Compute the power and the energy used up in 30 min of operations.

13.6.* A 100 W lamp is connected to a mains of 220 V. Determine the filament resistance under working conditions, the current through the lamp and the monthly energy consumption if the lamp is lighted every day for 5 hours.

13.7.* A heating device of power 600 W is used on the average 3 hours a day at the rate of 4 kop/kW·h. What is the monthly cost of the electric energy?

13.8. An arc steel melting furnace draws a current of $3.0 \cdot 10^4$ A at 220 V. What is the power developed by the furnace? How much energy is used up in 5 hours of operation?

13.9. A battery is rated at 54 A·h at a voltage of 12 V across its terminals. If the efficiency of the battery is 81 per cent, what is the energy required to charge it?

13.10. A lathe motor draws 5.0 A of current at a voltage of 220 V. What is the energy used in 5 hours of operation of the motor? What is the cost of the energy at the rate of 4 kop/kW·h?

13.11. Three lamps with resistance 240 Ω each are connected in parallel to a 120 V mains. Determine the power

---

* A month is considered to have 30 days.

developed by all the lamps, the total current and energy used in 8 hours of lighting of the lamps.

13.12. An electric motor whose winding has a resistance of 0.40 Ω is rated at 300 V, 50 A. Determine the energy used up in 5 hours, mechanical work done and the heating of the winding.

13.13. A 500 W lamp for epidiascope is rated at 110 V. Determine the lamp resistance under operating conditions. Determine the additional resistance required to connect the lamp to a 127 line voltage.

13.14. An arc lamp is placed in series with a resistance of 7.3 Ω and connected to a mains with a voltage of 110 V. The lamp develops a power of 410 W. Compute the current through the lamp and its resistance under operating conditions.

13.15. There are 4 electric motors of 5.0 h.p. in a shop. They operate 8 hours a day for 22 working days. What is the cost of the electric energy consumed at the rate of 4 kop/kW·h?

13.16. An electric locomotive is equipped with 6 electric motors with mechanical power 350 kW and efficiency 92 per cent each. If the voltage in the line is 3 000 V, what is the total current through all the electric motors?

13.17. The power developed by a rheostat is 30 W, the voltage across its terminals is 15 V. The rheostat is made of nickeline wire with cross-sectional area 0.50 mm^2. What is the length of the wire?

13.18. Two incandescent lamps of powers of 100 and 80 W are rated at a voltage of 120 V. What power will each lamp develop if connected not in parallel, but in series? What lamp will glow brighter? How will the voltage distribute between the lamps?

13.19. A fully engaged rheostat is connected in series with an electric arc to a 120 V line voltage. The voltage drop across the arc electrodes is 45 V, the current flowing through the circuit is 12 A. Determine the voltage drop across the rheostat, its resistance, and the distribution of powers between the arc and rheostat.

13.20. The voltage across the terminals of a generator is 132 V, and across the load 127 V. If the power dissipated by the load is 5.0 kW, what is the voltage drop across the main line, and the resistance of the latter?

13.21. An electric fire-place is made of nickeline wire of length 50.0 mm and cross-sectional area 1.4 mm^2. The power supply has a voltage of 120 V. Determine the power developed by the fire-place and the cost of energy used in 2 hours at the rate of 4 kop/kW·h.

13.22. An electric motor with mechanical power 3.3 kW and efficiency 85 per cent is rated at 220 V. Compute the current and resistance of the armature winding of the electric motor.

13.23. An electric water heater for an aquarium when connected to a 12 V source with internal resistance 3.2 Ω develops a power of 10 W. What is the current flowing through the circuit and the efficiency of the appliance?

13.24. A 220 V generator supplies 11 kW to the external circuit. Conducting wires are made of copper and have a length of 50 m. What is the minimum cross-sectional area of the wires required for the voltage drop across them to be not higher than 2 per cent of the voltage specified?

13.25. Under working conditions, the resistance of an electric lamp is 420 Ω. A number of these lamps are connected in parallel to a 127 V line voltage, their total power being 1.52 kW. How many lamps are there? Ignore the resistance of conducting wires.

13.26. A load dissipates 10 kW at 400 V. The distance from the generator to the load is 500 m. If the cross-sectional area of the wires is 26 mm^2, what is the voltage drop across the conducting copper wires?

13.27. A 138 V generator with internal resistance 0.050 Ω supplies parallel-connected lamps. The resistance of each lamp is 300 Ω, the voltage across the terminals is 120 V. The resistance of conducting wires is 0.25 Ω. How many lamps are there in the circuit? What is the net power?

13.28. An electric locomotive travels at a constant speed of 43.2 km/h, the average pull being 43.7 kN. If the voltage across the commutator of a motor is 1 500 V and the motor efficiency is 92 per cent, what is the current through the motor?

13.29. An electric motor rated at 120 V, 7.5 A has a winding with a resistance of 2.2 Ω. Determine the power loss in the motor winding and its efficiency.

13.30. A power of 100 kW is to be transmitted from

a 20 000 V generator to the user separated by a distance of 2.5 km. The power loss in the line should not be higher than 2 per cent. What is the minimum cross-sectional area of copper wires required?

13.31. A 220 V mains supplies in parallel 5 electric motors with an electric power of 1.5 kW each. The length of copper conducting wires is 250 m, and their cross-sectional area 25 mm^2. What is the current through the circuit, the voltage across the generator terminals, and the power loss in conducting wires?

13.32. Electric motors of tram cars are rated at 112 A, 550 V. If the motors develop a pull of 3 600 N and their efficiency is 70 per cent, what is the speed of the tram?

13.33. A high-speed lift weighing 15.7 kN goes up at 1.0 m/s. The supply line is 220 V, the motor efficiency is 92 per cent. What is the power of the electric motor driving the lift? What is the current flowing?

13.34. A traction electric motor of a lifting crane draws a current of 10 A at a voltage of 220 V, and in 80 min lifts a weight of 26 t to a height of 30 m. Determine the power developed, the power lost, and the efficiency of the equipment.

13.35. A traction electric motor of a diesel-electric tractor is rated at 360 A and 470 V, its efficiency being 72 per cent. If the travelling speed is 2.0 km/h, what is the maximum pull developed by the tractor?

13.36. An electromagnetic lifting crane has two coils drawing a current of 30 A at 220 V. The total crane efficiency is 78 per cent, the lifting speed is constant and equal to 0.50 m/s. What is the maximum weight per coil?

13.37. A cargo cage of mass 1.8 t travels uniformly to a height of 25 m in 1.0 min. The cage is driven by a motor of 90 per cent efficiency, the voltage across the terminals of the motor is 220 V. What is the power developed by the motor and the energy used in the lifting? What is the current through the motor?

13.38. An electric locomotive is powered by 8 pull motors of 92 per cent efficiency series-connected in two. The contact system voltage is 3 000 V, and the current flowing through the motor is 380 A. What is the average pull, if an average speed of the electric locomotive is 54 km/h?

13.39. A tram car is lighted by 5 lamps placed in series.

If the number of the lamps is reduced by one, would the electric energy consumption decrease?

13.40. What is the purpose of fuses? What endures the higher current: the fuse, or the circuit, into which it is inserted?

13.41. How would the amount of the heat evolved change, if the resistance of the heating coil is reduced twice, and the current is increased twice?

13.42. When in a room a heavy-current appliance is switched on, the glow of lamps becomes weaker. Why?

13.43. Two lamps rated at similar voltage, but consuming dissimilar powers are connected to a mains in series. Why will one of them glow brighter?

13.44. Why is it not recommended to remove out of water a boiler that is still on?

13.45. When an electric stove is on for a prolonged period, the electric energy is consumed uninterruptedly, but the temperature of the heating coil does not grow beyond all bounds. Why? How would a minor shortening of the failed coil affect the functioning of the stove?

13.46. What amount of heat will evolve in a 6.0 Ω rheostat, if it passed 600 C of electricity in 5.0 min? Give the answer in joules and calories.

13.47. A heating element of a boiler rated at 220 V gives off 138 kcal in 10 min. What is the resistance of the element?

13.48. Two conductors with resistances 5.00 and 7.00 Ω are connected in parallel to a source. In the first one 4.20 kcal of heat is liberated. What amount of heat (in joules) is released in the second conductor during the same time interval?

13.49. Two identical sources of e.m.f. 6.0 V and internal resistance 1.0 Ω each are connected in series across a fully engaged rheostat of 10 Ω resistance. How much heat will evolve in one current source in 5.0 a min?

13.50. A current source is connected first across a resistance $R_1 = 2\ \Omega$, and then across $R_2 = 8\ \Omega$. In both cases the resistances release an identical amount of heat. Compute the internal resistance of the source.

13.51. An electric boiler uses 3.5 kW·h of electric energy to heat in 35 min some water with initial temperature of

10°C. Neglect the heat losses. How much water is to be boiled? What is the power of the boiler?

13.52. The heating of 2.0 l of water from 19°C to the boiling point required 0.225 kW·h of energy. A 120 V boiler supplied it in 18 min. What is the efficiency of the heating element of the boiler?

13.53. An amount of 3.0 l of water is heated from 18°C to the boiling point in a 800 W electric kettle with an efficiency of 87 per cent. How long will it take?

13.54. A boiler takes 5.0 min to heat 210 g of water from 14°C to the boiling point. The boiler is rated at 120 V. Ignoring the energy losses, determine the power and resistance of the boiler.

13.55. Two lamps with powers 60.0 and 100 W rated at 220 V are included in a circuit of the same voltage, first in parallel, and then in series. Ignoring the dependence of lamp resistance on temperature, determine the amount of heat released in each lamp in 30 s.

13.56. A 1.0 kW electric boiler rated at 220 V takes 12 min to heat 1.5 l of water by 88 degrees. If the rate is 4 kop/kW·h, what is the cost of the energy used and the current through the circuit? What is the efficiency of the boiler?

13.57. An electric boiler has two identical windings, which may be connected to a mains outlet separately, or in combination. How should the windings be connected to speed up the heating?

13.58. There are two 400 W heaters. Neglect the heat losses. How long will it take to heat 1.0 l of water by 80°C, if the heaters are connected (a) in series, and (b) in parallel?

13.59. A room loses heat at a rate of 823 kcal/h. What must the resistance of a 120 V electric fire-place be for the temperature in the room to be maintained constant?

13.60. A 120 V electric stove with Nichrome heating element of cross-sectional area 0.5 mm^2 and efficiency 80 per cent heats from 20°C to the boiling point and fully evaporates 1 l of water in 10 min. What is the length of the Nichrome conductor?

13.61. An electric heater that draws 5.0 A at 120 V heats 1.5 l of water from 16 to 100°C in 20 min. Determine the energy lost during the heating and the heater efficiency.

13.62. A heater made of 6 m nickeline wire draws a current of 5.0 A. It heats 1.5 l of water by 84°C in 14 min. Ignoring the heat losses, determine the diameter of the nickeline wire.

13.63. An electric kettle is rated at 120 V and has an efficiency of 80 per cent. It takes 22.5 min to heat 1.8 l of water from 10°C to 100°C. What is the resistance of the heating element of the electric kettle? What is the current flowing through the heating element?

13.64. An electric stove rated at 120 V, 600 W has an efficiency of 82 per cent. It heats some water from 18°C to the boiling point in 10 min. What is the amount of water? What is the resistance of the heating coil?

13.65. In aluminium calorimeter of mass 46 g contaning 180 g of water a heating coil of 2.0 Ω resistance is immersed. The coil is connected to a source of voltage 4.8 V. Ignoring the energy losses, determine the change in the water temperature in 5 min.

13.66. A calorimeter of mass 0.13 kg with specific heat 378 J/kg·K contains 0.30 kg of kerosene. A 3.0 Ω heating coil is placed in the kerosene. How long should a current of 2.0 A be passed through the coil for the temperature in the calorimeter to rise by 2.5 K?

13.67. An electric stove rated at 220 V, 3.0 A has a total efficiency of 80 per cent. How much ice at a temperature of —10°C can be melted in 10 min using the stove?

13.68. Two arc lamps, each rated at 45 V and 8 A, are connected in series to a 127 line voltage so that an excess voltage is absorbed by a fully engaged rheostat. Calculate the amount of heat given off by the rheostat in 30 min, the rheostat resistance and the length of 1.0 mm^2 nickeline wire of which the rheostat is made.

13.69. In a calorimeter confining 300 g of alcohol a 5.7 Ω coil is immersed. The coil is connected to a battery of three series-connected cells of e.m.f. 2.0 V and internal resistance 0.10 Ω each. The alcohol heats by 1.4 K in 3.0 min. What is the specific heat of the alcohol?

13.70. An electric heater rated at 110 V heats 120 g of water from 10 to 100°C in 10 min, the total energy losses being 40 per cent. The heater is made of nickeline wire of 0.20 mm diameter wound on a 1.5 cm dia porcelain cylinder. Determine the number of turns of the wire.

13.71. Two iron wires with resistances $R_1 = 1.0\ \Omega$ and $R_2 = 2.5\ \Omega$ at 0°C are connected in series to a current source. The first wire is heated to 847°C so that the power dissipated by it remains unvaried. The temperature of the second wire remains constant. Neglecting the internal resistance of the current source, find the temperature resistance coefficient of iron.

13.72. A 220 V power supply feeds simultaneously a 0.30 kW electric motor with efficiency 90 per cent, a 1.0 kW electric stove and 10 lamps of 150 W each. Determine the currents through the motor, stove, and lamps, the total current, and the power consumed.

13.73. The circuit of Fig. 13.73 is supplied by a battery of 24.8 V cells with internal resistance 0.40 Ω each connected as shown. The ammeter reads 2.0 A, $R_1 = 4.2\ \Omega$, $R_2 = 4.8\ \Omega$, $R_3 = 6.0\ \Omega$. Calculate the resistance R_4, the current flowing through it, and the voltage across the battery terminals. Find the total power and the power consumed by the external circuit.

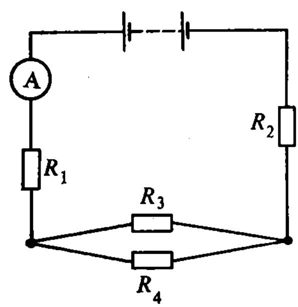


Fig. 13.73

13.74. A battery consists of five series-connected cells with e.m.f. 1.5 V and internal resistance 0.30 Ω each. At what current will the power consumed by the external circuit be maximal?

13.75. There are three incandescent lamps with powers 25, 25, and 50 W rated at a voltage of 110 V. How will you connect them so that when connected to a 220 V they would have a normal glow? What is the current through the lamps?

13.76. A cell is first short-circuited with a resistance of 0.64 Ω, and then with a resistance of 2.25 Ω. In both cases the current is the same. What is the internal resistance of the cell?

13.77. A source with e.m.f. 1.6 V and internal resistance 0.8 Ω is short-circuited with a conductor. The power con-

sumed by the external circuit is 0.60 W. What is the current and resistance of the conductor?

13.78. The short-circuiting of a current source causes a current of 1.5 A to flow. If short-circuited across an external resistance of 4.0 Ω, the power dissipated in the external circuit is 1.0 W. Find the electromotive force and the internal resistance of the current source.

13.79. A 2.15 kW electric stove with a Fechral element is connected to a 220 V power supply. The resistance of conducting wires is 0.50 Ω. Determine the stove resistance and the length of 2.0 mm Fechral wire.

13.80. An electric fire-place has two windings. If one of them is energized, the air temperature in a room rises by 1°C in 5 min, if the other is switched on, in 8 min. The heat losses are negligible. How long should the fire-place operate for the same heating effect to be obtained (a) with parallel connection of the windings, (b) with series connection of the windings?

13.81. A current of 3.0 A is passed over a time interval of 20 s through an aluminium wire of cross-sectional area 18 mm^2. Assuming that all the energy liberated goes into heating the wires, determine the temperature rise of the wire.

13.82. A copper wire of cross-sectional area 3.0 mm^2 is connected in series with a lead fuse with a cross-section of 1.0 mm^2. The initial temperature of the lead is 15°C. How far will the temperature of the wire rise before the fuse blows out?

SEC. 14. ELECTRIC CURRENT IN ELECTROLYTES. GALVANIC CELLS AND STORAGE BATTERIES

Example 45. Dies for disc records of surface area 3.0 dm^2 are produced using galvanoplastic technique. A wax mold is first plated with copper galvanically with a current density of 0.80 A/dm^2 during a time period of 30 min, the current efficiency being 90 per cent. Next the plating is carried out at a current density of 5.0 A/dm^2 for 20 hours, the current efficiency being 95 per cent. How much copper on the average is used per record die?

Given: $j_1 = 0.80\ \text{A/dm}^2 = 8.0 \cdot 10\ \text{A/m}^2$ is the current density in preliminary plating, $t_1 = 30.0$ min $= 1\,800$ s is the time period of the preliminary plating, $\eta_1 = 90.0$ per cent $= 0.90$ is the current efficiency in preliminary plating, $j_2 = 5.0\ \text{A/dm}^2 = 5.0 \cdot 10^2\ \text{A/m}^2$ is the current density in the secondary plating, $t_2 = 20.0$ h $= 72 \cdot 10^3$ s is the time period of the secondary plating, $\eta_2 = 95$ per cent $= 0.95$ is the current efficiency in the secondary plating, $k = 0.33 \times 10^{-6}$ kg/C is the electrochemical equivalent of copper, $S = 3.0 \cdot 10^{-2}\ \text{m}^2$ is the surface area of the die.

Determine: m—the amount of copper required to produce one die.

Solution. The current efficiency indicates the percentage (by current) of the theoretical figure accounted for by the substance deposited in electrolysis, i.e.

$$\eta = m/m_{\text{theor}}$$

where m_{theor} is obtained from Faraday's first law. The mass of copper separated in the first run can be found from the relationship

$$\eta_1 = \frac{m_1}{m_{1\,\text{theor}}}, \quad \text{where} \quad m_{1\,\text{theor}} = kI_1t_1$$

Using the relation $I = jS$, we have

$$m_1 = \eta_1 kj_1St_1$$

Similarly, we obtain an expression for m_2

$$m_2 = \eta_2 m_{2\,\text{theor}} \quad m_2 = \eta_2 kj_2St_2$$

The total copper separated is thus

$$m = m_1 + m_2 = kS\,(\eta_1j_1t_1 + \eta_2j_2t_2)$$

Substituting for the quantities in question gives

$$m = 0.33 \cdot 10^{-6}\ \text{kg/C} \cdot 0.03\ \text{m}^2\ (0.90 \cdot 80\ \text{A/m}^2 \cdot 1\,800\ \text{s} + 0.95 \cdot 500\ \text{A/m}^2 \cdot 72 \cdot 10^3\ \text{s}) \approx 0.34\ \text{kg}$$

Answer. About 0.34 kg of copper is required to produce one die.

Example 46. What is the minimum capacity of a storage battery required for 5.0 l of oxygen at 27°C and standard

atmospheric pressure to evolve in electolysis of acidified water?

Given: $V = 5.0\ l = 5.0\cdot10^{-3}\ \mathrm{m}^{-3}$ is the volume of oxygen evolved, $T = 300$ K is the temperature of oxygen, $p_0 = 101.3$ kPa is the standard atmospheric pressure, $\rho_0 = 1.43\ \mathrm{kg/m^3}$ is the density of oxygen at STP, $k = 8.29 \times 10^{-8}$ kg/C is the electrochemical equivalent of oxygen.

Determine: q—the capacity of the battery in ampere-hours.

Solution. The amount of electricity required for the electrolysis is obtainable from the relations

$$m = kq \qquad q = m/k$$

The mass of oxygen evolved is determined from the formula $m = \rho_0 V_0$, where V_0 is obtained from the Charles law $V_0 = VT_0/T$. Thus

$$q = \frac{VT_0\rho_0}{Tk}$$

Expressing q in ampere-hours we arrive at

$$q = \frac{VT_0\rho_0}{Tk\cdot 3\,600}$$

Substituting the data, we get

$$q = \frac{5.0\cdot10^{-3}\ \mathrm{m^3}\cdot 273\ \mathrm{K}\cdot 1.43\ \mathrm{kg/m^3}}{300\ \mathrm{K}\cdot 8.29\cdot10^{-8}\ \mathrm{kg/C}\cdot 3\,600} \approx 22\ \mathrm{A\cdot h}$$

Answer. The battery should have a minimum capacity of 22 A·h.

14.1. How would you account for the fact that the solution in which ions are available is electrically neutral?

14.2. Are there any free electrons in electrolytes?

14.3. There are series-connected baths with various concentration of the same electrolyte. (1) What can be said of the amount of a substance liberated at electrodes in these baths? (2) Will the concentration of unsaturated solution of copper sulfate change in electrolysis, if as the anode a rod of coal is used? Of copper?

14.4. What affects the specific conductivity of electrolytes?

14.5. What is the role played by sulfuric acid in electrolysis of water?

14.6. When is it more hazardous to touch a live wire: with dry or wet hands? Why?

14.7. Anhydrous sulfuric acid can be contained in iron containers, whereas diluted acid in glass containers only. Why?

14.8. An electrolysis of silver salt solution produces 300.0 mg of silver at the cathode. Determine the charge passed through the electrolyte.

14.9. If a current of 1.5 A is passed through an electrolyte, 137 mg of a substance will deposit on the cathode in 5 min. What substance is it?

14.10. In order to determine the electrochemical equivalent of copper a student is passing a current of 1.2 A through a copper sulfite solution over a time interval of 5.00 min, during which time the mass of the cathode increases by 120 mg. What will the value of the electrochemical equivalent of copper obtained from the above measurement be? Compare the result obtained with that listed in the table and calculate the percentage error of the measurement.

14.11. Silver is deposited from a solution of silver nitrate during a period of 1.5 min, so that for the first 30 s the current grows steadily from 0 to 2 A, and is kept constant during the rest of the time. Calculate the amount of silver deposited. Plot the curve $I = f(t)$.

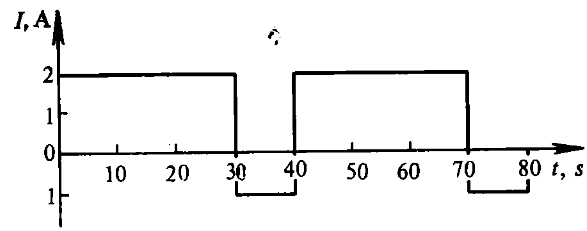


Fig. 14.12

14.12. Using the variation of current with time shown in Fig. 14.12 determine the amount of copper deposited in electrolysis of a copper sulfate solution.

14.13. Why do they reverse current in electroplating?

14.14. What amount of bivalent nickel can be separated electrolytically from a solution of nickel sulfate in 1 hour at a current of 1.5 A?

14.15. An ammeter in a circuit involving a plating bath reads 0.2 A. Is the reading of the ammeter correct, if on the cathode 250 mg of silver is liberated?

14.16. An electrolysis of copper sulfate solution produces 6 g of copper in 50 min. What is the current?

14.17. If the current in a bath is 3.0 A, how long will it take for a copper anode of dimensions $100 \times 50 \times 2$ mm^3 to be expanded?

14.18. In an electroplating bath with a solution of copper sulfate the current varies during a period of 1 min according to the law $I = 0.05\,t$. What amount of copper will deposit on the cathode during the time?

14.19. If the current in an electroplating bath varies as $I = 6 - 0.03\,t$ what is the mass of silver deposited from a solution of silver salt in 2 min?

14.20. In small electroplating baths not one, but two anodes are utilized with a job positioned in between. Explain why? What are the principles underlying the electrolytic polishing of metal products?

14.21. Using electrolysis, 1 kg of copper is produced. How many silver would be deposited, if the same amount of electricity were passed through an appropriate electrolyte?

14.22. In nickel plating on a job with a surface area of 120 cm^2 a 0.030 mm layer of bivalent nickel is formed. The voltage across the bath terminals is 1.8 V, the resistance of the solution is 3.75 Ω. How long does the process take? What is the energy consumed?

14.23. A mass of 1 980 kg of copper is refined electrolytically. The terminal voltage is 3 V. Determine the cost of the energy used at the rate of 4 kop/kW·h, neglecting the energy losses.

14.24. Aluminium is being deposited at a voltage of 5.0 V, the electric energy is being consumed at a rate of 1.0 kW·h. If the efficiency of the bath is 80 per cent, what is the mass of aluminium produced.

14.25. In copper refining a bath of copper sulfate is used in which a copper plate is immersed—anode—containing 10 per cent impurities. The voltage across the bath is 6.0 V,

If it is necessary to refine 1.00 kg of such copper, and if electricity costs 2 kop/kW·h, determine the energy needed.

14.26. An electroplating copper bath produced 1.98 g of copper in 40 min. The solution resistance is 1.3 Ω, the internal resistance of the battery is 0.3 Ω, and the polarization e.m.f. is 1 V. What is the e.m.f. required to supply the bath?

14.27. In electrolysis of a solution of zinc sulfate 2.04 g of zinc is deposited on the cathode in 50 min. If the voltage across the bath terminals is 4.2 V, the solution resistance is 1.8 Ω, what is the polarization electromotive force?

14.28. Why are nickel and chromium most commonly used for electroplating?

14.29. In nickel plating of jobs a current density of 0.40 A/dm^2 is used. What thickness of a layer of nickel (bivalent) may be achieved by passing the current for 8.9 hours?

14.30. In silver plating a current density of 0.7 A/dm^2 is used. How long is it necessary to pass the current for a 0.05 mm silver layer to form?

14.31. A part is silver plated using a current of 0.55 A/dm^2. Calculate the rate of growth of a silver layer on the part.

14.32. The electrolysis of a $NiSO_4$ solution occurs at a current density of 0.15 A/dm^2. What is the number of nickel atoms deposited on 1 cm^2 of the cathode surface in 2 min?

14.33. How long does it take for a copper anode to become thinner by 0.03 mm if the current density in electroplating is 2.0 A/dm^2?

14.34. In series with a copper voltameter an ammeter is placed that reads 1.5 A. An amount of 0.316 g of copper liberates on the cathode in 10 min. What correction should be introduced into the ammeter reading?

14.35. An electrolysis is carried out in a silver nitrate solution having a resistance of 0.8 Ω, so that 100 g of silver are obtained in 5 hours. What is the power consumed in heating the solution?

14.36. In electrolysis of a solution of sulfuric acid with a resistance of 0.4 Ω at STP 3.3 l of hydrogen is separated in 50 min. Compute the power consumed in heating the electrolyte.

14.37. A product is being silver plated in a solution of a salt with a resistance of 1.2 Ω. In 2 hours 40.32 g of silver is produced. Determine the current in the bath, the voltage across its terminals, and the energy consumption during the time of electrolysis.

14.38. A current is being passed through a gas voltameter at STP so that 0.2 m^3 of hydrogen is obtained in 10 hours. Determine the value of the current.

14.39. A current of 2.5 A is passed through a weak solution of sulfuric acid during 12 min. Find the volumes and masses at STP of the hydrogen and oxygen evolved.

14.40. A solution of sulfuric acid in electrolysis gives at STP 5.0 l of hydrogen in 2 hours 23 min. If the power consumed is 32.5 W what is the resistance of the solution?

14.41. Consider an electrolysis in water bath through which 4000 C of electricity is passed to give off 0.4 l of hydrogen at a pressure of $1.28 \cdot 10^5$ Pa. Calculate the temperature of the hydrogen.

14.42. Given the molar mass and valence of aluminium, calculate its electrochemical equivalent. Determine the amount of aluminium to be obtained using a current of 3 A in 24 hours.

14.43. The electrochemical equivalent of silver being $1.118 \cdot 10^{-6}$ kg/C compute the electrochemical equivalents of sodium, chlorine, and aluminium.

14.44. Knowing the molar mass and valence of oxygen, determine its electrochemical equivalent and mass evolved in passing a charge of 5.5 C.

14.45. A $1.8 \cdot 10^{-2}$ mm layer of bivalent nickel is obtained on a job in 50 min. What is the current density in the nickel plating?

14.46. To prevent corrosion of sea-going ships the method of galvanic-action protectors is used: on the steel surface of the hull zinc sheets are attached at places. Explain the principle underlying the method.

14.47. In manufacturing of electrodes for lead batteries highly pure lead is utilized. Why?

14.48. To a 120 V generator terminals 25 baths for silver plating and a rheostat are connected in series. Each bath must give 4.0 g of silver in 2.0 hours. The terminal voltage

of the bath is 4.2 V, and the total surface area eing silver plated is 23.8 dm^2. What is the current flowing through the baths? What is the thickness of the silver layer? What is the efficiency of the equipment?

14.49. Using electrolysis of a weak solution of sulfuric acid a volume of 5.0 l of hydrogen at 27°C and pressure $1.013 \cdot 10^5$ Pa is obtained. The efficiency of the plant is 80 per cent, and the process occurs at a bath voltage of 5.0 V. Determine the energy to be consumed.

14.50. Consider the electrolysis of zinc sulfate at a current of 2.5 A. Calculate the number of atoms of bivalent zinc deposited at the cathode in 5 min.

14.51. An electrolysis is being conducted at a current density of 0.15 A/dm^2. What number of bivalent metal atoms will separate out on a cathode with surface area 50.0 cm^2 in 1.0 min?

14.52. The passing of 60 C of electricity through an iron voltameter caused $11.6 \cdot 10^{-8}$ kg of iron to deposit on the cathode. Determine the valence of the iron.

14.53. The charge of hydrogen ion is $1.6 \cdot 10^{-19}$ C. Knowing the electrochemical equivalent of hydrogen, determine the mass of hydrogen atom.

14.54. A mass of 0.9 g of water is decomposed electrolytically using a current of 160 A. How long does it take? What are the masses of hydrogen and oxygen produced?

14.55. Through a salt of bivalent nickel a charge of $3.0 \cdot 10^4$ C is passed. The molar mass of the nickel is $58.68 \times \times 10^{-3}$ kg/mol. Compute the amount of nickel deposited in the electrolysis.

14.56. The Daniel cell is essentially a glass vessel with a solution of copper sulfate in which a copper plate (anode) is inserted. A porous vessel placed within the glass vessel contains a solution of sulfuric acid with a zinc rod (cathode) immersed in it. If the cell operates at a current of 0.5 A during a time period of 30 min, what will the amount of zinc spent be?

14.57. On a product with surface area 120 cm^2 nickel-plated during 5 hours a 0.155 mm layer of bivalent nickel deposits. The molar mass of nickel is $58.7 \cdot 10^{-3}$ kg/mol, the nickel density is taken to be $8.8 \cdot 10^3$ kg/m^3. What is the current of the electroplating?

14.58. An electrolysis of sulfuric acid produces $27 \cdot 75$ cm^3 of fire-damp* at STP. Calculate the electric charge that is passed.

14.59. A balloon with a volume of 250 m^3 is to be filled with hydrogen at a temperature of 27°C and pressure of 2 atm. Hydrogen is produced using electrolysis of a weak solution of sulfuric acid. What is the charge that should be passed to obtain the required amount of hydrogen?

14.60. There is a battery with e.m.f. 2 V and capacity 22 A h. What is the amount of electricity and the energy stored in the battery?

14.61. What is to be done, if before charging a battery the level of electrolyte in it is below normal, and no electrolyte is known to be spilled?

14.62. A battery has a capacity of 54 A h at a voltage of 2.0 V. Given the battery efficiency is 80 per cent, what is the energy required to charge the battery?

14.63. A battery has a capacity of 40 A h. It took 60 hours to charge it with a current of 0.80 A. What is the efficiency of the battery?

14.64. A battery composed of 6 series-connected cells has a capacity of 54 A h. What is the capacity of one cell? If the cells are connected in parallel, what will the battery capacity be?

14.65. A battery with an internal resistance of 0.01 Ω and initial e.m.f. of 1.8 V is being charged from a 2.7 V power supply. Compute the useful power consumed by the battery charging, the power dissipated as heat, and the total power at the outset.

SEC. 15. ELECTRIC CURRENT IN GASES. THERMIONIC EMISSION. ELECTRIC CURRENT IN VACUUM

Example 47. A flat air capacitor with a separation between the plates of 2.00 cm is charged to a voltage of 1 000 V and disconnected from a source. The surface area of each plate is 50.0 cm^2. In the air between the plates $2.00 \cdot 10^8$ pairs of univalent ions per second are formed by an ionizer. Assum-

---

* Fire-damp is a mixture of hydrogen with oxygen in the ratio 2 : 1 (by volume).

ing that all the ions reach the capacitor plates, how long will it take the capacitor to discharge? What would the saturation current be, if with the same ionizer the capacitor were connected to a d.c. source? How does the saturation current vary with the capacitor voltage?

Given: $d = 2.00$ cm $= 2.00 \cdot 10^{-2}$ m is the separation between the capacitor plates, $S = 50.0$ cm^2 $= 5.00 \cdot 10^{-3}$ m^2 is the surface area of each capacitor plate, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the permittivity of vacuum, $\varepsilon = 1$ is the dielectric constant of air, $U = 1\,000$ V $= 1.00 \cdot 10^3$ V is the capacitor voltage, $n_0 = 2.00 \cdot 10^8$ cm$^{-3} \cdot$s^{-1} $= 2.00 \cdot 10^{14}$ m$^{-3} \cdot$s^{-1} is the number of pairs of univalent ions per cubic metre of air per second produced in the air between the plates, $e = 1.602 \cdot 10^{-19}$ C is the charge of a univalent ion.

Determine: (1) t—the time taken by the capacitor to discharge,

(2) I_s—the saturation current in connecting the capacitor to the source,

(3) I_s vs V dependence.

Solution. (1) The time required for the capacitor to discharge is obtained, knowing the charge q of the capacitor, and the total charge of ions of like sign q_t formed within the space between the capacitor plates per second, using the ionizer.

$$t = q/q_t$$

To determine the charge q we will use the relations $C = q/U$ and $C = \varepsilon_0 \varepsilon S/d$, hence

$$q = \frac{\varepsilon_0 \varepsilon S U}{d}$$

The charge of one species of ions q_t formed in 1 s can be obtained from the formula $q_t = n_t e$, where n_t is the number of ion pairs formed in 1 s between the plates. If the volume of a dielectric between the plates V is expressed in terms of the plate area and thickness of the dielectric layer $V = Sd$, then

$$q_t = n_0 S d e$$

Substituting for q and q_t in the relationship $t = q/q_t$ we get

$$t = \frac{\varepsilon_0 \varepsilon S U}{dn_0\, dSe} = \frac{\varepsilon_0 \varepsilon U}{en_0 d^2}$$

It is seen that the time of capacitor discharging is independent of the area of its plates. Finally, the time is

$$t = \frac{8.85 \cdot 10^{-12}\ \text{F/m} \cdot 1 \cdot 1.00 \cdot 10^3\ \text{V}}{1.602 \cdot 10^{-19}\ \text{C} \cdot 2.00 \cdot 10^{-14}\ \text{m}^{-3} \cdot \text{s}^{-1} \cdot (2.00 \cdot 10^{-2})^2\ \text{m}^2}$$
$$= 6.91 \cdot 10^{-1}\ \text{s}$$

The shortness of the discharging time is due to the fact that we neglect the ion recombination.

(2) The saturation current is obtainable from the relationship

$$I_s = q_{\max}/t$$

where $q_{\max}$ is the maximum charge reaching the capacitor plates: $q_{\max} = n_t e t = n_0 S d e t$; t is the time taken by the charge to reach the plates (through the circuit). Then

$$I_s = \frac{n_0 S d e t}{t} = n_0 S d e$$

As is seen from the formula, the saturation current is equal to the charge of like ions formed by the ionizer between the capacitor plates in a unit time. Now we proceed to calculate the saturation current I_s

$$I_s = 2.00 \cdot 10^{14}\ \text{m}^{-3} \cdot \text{s}^{-1} \cdot 5.00 \cdot 10^{-3}\ \text{m}^2 \cdot 2.00 \cdot 10^{-2}\ \text{m} \cdot 1.602$$
$$\times 10^{-19}\ \text{C} = 3\,204 \cdot 10^{-9}\ \text{A}$$

(3) A glance at the expression for the saturation current indicates that it is independent of voltage, and hence does not obey Ohm's law. The saturation current is only controlled by the ionizer intensity and the volume of interelectrode space.

Answer. (1) The time of capacitor discharging is $6.91 \times \times 10^{-1}$ s. (2) The saturation current is $3.204 \cdot 10^{-9}$ A. (3) The saturation current is voltage-independent.

15.1. What is the difference between the ionization of gases and ionization of liquids?

15.2. When exposed to a constant ionizer, the number of ions in a gas only grows to a certain level and then remains constant. Why?

15.3. What is the minimum kinetic energy required for cathode rays to ionize in a tube helium atoms having an ionization potential of 24.5 V? The ionization potential refers to a potential difference in an electric field that is required to be traversed by an electron to acquire the energy needed to ionize atoms.

15.4. A univalent ion is accelerated from rest by a potential difference equal to the ionization potential φ_i and collides with an atom. Is its kinetic energy adequate for a collision ionization of the atom?

15.5. Which particles—univalent ions or electrons—require a higher potential difference to be traversed to obtain the kinetic energy sufficient for a collision ionization?

15.6. Lithium atoms require an energy of $8.6 \cdot 10^{-19}$ J for ionization. What is their ionization potential?

15.7. If the ionization potential of nitrogen atoms is 14.47 V, will the nitrogen atoms be ionized by electrons with a kinetic energy of $2.2 \cdot 10^{-18}$ J?

15.8. If the ionization potential of neon atoms is 21.5 V, what is the minimum velocity needed for an electron to ionize a neon atom?

15.9. Helium atoms have an ionization potential of 24,580 V. At what temperature is the average translational energy of helium atoms (*a*) equal to the ionization energy, (*b*) sufficient for collision ionization of a stationary atom?

15.10. The ionization potential of hydrogen atoms is 13.54 V. Is the average translational energy of hydrogen atoms at $1.00 \cdot 10^4$ K sufficient for their collision ionization?

15.11. Potassium atoms are ejected between the capacitor plates spaced 1.2 cm apart and with a voltage of 12 kV applied across them. The mean free path of electrons in the air at a standard pressure is taken to be 5.0 μm, and the ionization potential of potassium atoms—4.32 V. Will the potassium atoms be ionized by collision with the electrons?

15.12. A flat capacitor is located in a glass tube filled

with hydrogen and neon under lower pressure. The mean free path of electrons here is taken to be 92 μm. The plates of the capacitor are separated by a distance of 2.5 cm, the voltage across the plates is 4 000 V. The ionization potential of hydrogen and neon atoms are 13.54 and 21.5 V, respectively. Will the hydrogen and neon atoms be ionized within the capacitor?

15.13. A flat air capacitor with the plates 2.00 cm apart and a surface area of plates of 10.0 cm^2 is charged to a voltage of 2 000 V, and then disconnected from the energy source. An ionizer placed between the plates produces 10^9 pairs of univalent ions per second. Assuming that all the ions produced reach the capacitor plates, determine the voltage across the capacitor in 5.53 s after the ionizer has been switched on.

15.14. Is it possible for a saturation current to be reached under the conditions of independent conduction in gases?

15.15. Consider a flat air capacitor with the plates separated by 2.00 cm and having a surface area of 10.0 cm^2, that is connected to a d.c. source. An ionizer between the plates gives rise to $2.5 \cdot 10^8$ pairs of univalent ions per second. What is the saturation current?

15.16. What is the difference between self-sustained gaseous conduction and induced? Under what conditions does an induced discharge change into a self-sustained one?

15.17. What conditions are to be met for an electric arc to be drawn at atmospheric pressure? How will the interelectrode voltage change at the moment the arc strikes?

15.18. A rheostat is normally connected in series with a subcircuit containing an arc discharge. Explain why?

15.19. What will happen to an electric arc if we cool (*a*) the cathode, (*b*) the anode?

15.20. How will the temperature of an electric arc vary if the gas pressure is increased?

15.21. What will happen if an arc discharge is placed in a magnetic field perpendicular to the current?

15.22. A spark discharge occurs at a voltage of the order of thousands of volts, whereas to obtain an arc discharge 40-50 V is sufficient. Why?

15.23. Discuss the instantaneous and average power at a spark discharge.

15.24. Why does the spark discharge lend itself for processing of the most high-melting metals? To what pole of an energy source should they be connected?

15.25. Can you think of a way to plate metallic parts with another metal using a spark discharge?

15.26. Show the dependence of the corona discharge energy losses on voltage applied? On weather?

15.27. In high-voltage transmission lines wires with larger diameter exhibit smaller losses to corona discharge. Explain.

15.28. One of the ways to diminish corona discharge is to connect several spaced wires to each pole of a high-voltage source. Explain the phenomenon.

15.29. Name the state of matter in an electric arc, in gas-discharge tubes, in upper strata of the atmosphere.

15.30. Why does a rarefication of a gas improve its conductivity? Is it always the case?

15.31. How can gas atoms be excited?

15.32. Under what conditions will an atom emit light?

15.33. In discharge in rarefied gases, why does each gas glow its own colour?

15.34. Why are aurorae polares more frequent and intensive during solar activity maximum periods? Why do the aurorae polares not occur in equatorial latitudes, and why are they but rarely observed in middle latitudes?

15.35. Why are aurorae polares accompanied by magnetic storms (continuous vibration of a compass needle)?

15.36. What is the role played by a rarefied gas in producing cathode rays in the tube? Is it possible to have cathode rays in a tube with the gas completely pumped out?

15.37. What is the direction of the magnetic field deflecting the cathode rays in the tube of Fig. 15.37?

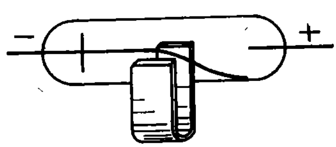


Fig. 15.37

15.38. Give some examples of applications of electron beams in vacuum.

15.39. Barium has a work function of 2.49 eV. What is the surface-energy barrier in barium? What is the threshold energy required for electrons to do this work?

15.40. The surface-energy barrier for tungsten is 4.54 V. What is the temperature corresponding to the average kinetic energy of electrons equal to the work function for this metal?

15.41. The working temperature of the tungsten cathode is 2 500 K. Is it possible for electrons with the average kinetic energy corresponding to the temperature to do work on escaping?

15.42. What is the minimal velocity component normal to a cathode surface at which electrons can be released from the tungsten surface?

15.43. What controls the number of electrons ejected by the cathode in a unit time?

15.44. Can you think of a way of increasing the cathode emission with its temperature and dimensions unaltered?

15.45. A cathode surface emits $5 \cdot 10^{16}$ electrons per second. What is the limiting value for the saturation current?

15.46. For a saturation current of 12 mA, how many electrons per second are liberated from the cathode surface?

15.47. How can electrons be ejected from a cold cathode? What is the name of the phenomenon?

15.48. Is it possible to change the saturation current in a cathode-ray tube? If so, how?

15.49. In a vacuum tube the space charge with the broken anode circuit and hot cathode remains constant though electrons keep liberating from the cathode. Explain why?

15.50. A voltage of 300 V is applied to a tube, the anode current being 10 mA. Determine (1) the amount of heat per second released by the anode, (2), the voltage drop across a resistance of 5 kΩ included in the anode circuit.

15.51. A vacuum valve can pass a maximum current of 100 mA, its internal resistance is 100 Ω, and the power consumed by the circuit from the energy source is 10 W. What is the minimal load resistance? What is the power dissipated by the anode?

15.52. A cathode is a filament of length 5.0 cm and diameter 0.16 mm. Its surface emits $1.50 \cdot 10^{17}$ electrons per square centimetre per second. Assuming that every fifth electron reaches the anode, determine the voltage drop across a resistance of 5.00 kΩ connected into the anode circuit of the tube.

15.53. Which of the plots $I_a = f(U_a)$ in Fig. 15.53 corresponds to the higher cathode temperature?

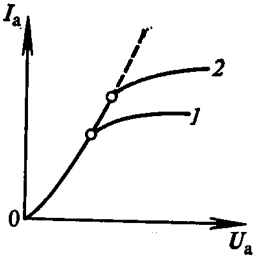


Fig. 15.53

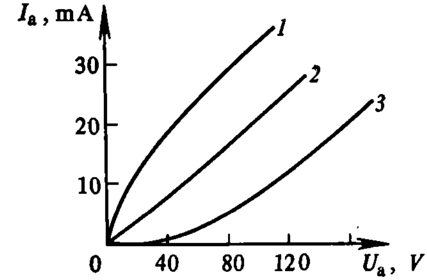


Fig. 15.54

15.54. Fig. 15.54 shows a family of curves representing the variation of the anode current with the anode voltage for a triode. Which of the curves corresponds to the grid-to-cathode voltage: (1) $U_g > 0$, (2) $U_g = 0$, and (3) $U_g < 0$?

15.55. Which of the plots of $I_a = f(U_g)$ in Fig. 15.55 corresponds to the higher anode voltage?

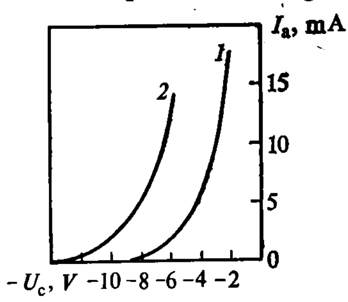


Fig. 15.55

15.56. Two identical diodes are connected as presented in Fig. 15.56 (*a*), (*b*), and (*c*). The anode voltage U_a being equal in all these cases, when is the current in the circuit larger?

15.57. Why with gas-filled tubes can one obtain larger currents than with vacuum ones?

15.58. Why is a high vacuum necessary in the cathode-ray tube?

15.59. In the cathode-ray tube along the path of the electron beam two flat capacitors are located so that their plates are mutually perpendicular. Explain why? What could substitute for these capacitors?

15.60. What is the purpose and name of the elements of the circuit in Fig. 15.60?

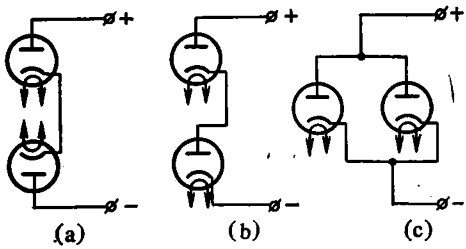


Fig. 15.56

15.61. To produce cathode rays in a tube use is made of a flat capacitor with plates 4.5 cm long and 1.8 cm apart. A cathode-ray beam directed parallel to the capacitor plates

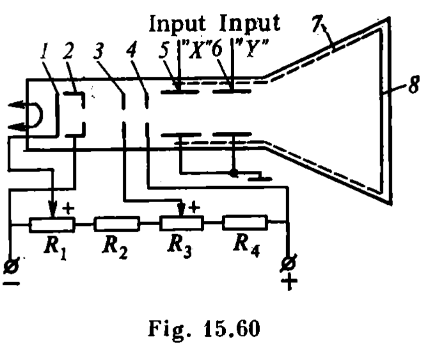


Fig. 15.60

is deflected by the latter by 1.2 mm. The velocity of electrons in cathode-rays is taken to be 50,000 km/s. What is the voltage across the capacitor plates?

SEC. 16. ELECTRIC CURRENT IN SEMICONDUCTORS

16.1. How do impurities affect the electric conductivity of absolutely pure conductors and insulators?

16.2. Describe the variation of conductivity of pure semiconductors with temperature. Is the superconductivity effect observed with them?

16.3. Describe the thermistor operation.

16.4. A subcircuit incorporates a metallic and semiconducting resistors connected in series. Assuming that the voltage across the subcircuit is constant, describe the variation of the current in the subcircuit on heating the resistor which is (*a*) metallic, (*b*) semiconducting.

16.5. What mobile carriers of electric charge are available in a pure semiconductor, and what is their ratio?

16.6. What is required for an electron-hole pair to be generated? What are the causes of the formation of the pairs?

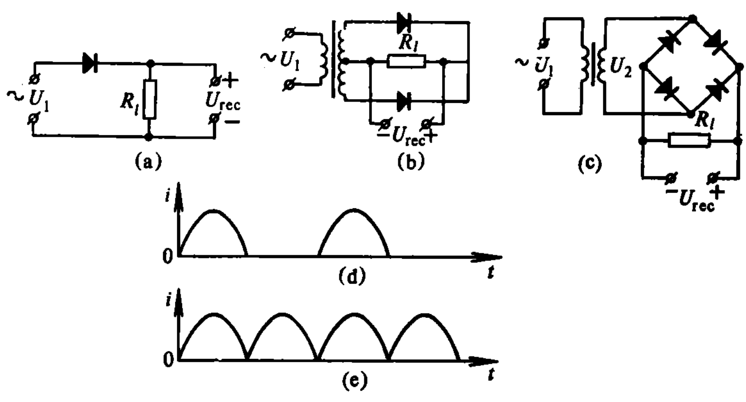


Fig. 16.16

16.7. What will happen in merging of electrons and holes?

16.8. Under steady-state conditions the number of free charge carriers in a semiconductor remains constant, although electron-hole pairs keep generating. Explain.

16.9. (1) Describe the method of creating in semiconducting materials the situation where prevails (*a*) electron conduction, (*b*) hole conduction. (2) In germanium semiconductor, which of the impurities—phosphorus, arsenic, antimony, gallium, boron, indium—are responsible for predominantly electron conductivity, and hole conductivity?

16.10. Discuss the temperature dependence of the resistivity of impurity semiconductors.

16.11. Why cannot free charge carriers stay near the *p-n* junction?

16.12. At equal voltage, why is the forward current through the *p-n* junction much larger than the reverse current?

6.13. A semiconductor rectifier shall not be connected to an a.c. power supply without a load. Justify why.

16.14. Consider a circuit incorporating a load resistor $R_1 = 100$ kΩ and a semiconductor diode with a reverse current of 150 μA and permissible reverse voltage of no higher than 100 V. Find the maximum supply voltage of the circuit.

16.15. The rectifying effect of the *p-n* junction is drastically reduced on substantial raising the temperature. Why? At what temperature germanium devices can be used? Silicon?

16.16. Discuss the principle of rectifiers connected as shown in Fig. 16.16 (*a*), (*b*), and (*c*). Which of the current vs time plots in Fig. 16.16 (*d*), (*e*) corresponds to the arrangements presented in Fig. 16.16 (*a*), (*b*), and (*c*)?

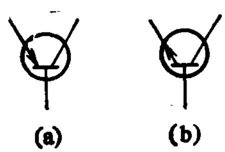


Fig. 16.21

16.17. Why should the base width in a transistor be small?

16.18. The impurity concentration in the emitter of a transistor is by far higher than in the base. Explain.

16.19. How are the currents of emitter, base, and collector related?

16.20. On a transistor the voltage is increased by the same amount across emitter-base, and base-collector. Will the current increase in the collector circuit be the same in both cases?

16.21. What transistor types—*p-n-p* or *n-p-n*—are represented symbolically in Fig. 16.21? Draw schematically the simplest connection configurations for these transistors, and indicate on them the emitter, the base, and the collector.

SEC. 17. ELECTROMAGNETISM *

Example 48. Fig. 48 represents two straight long parallel conductors spaced 50 cm apart and carrying oppositely directed currents: the first, 20 A, the second, 24 A. A point A is separated from the first conductor by a distance of 40 cm, and from the second by 30 cm. Calculate the magnetic field induction and strength.

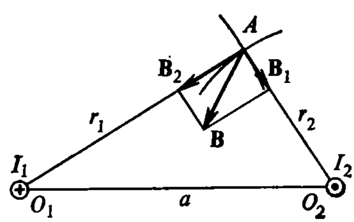


Fig. 48

Given: $I_1 = 20$ A is the current flowing in the first conductor, $I_2 = 24$ A is the current through the second conductor, $a = 0.50$ m is the separation between the conductors, $r_1 = 0.40$ m is the distance from point A to the first conductor, $r_2 = 0.30$ m is the distance from point A to the second conductor, $\mu_0 = 4\pi \cdot 10^{-7}$ $\Omega \cdot$s/m is the permeability of vacuum.

Determine: B—the magnetic field induction at point A, H—the magnetic field strength at point A.

Solution. The magnetic field induction at point A is the vectorial sum of inductions $\mathbf{B}_1$ and $\mathbf{B}_2$ due to the current I_1 and I_2, respectively. The magnitude of the induction due to each individual current is given by

$$B = \frac{I}{2\pi r}\mu_0$$

As the total magnetic field induction $\mathbf{B}$ is the vectorial sum of $\mathbf{B}_1$ and $\mathbf{B}_2$, its magnitude here is obtainable from the Pythagorean theorem

$$B = \sqrt{B_1^2 + B_2^2}$$

In fact, the triangle AO_1O_2 is right-angled, as $50^2 = 30^2 + 40^2$, and the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ are tangent to the magnetic induction lines, i.e. to circles of radii AO_1 and AO_2.

The field strength at point A is determined by the relation $B = \mu_0 H$.

* Throughout Section 17 the relative magnetic permeability μ is taken to be 1, unless stated otherwise.

We substitute for B_1 and B_2 in the formula $B = \sqrt{B_1^2 + B_2^2}$, and proceed to calculate B

$$B = \sqrt{\frac{I_1^2}{4\pi^2 r_1^2}\mu_0^2 + \frac{I_2^2}{4\pi^2 r_2^2}\mu_0^2} = \frac{\mu_0}{2\pi}\sqrt{\frac{I_1^2}{r_1^2} + \frac{I_2^2}{r_2^2}},$$

$$B = \frac{4\pi\cdot 10^{-7}\ \Omega\cdot\text{s/m}}{2\pi}\sqrt{\frac{20^2\text{A}^2}{0.16\ \text{m}^2} + \frac{24^2\text{A}^2}{0.09\ \text{m}^2}} = 188\cdot 10^{-7}\ \text{T}$$

We now find H

$$H = \frac{B}{\mu_0} \qquad H = \frac{188\cdot 10^{-7}\ \text{T}}{4\cdot 3.14\cdot 10^{-7}\ \Omega\cdot\text{s/m}} = 15\ \text{A/m}$$

Answer. The induction at point A is $1.9\cdot 10^{-5}$ T, and the magnetic field strength is 15 A/m.

Example 49. In a uniform magnetic field of induction 0.25 T there is a flat coil of 25 cm radius with 75 turns (Fig. 49). The plane of the coil makes an angle of 60° with the magnetic induction lines. The current flowing through the coil is 8.0 A. Determine the torque acting on the coil. Calculate the work required for the coil to be removed from the magnetic field.

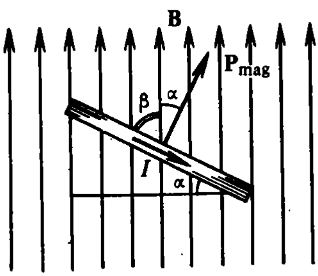


Fig. 49

Given: $B = 0.25$ T is the magnetic field induction, $r = 0.25$ m is the radius of the coil, $N = 75$ is the number of turns of the coil, $\beta = 60°$ is the angle between the coil plane and direction of magnetic field, $I = 8.0$ A is the current through the coil.

Determine: M_c—the torque acting on the coil,
A—the work done in removing the coil from the magnetic field.

Solution. The torque acting on a current-carrying loop in a magnetic field is given by

$$M_1 = P_{mag} B \sin\alpha$$

where α is the angle between the vectors $\mathbf{B}$ and $\mathbf{P}_{mag}$ that takes on values in the range $^1/_2\ \pi - \beta$. The coil having N turns, the torque exerted on it is

$$M_c = M_1 N$$

To obtain the required magnetic moment of the current-carrying loop we make use of the relationship

$$P_{mag} = I\pi r^2$$

The work performed will be

$$A = NI\Phi$$

where Φ is the magnetic flux, defined by

$$\Phi = B\pi r^2 \cos\alpha = B\pi r^2 \cos(^1/_2\ \pi - \beta)$$

Substituting for P_{mag} and α in the relation $M_c = P_{mag} BN \sin\alpha$ we go on to calculate M_c

$$M_c = \pi r^2 IBN \sin(^1/_2 \pi - \beta)$$
$$M_c = 3.14 \cdot 0.25^2\ \mathrm{m^2} \cdot 8.0\ \mathrm{A} \cdot 0.25\ \mathrm{T} \cdot 75 \cdot 0.5 = 15\ \mathrm{N \cdot m}$$

We now find the work

$$A = NIB\pi r^2 \cos(^1/_2 \pi - \beta)$$
$$A = 75 \cdot 8.0\ \mathrm{A} \cdot 0.25\ \mathrm{T} \cdot 3.14 \cdot 0.25^2\ \mathrm{m^2} \sqrt{^3/_2} = 25\ \mathrm{J}$$

Answer. The torque acting on the coil is 15 N·m, the work needed to remove the coil out of the field is 25 J.

Example 50. A beam of electrons travelling with the same speed (Fig. 50) passes between the plates of a flat capacitor

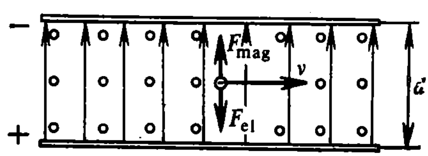


Fig. 50

separated by a distance of 2.4 cm. The electrons fly parallel to the plates. Inside the capacitor there is a magnetic field

with induction $6.20 \cdot 10^{-4}$ T directed at the reader (the induction lines in the figure are indicated by circles), i.e. perpendicular to the vector of electron velocity. With no voltage across the capacitor (the capacitor is discharged) the Lorentz force makes the electrons move in a circle of radius 1.8 cm. If the capacitor is charged to a voltage of 29.3 V, the electrons move within it rectilinearly, parallel to the plates. Calculate the specific charge of electron (the ratio of electron charge to its mass) and the mass of electron, given that its charge is $1.6 \cdot 10^{-19}$ C.

Given: $d = 0.024$ m is the separation between the capacitor plates, $B = 6.20 \cdot 10^{-4}$ T is the magnetic field induction, $r = 1.8 \cdot 10^{-2}$ m is the radius of circle, $U = 29.3$ V is the voltage across the capacitor, $e = 1.6 \cdot 10^{-19}$ C is the electron charge.

Determine: e/m_e—the specific electron charge,

m_e—the electron mass.

Solution. In the absence of an electric field the Lorentz force $F_{mag} = Bve$ to which the electrons are exposed is of centripetal nature: $F_{cp} = m_e v^2/r$. By equating these two forces we may obtain the quantity e/m_e.

The unknown travelling speed v may be found as follows. With an electric field the electrons move along a straight line. Hence the magnetic and electric forces acting on an electron are equal in magnitude $F_{mag} = F_{el}$. Recalling that $F_{el} = eE = eU/d$ we get the electron velocity

$$Bve = eU/d \quad v = U/dB$$

$$v = \frac{29.3 \text{ V}}{2.4 \cdot 10^{-2} \text{ m} \cdot 6.20 \cdot 10^{-4} \text{ T}} = 1.97 \cdot 10^{6} \text{ m/s}$$

From the equality $F_{mag} = F_{cp}$, we determine e/m_e

$$Bve = m_e v^2/r \qquad e/m_e = v/Br$$

$$\frac{e}{m_e} = \frac{1.97 \cdot 10^{6} \text{ m/s}}{6.20 \cdot 10^{-4} \text{ T} \cdot 1.8 \cdot 10^{-2} \text{ m}} \approx 1.77 \cdot 10^{11} \text{ C/kg}$$

Finally, we determine the mass of an electron

$$m_e = \frac{1.6 \cdot 10^{-19} \text{ C}}{1.77 \cdot 10^{11} \text{ C/kg}} = 9.1 \cdot 10^{-31} \text{ kg}$$

Answer. The specific electron charge is about $1.8 \cdot 10^{11}$ C/kg, the electron mass is $9.1 \cdot 10^{-31}$ kg.

Example 51. Ions of two potassium isotopes with masses 39 amu and 41 amu accelerated in an electric field of 500 V in vacuum are ejected into a uniform magnetic field with an induction of 0.16 T oriented perpendicular to the field lines. The charge of each ion is $1.6 \cdot 10^{-19}$ C. Determine the difference in the radii of trajectories of isotopes in the magnetic field.

Given: $m_{39} = 39 \cdot 1.67 \cdot 10^{-27}$ kg is the mass of isotope K^{39} ion, $m_{41} = 41 \cdot 1.67 \cdot 10^{-27}$ kg is the mass of isotope K^{41} ion, $U = 500$ V is the voltage of accelerating field, $B = 0.16$ T is the induction of the magnetic field, $q = 1.6 \cdot 10^{-19}$ C is the ion charge.

Determine: $(r_2 - r_1)$—the difference between the radii of potassium isotope trajectories.

Solution. Because the Lorentz force acting on potassium isotope ions in a magnetic field is centripetal in this case, the radius of trajectories might be found from the relation

$$Bvq = mv^2/r$$

The velocity of isotope ions is obtainable from the kinetic energy formula, considering that the energy is derived due to the work done by the electric field

$$\frac{1}{2} mv^2 = qU$$

It follows from this relationship that the velocity of isotope ions is

$$v = \sqrt{2qU/m}$$

The value of r is obtained from the relationship for the centripetal force

$$r = \frac{mv}{Bq} = \frac{m}{Bq} \sqrt{\frac{2qU}{m}} = \frac{1}{B} \sqrt{\frac{2qUm^2}{q^2 m}} = \frac{1}{B} \sqrt{\frac{2Um}{q}}$$

Let us now calculate the radii of trajectories of isotope ions and their difference

$$r_1 = \frac{1}{0.16\ \text{T}} \sqrt{\frac{2 \cdot 500\ \text{V} \cdot 39 \cdot 1.67 \cdot 10^{-27}\ \text{kg}}{1.6 \cdot 10^{-19}\ \text{C}}} = 0.1257\ \text{m} \approx 12.6\ \text{cm}$$

$$r_2 = \frac{1}{0.16\ \mathrm{T}} \sqrt{\frac{2 \cdot 500\ \mathrm{V} \cdot 41 \cdot 1.67 \cdot 10^{-27}\ \mathrm{kg}}{1.6 \cdot 10^{-19}\ \mathrm{C}}} = 0.129\ \mathrm{m} = 12.9\ \mathrm{cm}$$

$$\Delta r = r_2 - r_1,\ \Delta r \approx 12.9\ \mathrm{cm} - 12.6\ \mathrm{cm} \approx 0.3\ \mathrm{cm}$$

Answer. The radii of trajectories of potassium isotope ions differ by about 0.3 cm.

Magnetic Field Due to Current. Magnetic Induction. Magnetic Field Strength. Magnetic Moment of Current-Carrying Loop

17.1. Does electric current always cause a thermal effect? Chemical effect? Magnetic field?

17.2. A reference system K' travels uniformly and rectilinearly with respect to a charged body that is at rest relative to a system K. In which reference system does the field of the charged body belong (*a*) electric, (*b*) magnetic?

17.3. A charged particle flies past a molecule. Is there at the moment a magnetic field in reference to the molecule? Is the charge sign critical for the result?

17.4. If measurements have indicated that in a gas-filled space there is no magnetic field, might it be regarded as a proof of the absence of ions in the gas?

17.5. Why is the magnetic field often referred to as vortex field?

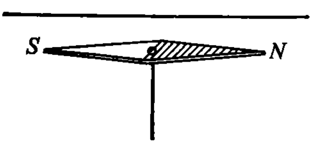


Fig. 17.6

17.6. Fig. 17.6 depicts a conductor with a magnetic needle located beneath. What is the direction of the current flowing through the conductor if the south pole of the needle deflects to the reader? What will happen to the deflection of the needle, if the latter is placed over the conductor?

17.7. Draw the patterns of magnetic fields for the cases represented in Fig. 17.7 (*a*), (*b*), (*c*), (*d*), (*e*), (*f*), and (*g*).

17.8. In Fig. 17.8 determine the directions of currents in the conductors, the direction of the magnetic field lines and forces acting on the current-carrying conductors.

17.9. In Fig. 17.9 indicate the direction of the force acting on the second conductor that carries a current, the directions of currents flowing through the conductors.

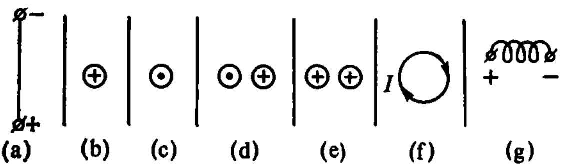


Fig. 17.7

17.10. What is the direction of currents at diametrically opposite points of a ring conductor? Discuss the action of the intrinsic magnetic field due to the current flowing through the ring conductor on ring conductor proper.

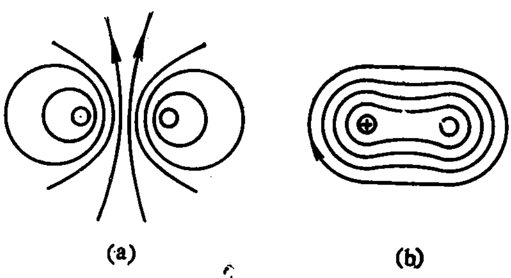


Fig. 17.8

17.11. A flexible wire made into a coil is suspended at one end. What will happen, if a current is passed through the coil?

17.12. In Fig. 17.12 determine the magnetic poles of the coil in which there is a current, and indicate the direction of the current and poles of the source.

17.13. Is it possible to wind a solenoid such that if a d.c. source is connected across it, its both ends will be (*a*) south poles, (*b*) north poles?

17.14. Is it possible for a steel rod to have at its both ends like magnetic poles? Can a permanent magnet have an even number of magnetic poles? Odd number?

17.15. A steel magnified needle suspended so that it is free to rotate in horizontal and vertical planes, finally takes a horizontal position. Does it always retain its horizontal position in the geomagnetic field?

17.16. What position will a magnetic needle assume at magnetic poles?

17.17. Is it always the case that the direction to north according to the magnetic needle coincides with the geographic meridian?

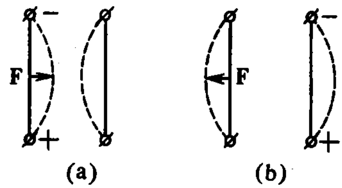


Fig. 17.9

17.18. A conductor attracts a parallel conductor 2.8 m long carrying a current of 58 A with a force of $3.4 \cdot 10^{-3}$ N. The conductors are 12 cm apart. Determine the current through the first conductor. What are the directions of currents flowing in both conductors?

17.19. Two parallel 320 cm conductors with equal currents are spaced at 8.7 cm and attracted with a force of $2.5 \cdot 10^{-2}$ N. Compute the currents through the conductors.

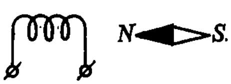


Fig. 17.12

17.20. Two very long parallel conductors are in vacuum at a separation of 4.0 cm. The current through one of them is 25 A, and through the other 5.0 A. Calculate the length of a segment of the conductor subjected to a force of $1.2 \times \times 10^{-3}$ N.

17.21. Two parallel conductors carrying a current of 100 A each are in a vacuum. A 75 cm conductor segment is acted upon by a force of $5.0 \cdot 10^{-2}$ N. Calculate the separation of the conductors.

17.22. Three very long parallel conductors are in a vacuum. They lie in one plane spaced by 0.50 m. Each of the conductors carries a current of 100 A so that in the first and the second one the current has the same direction.

What is the force acting on a metre of the first, second (middle), and third conductors?

17.23. Fig. 17.23 (*a*), (*b*) shows current-carrying conductors in magnetic fields. Determine the direction of the magnetic field for the case (*a*) and the direction of Ampere force for the case (*b*).

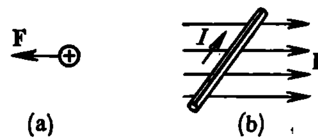


Fig. 17.23

17.24. A straight conductor of length 128 cm in which there is a current of 18 A is placed in a magnetic field with induction 0.82 T. The conductor is oriented perpendicular to the magnetic induction lines. What is the force acting on the conductor?

17.25. A 88 cm straight conductor is placed perpendicular to the magnetic induction lines in a uniform field. The conductor is exposed to a force of 1.6 N at a current of 23 A flowing through it. What is the magnetic induction of the field?

17.26. In a uniform magnetic field with an induction of 0.25 T a 1.4 m long straight conductor is placed. It is acted upon by a force of 2.1 N. If the current through the conductor is 12 A, what is the angle between the direction of the current and that of the magnetic field?

17.27. A straight conductor carrying a current of 14.5 A is subjected to a force of 1.65 N in a uniform magnetic field with induction 0.34 T. If the conductor makes an angle of 38° with the magnetic induction lines, what is the length of the conductor?

17.28. A conductor of mass 102 g and length 0.20 m is suspended horizontally from two dynamometers and placed in a uniform horizontal magnetic field with an induction of 0.50 T that is normal to the conductor. What will the change in the reading of each dynamometer be, if a current of 5.0 A is passed through the conductor? What is the current required for the conductor to be weightless?

17.29. In a vertical, uniform magnetic field a 0.20 m long conductor weighing 0.20 N is suspended horizontally from two thin strings. The magnetic field induction is 0.50 T. If there is a current of 2.0 A through the conductor, what angle with the vertical will the strings make?

17.30. A conducting rod of mass 0.10 kg and length 0.25 m lies on horizontal surface normal to the uniform horizontal magnetic field with induction 0.20 T. If the rod carries a current of 10 A, what is the force required to be applied perpendicular to the rod in a horizontal plane for the rod to move uniformly? The coefficient of friction is 0.10.

17.31. Fig. 17.31 shows a conductor of weight 1.0 N and length $l = 0.50$ m located in a plane making an angle of 30° with the horizon so that it is perpendicular to a uniform, horizontal magnetic field with induction $B = 0.10$ T. Given that the conductor carries a current of $I = 10.0$ A and the coefficient of static friction is 0.10, what is the force needed to be applied parallel to the inclined plane to sustain the conductor at rest?

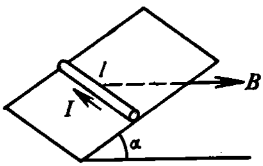


Fig. 17.31

17.32. A point is at a distance of 9.2 cm from a straight conductor in which there is a current of 13.2 A. Find the magnetic field strength and induction at the point.

17.33. The induction of magnetic field at a point separated by a distance of 4.5 cm from a straight current-carrying conductor is $2.8 \cdot 10^{-4}$ T. Determine the magnetic field strength at the point and the current through the conductor.

17.34. At a certain point the magnetic field strength due to a current of 12 A is 12.7 A/m. Calculate the magnetic field induction at the point and its separation from the conductor.

17.35. A thin infinite cylindrical conductor of radius R carries a current I. Determine the magnetic field strength and induction (1) inside the conductor, (2) at a point located at a distance R from the conductor surface.

17.36. An infinite straight cylindrical conductor of 2.0 cm diameter carries a current of 100 A. Assuming the current density uniform over the conductor cross-section, determine the magnetic field strength (1) on the conductor axis, (2) at a point spaced 0.50 cm apart from the conductor axis, (3) on the conductor surface, (4) at a point separated from the conductor surface by a distance of 1.0 cm.

17.37. Two straight, long, parallel conductors spaced in

air 20 cm apart carry currents of 24 and 16 A. Find the locus where the magnetic field strength is zero at equal and opposite currents through the conductors.

17.38. In each of two long, parallel conductors there is a current of 20 A. The conductors are spaced at 15 cm. For the cases of like and unlike directions of the current, determine the magnetic field strength at a point equidistant as to both conductors.

17.39. Two long, straight, isolate conductors lie in one plane at right angle to each other and carry equal currents I. Discuss the variation of the magnetic field induction along bisectors of angles made by the conductors with the distance x to the point of intersection of the conductor. Express the dependence with a formula.

17.40. Three parallel, straight, long conductors are located in air at equal distances (15 cm) from each other as indi-

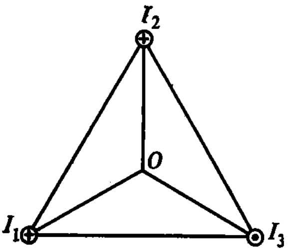


Fig. 17.40

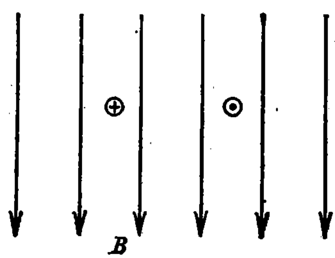


Fig. 17.41

cated in Fig. 17.40. The conductors carry equal currents $I = 12$ A directed as shown. Find the magnetic field strength and induction at a point O equidistant from all the three conductors.

17.41. Fig. 17.41 gives two parallel long conductors 5.0 cm apart located in a uniform magnetic field of induction $1.6 \cdot 10^{-4}$ T in a plane normal to magnetic induction lines. The conductors carry opposite currents of 20 A. Calculate the forces acting on the 1.5 m segments of the conductors. What will the forces be, if the currents in both conductors are reversed?

17.42. A straight long conductor is located normal to the lines of a magnetic field induction of $2.0 \cdot 10^{-4}$ T and carries a current of 50 A. Find the locus of zero induction. Calculate the force acting in air on a 50 cm segment of the conductor.

17.43. A conductor of flexible loop-shaped wire lies on a flat smooth surface. What configuration will it assume if a strong current is passed through it?

17.44. A current-carrying loop suspended from strings repels from the south pole of a magnet. Indicate the direction of current in the ring.

17.45. A ring conductor of radius 6.4 cm carries a current of 12.4 A. Determine the magnetic field strength and induction.

17.46. A current flowing through a ring of radius 5.8 cm causes at the centre a magnetic field of induction $1.3 \cdot 10^{-4}$ T. What is the magnetic field strength at the centre of the ring and the current through the conductor?

17.47. The magnetic field strength at the centre of a ring with current 11 A is found to be 120 A/m. Compute the diameter of the ring and the magnetic field induction at the centre.

17.48. Two conductors are concentric rings with radii 20 and 10 cm. The outer conductor carries a current of 10 A, the inner 6.0 A. Find the magnetic field strength at the centre of rings for equal and opposite currents.

17.49. Two ring conductors of equal radius having a common centre at O are located in mutually perpendicular planes. The magnetic induction of the resultant field at the centre is $B_0 = 2.0 \cdot 10^{-4}$ T. The magnetic induction due to the first conductor at the same point is $B_1 = 1.6 \cdot 10^{-4}$ T. If $I_1 = 8.0$ A, determine the magnetic induction due to the second conductor at point O and the current through it.

17.50. A ring conductor of radius 5.2 cm that carries a current $I_1 = 13.4$ A and a straight conductor that carries a current $I_2 = 22$ A lie in one plane (Fig. 17.50). The straight conductor is separated from the ring centre by a distance of 8.3 cm. The conductors are in air. Calculate the magnetic field strength and induction at the ring centre. What will the strength and induction be at the same point, if the current through the straight conductor is reversed?

17.51. Fig. 17.51 shows a loop carrying a current of 12 A. Given that the loop radius is 6.0 cm, what is the magnetic field strength at the centre of the loop?

17.52.* A 85 cm solenoid without a core has 750 turns, in which there is a current of 5.6 A. Calculate the magnetic field strength and induction inside the solenoid.

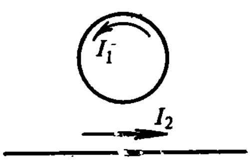


Fig. 17.50

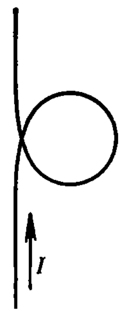


Fig. 17.51

17.53. A solenoid without a core has a length of 64 cm and contains 820 turns, the magnetic field induction inside it being $1.2 \cdot 10^{-3}$ T. What is the current flowing through the solenoid?

17.54. How many turns per centimetre of length must a coreless solenoid contain for the magnetic field induction inside to be no less than $8.2 \cdot 10^{-3}$ T at a current of 4.3 A?

17.55. A long solenoid of insulated 0.20 mm dia wire compactly wound in two layers carries a current of 0.52 A. What is the magnetic field strength inside the solenoid?

17.56. A ring conductor of radius 30 cm carries a current of 20 A. What is the magnetic moment of the ring?

17.57. A rectangular frame of wire has a length of 25 cm and a width of 12 cm. Its magnetic moment is 0.45 $A \cdot m^2$. What is the current in the frame? What is the maximum force couple acting on the frame in a uniform magnetic field with

* In problems 17.52-17.55 the diameter of solenoid is considered small as compared with its length.

induction 0.20 T? What is the angle between the vectors **P** and **B**?

17.58. Consider a flat coil with 40 turns. Determine its radius, if at a current of 3.5 A it has a magnetic moment of 1.33 A·m^2.

17.59. A small ball of radius q suspended from a string of length l moves uniformly in a circle horizontally so that the spring describes a conical surface that forms an angle of α with the vertical. Determine the magnetic field induction at the circle centre due to the movement of the ball and its magnetic moment.

17.60. In the unexcited hydrogen atom the electron travels in an orbit of radius $0.53 \cdot 10^{-10}$ m at a speed of $2.0 \cdot 10^6$ m/s. Regarding the electron orbiting as a circular current, compute the magnetic field induction at the orbit centre and the magnetic moment due to the electron orbiting.

17.61. In a hydrogen atom the electron rotates in an orbit of radius $2.12 \cdot 10^{-12}$ m. Determine the magnetic field induction at the orbit centre and the magnetic moment.

Magnetic Flux. Flux Linkage and Inductance. Work Done by Magnetic Forces

17.62. A flat plane of surface area 280 cm^2 is oriented normal to the magnetic induction lines in a uniform magnetic field of 250 A/m in the air. What is the magnitude of the magnetic flux through the plane?

17.63. A flat plane of area 2.4 m^2 is exposed in air to a uniform magnetic field of $1.2 \cdot 10^4$ A/m, whose direction makes an angle of 30° with the plane. What is the magnetic flux crossing the area?

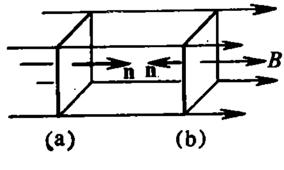


Fig. 17.64

17.64. A frame with an area of 100 cm^2 is located in a uniform magnetic field perpendicular to the magnetic induction lines so that the vector **n** normal to the frame plane coincides with the magnetic induction vector (Fig. 17.64, *a*). The magnetic field induction is $B = 0.20$ T. What is the

change in the magnetic flux crossing the frame, if the latter is turned 180° (Fig. 17.64, *b*)?

17.65. Consider a coil in which there is a flux linkage of 0.12 Wb at a current of 8.6 A. What is the inductance of the coil?

17.66. A coil generates a magnetic flux of 0.015 Wb when a current of 5.0 A is passed through its turns. The coil's inductance is 60 mH. How many turns are there in the coil?

17.67. There is a coreless coil through which a current of constant magnitude flows. The number of turns is doubled, the overall dimensions being the same. By how many times will the coil inductance change? By how many times will the flux linkage change?

17.68. A solenoid without a core has a length of 1.6 m and a radius of 4.8 cm. Its 1 400 turns carry a current of 6.3 A. Calculate the magnetic flux and flux linkage in the solenoid, and the solenoid inductance.

17.69. Consider a coreless solenoid of N turns of a compactly wound insulated wire with resistivity ρ, the constant voltage being U and the current I. The magnetic field strength within the solenoid is H. Assuming the uniform field and neglecting the insulation thickness, compute the inductance and flux linkage for the solenoid.

17.70. A 0.40 m conductor carrying a current of 21 A is located in a uniform magnetic field with induction 1.2 T. The conductor moving perpendicular to the magnetic induction lines, what is the work done in carrying the conductor by 0.25 m?

17.71. A conductor 0.50 m long that carries a current of 20.0 A is exposed to a magnetic field of induction 0.50 T making an angle of 30° with the magnetic induction vector. It is displaced by 2.0 m perpendicular to the direction of the current and the magnetic induction vector. What is the work done in the process?

17.72. A loop of 25 turns is placed in a magnetic field so that an external magnetic flux of 0.012 Wb traverses it. When a current of 8.4 A is passed through the turns, the external magnetic flux intersecting it is 0.077 Wb. Assuming the current in the circuit unchanged, determine the work done in turning the loop.

17.73. In a uniform magnetic field of induction 0.060 T

there is a rectangular frame 5.0 cm wide and 8.0 cm long. The frame has 200 turns and may rotate about an axis normal to the magnetic induction lines. When a current of 0.50 A flows through the turns, the frame orients perpendicular to the magnetic induction lines. Assuming the current in the circuit to be constant, what is the work required to turn the frame from the above position by 1/4 turn? By 1/2 turn? By a whole turn?

17.74. Will the solenoidal magnetic field strength and induction change, if an aluminium core is inserted in the solenoid?

17.75. A closed core (toroid*) of length 20 cm and cross-sectional area 3.1 cm^2 supports a coil of 1 000 turns. The winding carries a current of 0.16 A. Table XXI lists the variation of the magnetic field induction with the magnetic field strength for a ferromagnetic material of the core. What is the magnetic flux in the core? By how many times will the magnetic flux in the core increase, if the current is increased two times? Four times?

17.76. A mild iron core of 18 cm diameter has a winding of 1 200 turns. The cross section of the toroid is 2.4 dia circle. The induction vs strength dependence for the toroid is to be determined from the graph in Table XXI. What is the current required to be passed through the winding turns so that a magnetic flux of $5.4 \cdot 10^{-4}$ Wb is caused in the toroid? How will the magnetic flux through the toroid change, if the current is doubled?

Lorentz Force

17.77. Into a uniform magnetic field with induction 0.085 T an electron enters with a velocity of $4.6 \cdot 10^7$ m/s transversely to magnetic induction lines. Determine the force acting on the electron in the magnetic field, and the radius of a circle in which it moves.

17.78. An electron moves in a uniform magnetic field in vacuum perpendicular to magnetic induction lines in a

* Toroid refers to a doughnut-shaped body.

circle of radius 10 cm. Determine the velocity of the electron, if the field strength is $1.6 \cdot 10^2$ A/m.

17.79. Can it be considered that in a gas-filled tube the electrons in a uniform magnetic field in a plane perpendicular to the magnetic induction lines travel in a circle?

17.80. A proton moves in vacuum in a uniform magnetic field with strength 100 A/m in a plane perpendicular to magnetic induction lines. Determine the trajectory of the proton, if its velocity is $1.2 \cdot 10^3$ m/s. What is the period of revolution of the proton in the magnetic field?

17.81. A proton in vacuum accelerates from rest in an electric field with a potential difference of 1.5 kV and enters a uniform magnetic field at a right angle to the magnetic induction line. In the magnetic field it moves in a circle of radius 56 cm. Determine the magnetic field strength.

17.82. An electron accelerates in vacuum from rest under the action of an electric field and enters a uniform magnetic field perpendicular to the magnetic induction lines, moving in a circle of radius $7.58 \cdot 10^{-3}$ m with a period of $5.96 \cdot 10^{-10}$ s. Determine the accelerating potential difference and the magnetic field induction.

17.83. A uniform electric field of strength 100 V/cm is perpendicular to a uniform magnetic field with induction 0.020 T. The electron is ejected in these fields normal to the vectors **E** and **B**. At what velocity will the electron move rectilinearly? At what velocity will protons move rectilinearly?

17.84. An electron travels with a velocity of $2.5 \cdot 10^6$ m/s in vacuum in a uniform magnetic field of strength 75 A/m so that the velocity vector makes an angle of 30° with the field direction. Determine the radius of turns of the electron trajectory and the distance covered along the magnetic induction lines in three turns.

17.85. An electron ejected into a uniform magnetic field at an angle of 60° moves in a spiral of diameter 10.0 cm with a period of $6.0 \cdot 10^{-5}$ s. Determine the electron velocity. the magnetic induction and the spiral pitch.

17.86. Singly-charged neon ions with mass numbers 20 and 22 and kinetic energy $6.2 \cdot 10^{-16}$ J come into a uniform magnetic field of induction 0.24 T in vacuum at a right angle to induction lines, and having travelled in a semicircle,

leave the field in two beams as shown in Fig. 17.86. What is the separation between the beams?

17.87. A beam of singly-charged ions of a silicon isotope with a mass number of 28 in vacuum enters a magnetic field of induction 0.18 T perpendicular to magnetic induction lines and moves in a circle of radius 21 cm. Determine the kinetic energy of silicon isotope ions.

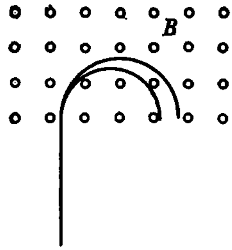


Fig. 17.86

17.88. Singly-charged argon ions accelerate from rest in an electric field with a voltage of 800 V, and then get into a uniform magnetic field with induction 0.32 T normal to the magnetic induction lines where they split into two beams travelling in a vacuum in a circle with radii 7.63 and 8.05 cm. What are the mass numbers of argon isotopes?

SEC. 18. ELECTROMAGNETIC INDUCTION

Example 52. A 1.2 m straight conductor of Fig. 52 is connected by flexible wires to a source of e.m.f. of 24 V and resistance of 0.5 Ω. It travels at a speed of 12.5 m/s

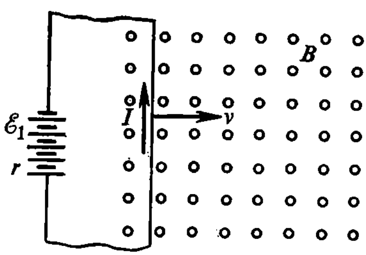


Fig. 52

in a uniform magnetie field with induction 0.8 T perpendicular to the field lines which are directed at the reader. The external circuit has a resistance of 2.5 Ω What is the

current flowing through the conductor? By how many times will the current change if the conductor comes to rest?

Given: $l = 1.2$ m is the length of the conductor, $\mathscr{E}_1 = 24$ V is the electromotive force of the source, $r = 0.5\ \Omega$ is the internal resistance of the energy source, $B = 0.8$ T is the magnetic field induction, $v = 12.5$ m/s is the velocity of the conductor, $R = 2.5\ \Omega$ is the resistance of the external circuit.

Determine: I_1—the current through the circuit,
I_2/I_1—the factor by which the current will change when the conductor comes to rest.

Solution. The current can be found from Ohm's law

$$I = \frac{\mathscr{E}}{r+R}$$

Here $\mathscr{E}$ is the e.m.f. for the electric circuit.

As the conductor moves in the magnetic field, in the circuit, besides the storage battery e.m.f. $\mathscr{E}_1$, there is produced the induced e.m.f. $\mathscr{E}_{\text{ind}} = Bvl$ directed so as to oppose $\mathscr{E}_1$ which may be established using the right-hand rule. Hence

$$\mathscr{E} = \mathscr{E}_1 - \mathscr{E}_{\text{ind}} \qquad I_1 = \frac{\mathscr{E}_1 - \mathscr{E}_{\text{ind}}}{r+R}$$

When the motion ceases $\mathscr{E}_{\text{ind}}$ reverts to zero, therefore $\mathscr{E}$ is equal to $\mathscr{E}_1$.

Now we find $\mathscr{E}_{\text{ind}}$

$$\mathscr{E}_{\text{ind}} = Bvl$$

$$\mathscr{E}_{\text{ind}} = 0.8\ \text{T} \cdot 12.5\ \text{m/s} \cdot 1.2\ \text{m} = 12\ \text{V}$$

During the conductor motion the current will be

$$I_1 = \frac{24\ \text{V} - 12\ \text{V}}{2.5\ \Omega + 0.5\ \Omega} = \frac{12\ \text{V}}{3\ \Omega} = 4\ \text{A}$$

For a conductor at rest the current is

$$I_2 = \frac{24\ \text{V}}{2.5\ \Omega + 0.5\ \Omega} = 8\ \text{A}$$

Next we seek the current ratio

$$I_2/I_1 = 8\ \text{A}/4\ \text{A} = 2$$

Answer. The current through the conductor in motion is 4 A. When the conductor stops the current doubles.

Example 53. In a coreless solenoid with 800 turns the current varies from 2.5 A to 14.5 A in 0.15 s, its magnetic flux increasing by $2.4 \cdot 10^{-3}$ Wb. What is the mean self-induced e.m.f. produced in the solenoid in the process? What is the energy of the magnetic field through the solenoid at a current of 5.0 A?

Given: $i_1 = 2.5$ A is the initial current, $i_2 = 14.5$ A is the final current, $N = 800$ is the number of turns in the solenoid, $\Delta\Phi = 2.4 \cdot 10^{-3}$ Wb is the change in the magnetic flux, $\Delta t = 0.15$ s is the current variation time, $I = 5.0$ A is the current of constant magnitude.

Determine: $\mathscr{E}_{av}$ — the average self-induced e.m.f. owing to the change in the current through the solenoid,

W_{mag} — the energy of magnetic field of the solenoid.

Solution. The average self-induced e.m.f. may be obtained from the formula

$$\mathscr{E}_{av} = -L \frac{\Delta i}{\Delta t}$$

The solenoid inductance can conveniently be found as follows. The flux linkage $\Psi = \Phi N$ in the solenoid varies directly with the current i, i.e. $\Phi_1 N = Li_1$ and $\Phi_2 N = Li_2$, hence $N \, \Delta\Phi = L \, \Delta i$.

We now find the solenoid inductance

$$L = \frac{N \, \Delta\Phi}{\Delta i} \quad L = \frac{800 \cdot 2.4 \cdot 10^{-3} \text{ Wb}}{14.5 \text{ A} - 2.5 \text{ A}} = 0.16 \text{ H}$$

And now we calculate $\mathscr{E}_{av}$

$$\mathscr{E}_{av} = -0.16 \text{ H} \cdot \frac{12 \text{ A}}{0.15 \text{ s}} = -13 \text{ V}$$

Note that the self-induced e.m.f. here may be also determined using the principal relation for the induced e.m.f.

$$\mathscr{E}_{ind} = -\frac{\Delta\Psi}{\Delta t} = -N \frac{\Delta\Psi}{\Delta t} \quad \mathscr{E}_{end} = -800 \cdot \frac{2.4 \cdot 10^{-3} \text{ Wb}}{0.15 \text{ s}} =$$

$$= -13 \text{ V}$$

The minus sign indicates that the induced e.m.f. opposes the increase of the field.

The magnetic energy is given by

$$W_{mag} = LI^2/2$$

We now proceed to calculate it

$$W_{mag} = \frac{0.16\ \text{H} \cdot 25\ \text{A}^2}{2} = 2\ J$$

Answer. The average self-induced e.m.f. in the solenoid is 13 V, the magnetic energy of the solenoid at a current of 5 A is 2 J.

Induced EMF. Induced Current. Lenz's Law

18.1. Figure 18.1 (*a*) shows a d.c. source connected across a galvanometer so that the pointer of the device deflects to the left. Discuss the direction of deflection of the galvanometer pointer (1) in inserting a permanent magnet with a velocity v into the coil with north pole on (Fig. 18.1, *b*); (2) in stopping the magnet inside the coil; (3) in removing the magnet with a velocity v from the coil with the south pole on (Fig. 18.1 (*c*)).

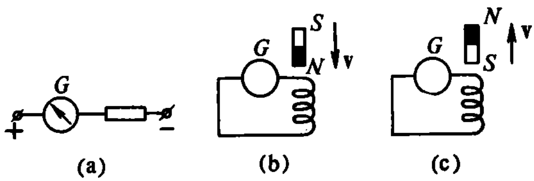


Fig. 18.1

18.2. In what direction should the permanent magnet of Fig. 18.2 be moved relative to the coil so that the magnetic needle would orient as shown?

18.3. Two closed loops *1* and *2* in Fig. 18.3 are located one after the other parallel to each other. The dependence of the pointer deflection on the current direction being as

in Fig. 18.1 (*a*), determine the direction of deflection of the galvanometer pointer in moving the rheostat slide in the loop *1* as shown.

18.4. Consider a coil with (*a*) closed-circuit winding, and (*b*) open-circuit winding. Determine if the work done in inserting the magnet in the coil is similar in both cases.

Fig. 18.2

18.5. Two similar magnets simultaneously start falling from the same height through fixed conducting coils: the first through a closed ring, the second through an open ring. Which magnet will fall first? Why?

18.6. A closed ring performs a translational motion in a uniform magnetic field: (*a*) along the magnetic induction lines, (*b*) perpendicular to them. Will an induced current arise in the ring?

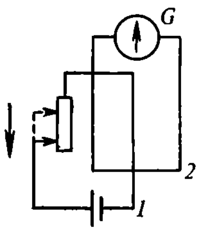


Fig. 18.3

18.7. A conducting loop performs a translatory motion in a magnetic field (*a*) uniform, (*b*) nonuniform. Will an induced e.m.f. arise in these cases?

18.8. Will a change in magnetic induction in a conducting loop always give rise to an induced e.m.f.? Induced current?

18.9. Is it always the case that a change in magnetic induction in a conducting loop oriented perpendicular to magnetic induction lines causes an induced e.m.f. in it?

18.10. A metal ring of radius 4.8 cm is in a magnetic field of induction 0.012 T normal to the magnetic induction lines. To remove it from the magnetic field requires 0.025 s. What is the average e.m.f. induced in the ring in the process?

18.11. A 18 $\times$ 5.0 cm wire rectangular frame is located in a uniform magnetic field perpendicular to the magnetic induction lines. When the field is switched off an average e.m.f. of $4.5 \cdot 10^{-3}$ V is induced in the frame in 0.015 s. Compute the induction of the field.

18.12. A frame with 25 turns is in a magnetic field. The

magnetic flux in the frame changes from 0.098 to 0.013 Wb in 0.16 s. Determine the induced e.m.f. caused in the frame.

18.13. In a coil containing 75 turns the magnetic flux is $4.8 \cdot 10^{-3}$ Wb. In what time would this flux vanish for an average induced e.m.f. of 0.74 V to arise in the coil?

18.14. A magnetic flux inside a coil changes from 0.024 Wb to 0.056 Wb in 0.32 s giving rise to an average induced e.m.f. of 10 V. How many turns must the coil have in this case?

18.15. A wire frame of 40 turns encloses an area of 240 cm^2. A uniform magnetic field is produced around it that is per-

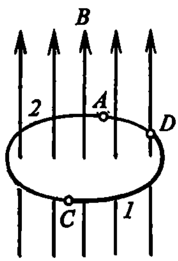


Fig. 18.16

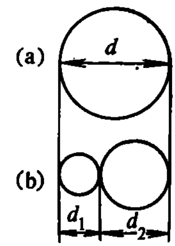


Fig. 18.17

pendicular to its plane. As the frame turns by 1/4 turn in 0.15 s an average induced e.m.f. of 160 mV is produced in it. Determine the magnetic field induction.

18.16. The wire loop *CDAC* in Fig. 18.16 has a resistance of 0.10 Ω and in it the magnetic flux grows steadily by $2.0 \cdot 10^{-1}$ Wb. What is the charge that passes through the loop as the magnetic field varies? What is the direction of the induced current? What is the direction of the induced current when the magnetic flux decreases?

18.17. A wire ring of diameter $d = 10.0$ cm and resistance $R = 0.20\ \Omega$ is located perpendicular to the lines of magnetic induction $B = 2.0$ T of a uniform magnetic field (Fig. 18.17, *a*). In $\Delta t = 0.10$ s the configuration of the ring becomes as shown in Fig. 18.17 (*b*), the diameter of the left ring being $d_1 = d/4$. What is the average induced e.m.f. in the loop and what is the charge q that will flow through the circuit during the time in question?

18.18. The diameter of the conducting loop in Fig. 18.16 is $d = 0.40$ m. It lies perpendicular to the lines of the mag-

netic induction of a uniform magnetic field. The magnetic induction grows at a constant rate $\Delta B/\Delta t = 0.020$ Wb/s. Discuss the work in carrying a charge $q = 3.0$ C from point C to point D along (*a*) the path *1*, (*b*) the path *2* (CAD). Determine the works A_1 and A_2 ($CD = d$).

18.19. The magnetic flux in a conducting loop with $N = 100$ turns varies as $\Phi = (2 + 5t)\cdot 10^{-2}$ [Wb]. Discuss the time dependence of the induced e.m.f. in the loop. What is the current through the loop, if the resistance of the conductor is $R = 2.5\ \Omega$? What is the meaning of the sign in the answer?

18.20. The magnetic induction of a uniform magnetic field varies according to the relationship $B = (2 + 5t^2)\cdot 10^{-2}$ [T]. A loop of area $S = 1.00\cdot 10^{-2}$ m^2 is located at a right angle to the magnetic induction lines. Determine the time dependence of the magnetic flux and the induced e.m.f.. Determine the instantaneous value of the magnetic flux and induced e.m.f. after 5 s.

18.21. A rectangular conducting loop with sides $a = 20.0$ cm and $b = 10.0$ cm containing $N = 100$ turns is placed perpendicular to uniform magnetic field lines, whose induction obeys the law $B = (3 + 2t^2)+10^{-2}$ [T]. Determine: (1) the time dependence of the flux linkage and induced e.m.f. $\Psi = f(t)$, $e = f(t)$, and (2) instantaneous values of the flux linkage and induced e.m.f. after 10 s.

18.22. A conducting loop with area $S = 200$ cm^2 is placed in a uniform magnetic field so that the normal vector **n**

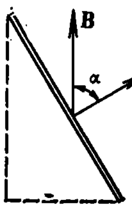


Fig. 18.22

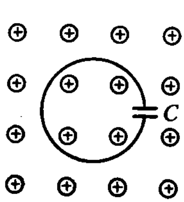


Fig. 18.23

makes an angle of $\alpha = 60°$ with the vector of magnetic induction **B** (Fig. 18.22). The magnetic induction varies according to the law $B = 2.0\cdot 10^{-2} \cos(4\pi t + \pi/6)$ [T].

Determine: (1) $\Phi = f(t)$, (2) $e = f(t)$, (3) the instantaneous value of the induced e.m.f. after 4 s.

18.23. A conducting loop of Fig. 18.23 has an area $S = 4.0 \cdot 10^2$ cm^2 and incorporates a capacitor with capacitance $C = 10.0$ μF. It is placed in a uniform magnetic field perpendicular to the magnetic induction lines, the magnetic induction varying as $B = (2 + 5t) \cdot 10^{-2}$ [T]. Determine: (1) the maximum charge of the capacitor, (2) the maximum energy of the capacitor electric field, (3) the capacitor plate—upper or lower—that is charged positively.

18.24. In a magnetic field with magnetic induction lines as shown in Fig. 18.24 a conductor travels with a velocity

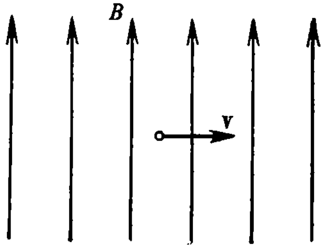


Fig. 18.24

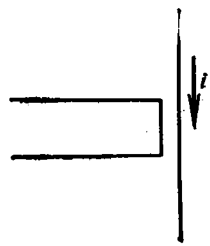


Fig. 18.25

v. What is the direction of the induced current through the conductor?

18.25. An arrow in Fig. 18.25 indicates the direction of the induced current in the conductor that travels to the reader in the field of a magnet. Which of the magnet poles is shown in the figure?

18.26. A conductor shown in Fig. 18.26 travels between the magnet poles at a right angle to the magnetic induction lines, the induced current through the conductor being directed to the reader. What is the direction of motion of the conductor?

18.27. A wire frame of Fig. 18.27 rotates about a straight conductor carrying a current I so that the conductor is a stationary rotation axis. Will there be any current through the frame? Answer the same question, if the rotation axis is one of the frame sides.

18.28. An automobile travels at 120 km/h. The length of the front axle is 180 cm, the vertical component of the geomagnetic field strength is 40 A/m. Find the potential difference across the axle.

18.29. A straight conductor travels at a speed of 25 m/s in a uniform magnetic field with induction 0.0038 T normal

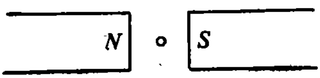


Fig. 18.26

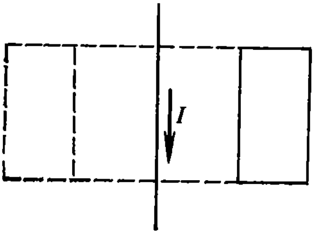


Fig. 18.27

to the magnetic induction lines. If a voltage of 28 mV is applied across the ends of the conductor, what is the length of the conductor?

18.30. A straight conductor of length 120 cm moves in a uniform magnetic field at an angle of 17° to magnetic induction lines with a velocity of 15 m/s. If in the conductor an induced e.m.f. of 6.2 mV is caused, what is the magnetic field induction?

18.31. A straight conductor of length 86 cm moves at a velocity of 14 m/s in a uniform magnetic field with an induction of 0.025 T. If in the conductor an e.m.f. of 0.12 V is produced, determine the angle between the vectors of induction of the field and the velocity.

18.32. A conductor of length 20.0 cm travels in a uniform magnetic field of $79.6 \cdot 10^3$ A/m at 2.0 m/s so that the conductor and the velocity vector lie in a plane perpendicular to the field strength lines. The velocity vector makes an angle $\beta = 30°$ with the conductor. Determine the potential difference across the conductor ends.

18.33. In a vertical uniform magnetic field with induction $B = 0.20$ T a conductor of length $l = 0.50$ m oriented horizontally performs a translatory motion with velocity $v = 10.0$ m/s so that the velocity vector makes an angle

of $\alpha = 30°$ with the vector of magnetic induction and an angle of $\beta = 60°$ with the conductor (Fig. 18.33). Determine the induced e.m.f. in the conductor.

18.34. A conducting rod of length $l = 0.20$ m rotates at 3 000 rpm in a uniform magnetic field with induction $B = 2.0 \cdot 10^{-2}$ T about an axis parallel to the magnetic induction vector and passing through the rod end at a right angle. Determine the induced e.m.f. in the rod.

18.35. A metal disc of radius $l = 10.0$ cm that is placed perpendicular to a uniform magnetic field with induction

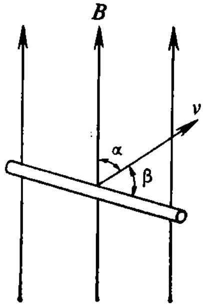


Fig. 18.33

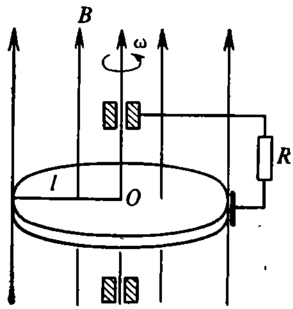


Fig. 18.35

$B = 0.50$ T rotates about an axis passing through the centre with a constant angular velocity ω (Fig. 18.35). Using sliding contacts—one on the disc axis and the other on its external circumference—the disc is connected with a resistance $R = 2.5\ \Omega$. Determine: (1) mechanical power consumed in rotating the disc at a current $I = 0.10$ A in the circuit, (2) the angular velocity of the disc. Ignore the losses to friction.

18.36. A straight conductor of length 1.4 m is in a uniform magnetic field with induction $7.4 \cdot 10^{-2}$ T. It rotates in a plane normal to the magnetic induction lines with an angular velocity of 75 rad/s. Determine the potential difference across the conductor ends, if the rotation axis passes through the middle of the conductor; through the end of the conductor; at 1/4 of the length of the conductor from one of the ends.

18.37. Figure 18.37 shows a conductor of resistance $r = 0.50\ \Omega$ travelling without friction at a velocity $v = 1.0$ m/s over two conducting horizontal parallel rods spaced at $l = 0.50$ m and short-circuited across a resistance $R = 1.5\ \Omega$. The conductor travels perpendicular to lines of force of a uniform vertically oriented magnetic field with induction $B = 0.40$ T. Determine: (1) the current in the circuit, (2) the force in the direction of motion to be applied

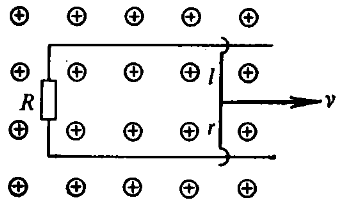


Fig. 18.37

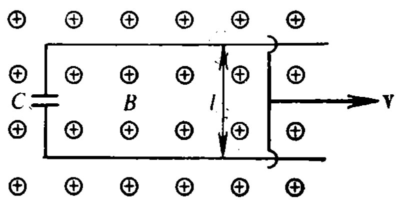


Fig. 18.39

to the conductor for the latter to move with a specified velocity, (3) the thermal power dissipated by the circuit. Ignore the resistance of guiding rods.

18.38. Using the data of problem 18.37, determine the mechanical power needed to move a conductor of mass $m = 100$ g with a coefficient of friction $k = 0.10$.

18.39. Figure 18.39 shows two parallel conducting rods spaced at $l = 0.20$ m at a right angle to a uniform magnetic field with induction $B = 0.10$. T, along which a conductor is moving so that its vector of velocity $\mathbf{v} = 0.50$ m/s is normal to it. Determine the charge q and energy W of the electric field of a capacitor with capacitance $C = 20.0\ \mu$F that is connected in the circuit.

18.40. Referring to Fig. 18.40, consider a conductor of length $l = 0.30$ m with a resistance $r = 1.0\ \Omega$ which moves along conducting guides at a velocity of $v = 5.0$ m/s normal to a uniform magnetic field with induction $B = 0.40$ T. The guides are short-circuited with resistances $R_1 = 3.0\ \Omega$ and $R_2 = 6.0\ \Omega$. Determine the current through the moving conductor and resistances R_1 and R_2. What mechanical power is needed for the motion of the conductor? Ignore the friction.

18.41. A square wire frame with a side $l = 0.20$ m and resistance $R = 1.0\ \Omega$ cuts the flux of a uniform magnetic field of width $b > l$, travelling at a constant velocity of $v = 4.0$ m/s at an angle $\alpha = 30°$ to the magnetic induction lines $B = 0.40$ T (vector **v** lies in the same plane as the frame). Determine the amount of heat Q liberated in the frame.

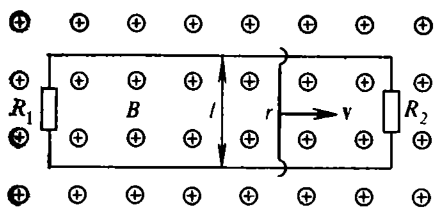


Fig. 18.40

18.42. Two vertical conducting rods short-circuited from above by a resistance $R = 2.0\ \Omega$ are located in a plane perpendicular to the magnetic induction vector $\mathbf{B} = 0.50$ T of a uniform magnetic field. A conductor of mass $m = 0.010$ kg slides down along the rods without friction and loss of contact. The rods are separated by a distance $l = 0.20$ m. Determine the quantity of heat given off in the circuit per second, and the steady-state velocity of the conductor.

18.43. Referring to Fig. 18.43, a horizontal connector of mass $m = 0.10$ kg and length $l = 1.0$ m slides without fric-

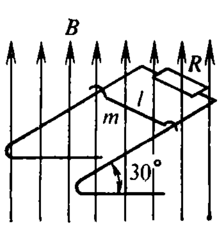


Fig. 18.43

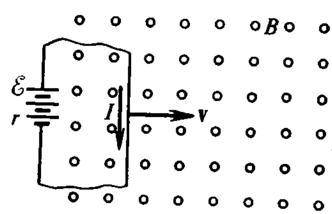


Fig. 18.44

tion along the two parallel conducting rods inclined at an angle $\alpha = 30°$ to the horizon. At the top the rods are short-circuited with resistance $R = 2.0\ \Omega$. The system is in a

vertical uniform magnetic field with induction $B = 1.0$ T. Ignoring the resistance of the rest of the circuit and the friction, what is the current in the circuit and the established velocity of motion?

18.44. Figure 18.44 shows a storage battery of e.m.f. $\mathscr{E} = 12$ V and internal resistance $r = 0.50\ \Omega$ that is connected with flexible wires to a conductor of length $l = 80$ cm located in a uniform magnetic field with induction $B = 0.45$ T directed at the reader. When the conductor travels perpendicular to the magnetic induction lines with a constant velocity v, in the circuit there is a current of 4.0 A. The resistance of the external circuit connected to the storage battery is 3.5 Ω. Determine the velocity of motion of the conductor.

18.45. A straight conductor with length 1.5 m is in a uniform magnetic field with induction 0.14 T directed at the reader. It is connected to a storage battery with e.m.f. 8.5 V. The impedance of the whole electric circuit is 3.2 Ω. What is the current in the circuit, when the conductor starts moving at a speed of 16 m/s as shown in Fig. 52?

18.46. A conductor travels as shown in Fig. 18.44. As the motion is reversed, the magnitude of the velocity being the same, the current in the circuit changes by 0.40 A. Given that the conductor is 0.50 m long, the uniform magnetic field has an induction of 0.10 T, and the circuit impedance is 1.0 Ω, what is the magnitude of the velocity?

18.47. In your opinion, is an electric field with closed field lines possible in the space? If so, under what conditions?

18.48. Are there any closed magnetic field (induction) lines in the space free of electric charges? If so, under what conditions?

18.49. What is the change in the magnetic field needed for the vorticity electric field to be constant? Variable?

18.50. The magnetic induction grows. Indicate the direction of magnetic lines of the vorticity electric field being generated. What rules are to be observed here?

18.51. On a vertically disposed coil there is a metal object. The object heats when a variable current flows through the coils and remains cold at direct current. Explain why.

18.52. Figure 18.52 shows two copper cubes composed of identical insulated plates and suspended from strings. The cubes rotate at a constant angular velocity between the

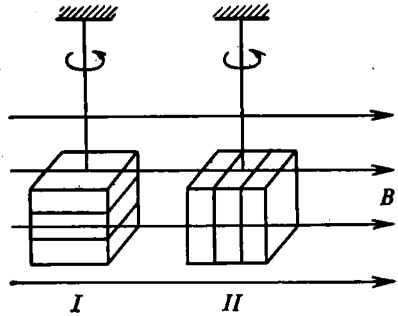


Fig. 18.52

poles of a deenergised electric magnet. Which cube will come to rest later on switching the electric magnet on?

18.53. Indicate in Fig. 18.52 the direction of vorticity currents in the cube *II* for the position shown.

18.54. An aluminium disc suspended from strings is free to rotate. As a magnet below starts to rotate fast, the disc begins to rotate too. Why? In what direction? Can it rotate with the same velocity as the magnet?

18.55. A copper plate is suspended from a string near a pole of a strong electric magnet whose winding carries a direct current. What will happen if the current is increased quickly? If the current through the winding is decreased quickly?

18.56. Can a strong electric magnet repel and attract one and the same object made of a nonferromagnetic conducting material? If so, account for the effect.

Self-Induction. Energy of Magnetic Field

18.57. When is the self-induced e.m.f. larger: in closing or breaking a d.c. current circuit?

18.58. Prove that the self-induced e.m.f. in closing a d.c. circuit is lower than the e.m.f. of the source.

18.59. If a current of 3.8 A in a coil with inductance 68 mH drops to zero in 0.012 s, what is the self-induced e.m.f. arising in it?

18.60. Determine the inductance of a coil if as the current in it falls off by 2.8 A in 62 ms, the coil develops an average self-induced e.m.f. of 14 V.

18.61. In a coil with inductance 240 mH the current grows from zero to 11.4 A producing an average self-induced e.m.f. of 30 V. How long does it take?

18.62. Find the instantaneous value of the self-induced e.m.f. produced in a circuit with inductance $L = 25$ mH in changing the current through it according to the law $i = (3 + 4t)\cdot 10^{-1}$ [A].

18.63. As the current in a circuit varies according to the law $i = (1 - 0.2t)$ [A] a self-induced e.m.f. of $2.0\cdot 10^{-2}$ V is produced. What is the inductance of the circuit?

18.64. A current of 7.5 A flowing through a coil with 120 turns gives rise to a magnetic flux of $2.3\cdot 10^{-3}$ Wb. Find the energy of the magnetic field of the coil.

18.65. Determine the inductance of a coil, if its magnetic field has an energy of 0.32 J at a current of 6.2 A.

18.66. The magnetic field of a coil with inductance 95 mH has an energy of 0.19 J. What is the current through the coil?

18.67. Across a coil with resistance 8.2 Ω and inductance 25 mH a constant voltage of 55 V is maintained. How much energy will be liberated in breaking the coil circuit? What is the average self-induced e.m.f. generated in the coil in the process, if the energy will be released during 12 ms?

18.68. What energy conversions occur in an electric circuit in increasing the current after the circuit is closed?

18.69. In a coreless solenoid with $N = 400$ turns wound on a cardboard cylinder of radius $r = 2.0$ cm and length $l = 0.40$ m the current follows the law $i = 0.20t$ [A]. Determine the energy of the magnetic field after ten seconds, and the self-induced e.m.f. in the coil.

18.70. The energy of a magnetic field in a solenoid increases by $1.0\cdot 10^{-2}$ J with the current raised by 2.0 A. The average current in the circuit being 5.0 A, what is the inductance of the solenoid?

18.71. In a solenoid a current I gives rise to a magnetic field with energy W. The resistance of the winding is R. What charge will the winding pass in uniform decreasing the current n-fold? What is the change in the magnetic field energy?

CHAPTER III

OSCILLATIONS AND WAVES

SEC. 19. MECHANICAL OSCILLATIONS AND WAVES. SOUND

Example 54. A ball of mass 100 g suspended from a massless spring with a stiffness of 10 N/m performs harmonic oscillation with an amplitude of $4.0 \cdot 10^{-2}$ m. If the oscillation is sustained and the initial phase is zero, determine (1) the displacement of the ball during the time $t_1 = 52.36 \cdot 10^{-3}$ s from the start of the oscillation, (2) the total energy of the harmonic motion and its kinetic energy at the instant it passes through the equilibrium, (3) the kinetic and potential energy at $t_2 = T/6$ from the beginning of the motion.

Given: $m = 0.10$ kg is the mass of the ball, $k = 10$ N/m is the stiffness of the spring, $\varphi_0 = 0$ is the initial phase of the harmonic motion, $t_1 = 52{,}36 \cdot 10^{-3}$ s and $t_2 = T/6$ are the times elapsed from the beginning of the oscillations, $A = 4.0 \cdot 10^{-2}$ m is the amplitude of oscillation.

Determine: x_1—the displacement of the ball at the time t_1,
W—the total energy of the harmonic motion,
E_{k0}—the kinetic energy at the instant the ball passes through the equilibrium,
E_{k2}—the kinetic energy of the harmonic motion,
E_{p2}—the potential energy of the harmonic motion at $t_2 = T/6$.

Solution. (1) The displacement of the ball in harmonic oscillation is given by

$$x = A \sin(\omega t + \varphi_0) \quad \text{or} \quad x = A \sin\left(\frac{2\pi}{T} t + \varphi_0\right)$$

As stated, $\varphi_0 = 0$, thus

$$x = A \sin\left(\frac{2\pi}{T} t\right)$$

The period of elastic harmonic oscillation is given by the relation $T = 2\pi \sqrt{m/k}$, where m is the mass of the body.

We find the displacement x_1 using the formula

$$x_1 = A \sin\left(\sqrt{k/m}\ t_1\right)$$

$$x_1 = 4.0 \cdot 10^{-2}\,\text{m} \cdot \sin\left(\sqrt{10\,\text{N}\cdot\text{m}^{-1}/0.10\,\text{kg}} \cdot 52.36 \cdot 10^{-3}\,\text{s}\right)$$
$$= 4.0 \cdot 10^{-2}\,\text{m} \cdot \sin(0.5236\,\text{rad}) \approx 2 \cdot 10^{-2}\,\text{m}$$

(2) The total energy of the harmonic motion is

$$W = \frac{1}{2} kA^2$$

As at the instant the pendulum passes through the equilibrium all the energy of the motion is kinetic, then

$$E_{k0} = W = \frac{1}{2} kA^2 = \frac{1}{2} mv_0^2$$

Next we proceed to calculate the total energy of the oscillatory motion and the maximum kinetic energy of an oscillating material point which is equal to the former. We have thus

$$W = \frac{1}{2} \cdot 10\,\text{N/m} \cdot (4.0 \cdot 10^{-2}\,\text{m})^2 = 8.0 \cdot 10^{-3}\,\text{J}$$

(3) The kinetic energy of the ball is obtainable from the relationship

$$E_k = \frac{1}{2} mv_0^2 \cos^2 \varphi$$

where $v = v_0 \cos \varphi$ is the instantaneous value of the velocity, v_0 is the maximum velocity, $\varphi = (2\pi/T)\,t + \varphi_0$ is the phase of oscillation. As stated, the initial phase is zero, and hence $\varphi = (2\pi/T)\,t$. As $(1/2)\,mv_0^2 = W$ is the total energy (see item 2), then

$$E_k = \frac{1}{2} mv_0^2 \cos^2 \varphi = \frac{1}{2} kA^2 \cos^2 \varphi = W \cos^2 \varphi$$

Substituting for φ, we get

$$E_{k2} = \frac{1}{2} kA^2 \cos^2\left(\frac{2\pi}{T}\, t_2\right) = W \cos^2\left(\frac{2\pi}{T}\, t_2\right) = W \cos^2 \frac{\pi}{3}$$

The potential energy at t_2 is

$$E_{p2}=\frac{1}{2}kx_2^2=\frac{1}{2}kA^2\sin^2\varphi=W\sin^2\varphi$$

where $x_2 = A\sin\varphi$. In this case

$$E_{p2}=\frac{1}{2}kA^2\sin^2\left(\frac{2\pi}{T}t_2\right)=W\sin^2\left(\frac{2\pi}{T}t_2\right)$$

The potential energy can be found from the energy conservation law

$$E_{p2} = W - E_{k2}$$

Substituting the data of the problem gives the kinetic and potential energies at the time t_2

$$E_{k2}=8.0\cdot10^{-3}\,\mathrm{J}\cdot\frac{1}{4}=2.0\cdot10^{-3}\,\mathrm{J}$$

$$E_{p2}=8.0\cdot10^{-3}\,\mathrm{J}-2.0\cdot10^{-3}\,\mathrm{J}=6.0\cdot10^{-3}\,\mathrm{J}$$

Answer. (1) The displacement of the ball is about $2\cdot10^{-2}$ m.
(2) The total energy of the oscillatory motion is $8.0\cdot10^{-3}$ J.
(3) At t_2 the kinetic energy is $2.0\cdot10^{-3}$ J, and the potential energy is $6.0\cdot10^{-3}$ J.

Example 55. A lift car, to whose ceiling a simple pendulum of length 1.0 m is suspended, begins to descend vertically with an acceleration $a_1 = g/4$. After a time $t_1 = 3.0$ s since the onset of the motion the car begins to move steadily, and then during a time interval of 3.0 s it slows down to rest. Determine: (1) the period of harmonic oscillations of the pendulum at each of the path sections, (2) the period of harmonic oscillations of the pendulum in horizontal motion of the suspension point with an acceleration of $a_4 = g/4$.

Given: $a_1 = g/4$ is the acceleration of the car in the first section of the trip, $t_1 = 3.0$ s is the time of travel in the first section, $v_2 =$ const is the car speed in the second section, $t_3 = 3.0$ s is the time of travel in the third section, $v_3 = 0$ is the speed of the car at the end of the third section, $a_4 = g/4$ is the acceleration of the suspension point at the horizontal section of the trip, $g \approx 9.8$ m/s^2 is the free-fall acceleration, $l = 1.0$ m is the length of the pendulum.

Determine: T_1, T_2, and T_3—the periods of oscillation of the pendulum in the respective sections of the trip, T_4—the period of oscillation in horizontal travel of the suspension point.

Solution. The period of harmonic oscillation of a simple pendulum is given by $T = 2\pi \sqrt{l/g}$. In uniform motion of the suspension point $T_2 = 2\pi \sqrt{l/g}$. In uniformly variable motion of the suspension point the period is given by $T = 2\pi \sqrt{l/g'}$, where the acceleration g' of the pendulum is

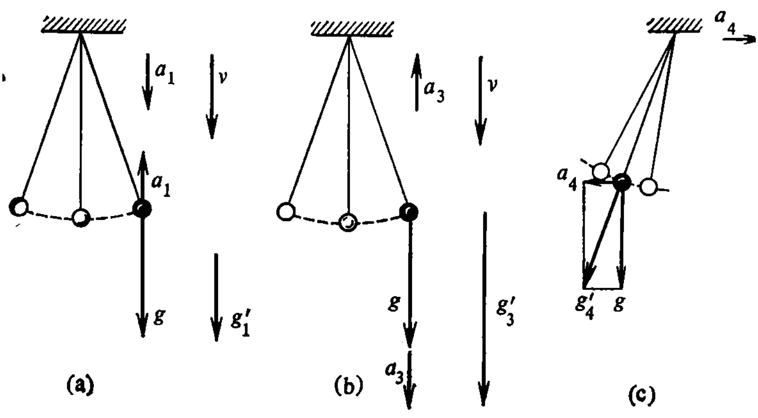


Fig. 55

given by the relationship $\mathbf{g}' = \mathbf{g} + \mathbf{a}$. The vector $\mathbf{a}$ is equal in magnitude to the vector of acceleration of the suspension point and opposite to the latter in direction.

As the suspension point moves down with the acceleration $\mathbf{a}_1$ (Fig. 55, *a*), the acceleration of the pendulum is $g_1' = g - a_1$.

As the suspension point slows down (Fig. 55, *b*), $g_3' = g + a_3$ where $a_3 = a_1$, because the suspension point slows down to rest during the same time interval, as it accelerates from rest (as stated $t_1 = t_3 = 3.0$ s).

For the horizontal section of the trip (Fig. 55, *c*) we have

$$g_4' = \sqrt{a_4^2 + g^2} = \sqrt{(g/4)^2 + g^2}$$

We now determine the periods of oscillation of the pendulum in each section of the trip

$$T_1 = 2\pi\sqrt{\frac{l}{g-a_1}} = 2\pi\sqrt{\frac{1.0\ \text{m}}{9.8\ \text{m/s}^2 - {}^1/_4\, 9.8\ \text{m/s}^2}} \approx 2.3\ \text{s}$$

$$T_2 = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{1.0\ \text{m}}{9.8\ \text{m/s}^2}} \approx 2.0\ \text{s}$$

$$T_3 = 2\pi\sqrt{\frac{l}{g+a_1}} = 2\pi\sqrt{\frac{1.0\ \text{m}}{9.8\ \text{m/s}^2 + {}^1/_4\, 9.8\ \text{m/s}^2}} \approx 1.8\ \text{s}$$

$$T_4 = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a_4^2}}}$$

$$= 2\pi\sqrt{\frac{1.0\ \text{m}}{\sqrt{(9.8\ \text{m/s}^2)^2 + ({}^1/_4\, 9.8\ \text{m/s}^2)^2}}} \approx 0.62\ \text{s}$$

Answer. The period of oscillation of the pendulum in the first section of the trip is $T_1 \approx 2.3$ s; in the second section $T_2 \approx 2.0$ s; in the third section $T_3 \approx 1.8$ s; for the horizontal section $T_4 \approx 0.62$ s.

Example 56. A plane wave excited by a vibrator obeying the law $x = 0.20 \sin 62.8\, t$ [m] propagates at a velocity $u = 10$ m/s. Write the equation for the plane wave. Determine: (1) the length of the running wave λ; (2) the displacement of particles during the period; (3) the displacement of points 1 and 2 located along the ray at distances $l_1 = 10.25$ m and $l_2 = 10.75$ m from the vibrator in 5 s after the beginning of vibrator operation; (4) the phase difference for points 1 and 2; (5) the length of the standing wave produced as the result of the interference of the wave from the vibrator, and the wave reflected from an obstacle.

Given: $x = 0.20 \sin 62.8t$ [m] is the equation describing the vibrator for operation, $u = 10$ m/s is the velocity of the wave propagation, $l_1 = 10.25$ m and $l_2 = 10.75$ m are the distances along the ray from the vibrator to points 1 and 2, $t = 5.0$ s is the time elapsed from the start of vibrator oscillation to the time for which the displacements of points 1 and 2 are determined.

Write: the equation of the plane wave for the given conditions.

Determine: λ—the length of the running wave,
s—the displacement of particles of the medium during the period of oscillation,
x_1 and x_2—the displacements of points 1 and 2,
$\Delta\varphi$—the phase difference for oscillations of points 1 and 2,
λ_{st}—the length of the standing wave.

Solution. The equation of the plane wave has the form

$$x = A \sin 2\pi\nu \left(t - \frac{l}{u}\right)$$

Now we write the plane wave equation for the vibrator

$$x = 0.20 \sin 62.8 \left(t - \frac{l}{10}\right) \text{ [m]}$$

In this case the initial phase of vibrator oscillation is zero.

(1) The length of the running wave is given by the relationship $\lambda = u/\nu$, and the unknown frequency ν is found from the equation of vibrator oscillation:

$$\lambda = \frac{u}{\nu} \text{ where } \nu = \frac{\omega}{2\pi}$$

$$\lambda = \frac{u \cdot 2\pi}{\omega} = \frac{10 \text{ m/s} \cdot 6.28}{62.8 \text{ s}^{-1}} = 1.0 \text{ m}$$

(2) The displacement of particles of the medium is obtained considering that they participate in the vibrational motion only. Such a displacement is zero.

(3) The displacement of points 1 and 2 after 5 s from the beginning of vibrator oscillation is determined from the plane wave equation, substituting for l_1 and l_2

$$x_1 = 0.20 \sin 62.8 \left(5 - \frac{10.25}{10}\right) \text{ m} = -0.20 \text{ m}$$

$$x_2 = 0.20 \sin 62.8 \left(5 - \frac{10.75}{10}\right) \text{ m} = 0.20 \text{ m}$$

(4) The phase difference for points 1 and 2 is found from the formula

$$\Delta\varphi = 2\pi\nu \frac{(l_2 - l_1)}{u}$$

$$\Delta\varphi = 62.8 \frac{\text{rad}}{\text{s}} \frac{(10.75 - 10.25)\ \text{m}}{10\ \text{m/s}} = 3.14\ \text{rad}$$

The points oscillate in antiphase.

(5) The length of the standing wave is determined, given that it is equal to that of the running waves, the interference of which is responsible for its formation.

$$\lambda_{st} = \lambda = 1.0\ \text{m}$$

Answer: $x = 0.20 \sin 62.8 \left(t - \frac{l}{10}\right)$. (1) The length of the running wave is 1.0 m. (2) The displacement of points of the medium during the period of oscillation is zero. (3) The displacements of points 1 and 2 are —0.20 and 0.20 m, respectively. (4) The phase difference is 3.14 rad. (5) The length of the standing wave is 1.0 m.

Example 57. As a sound source emitting waves of frequency $\nu_0 = 360$ Hz approaches a stationary receiver, the latter detects sound vibrations of frequency $\nu_1 = 400$ Hz (sound Doppler effect).

Taking the air temperature to be $t = 16$°C, determine: (1) the velocity of the sound source, (2) the frequency of sound vibrations excited in a stationary receiver as the source moves away with the same velocity, (3) the frequency of vibrations detected, if the source is stationary, and the receiver approaches it with a velocity equal to that of the source in the first case.

Given: $\nu_0 = 360$ Hz is the frequency of sound vibrations emitted by the source, $\nu_1 = 400$ Hz is the frequency of sound vibrations as received by a stationary receiver from an approaching sound source, $t = 16$°C is the air temperature.

Determine: v—the speed of the source,

ν_2—the frequency of sound vibrations excited in a stationary receiver by the receding source travelling at a speed v,

ν_3—the frequency of sound vibrations received by the sound receiver approaching the stationary source with speed v.

Solution. The temperature dependence of the sound velocity in air is known to be $u = 20\sqrt{T}$ [m/(s·K$^{1/2}$)], where $T = (273 + t)$ is the air temperature in Kelvin degrees. To determine v—the speed of the sound source approaching the stationary receiver—we establish the relationship between the vibration frequency as seen by the receiver, ν_1, the frequency of sound vibrations emitted by the source ν_0, the velocity of sound in a given medium, u, and the speed of the source, v. The distance covered by the vibrations emitted by the source and travelling in a stationary medium at a velocity u during the period of vibration T_0 is given by $\lambda_0 = uT_0$. In the same time the sound source travels through the distance $l = vT_0$, which reduces the distances between the closest point of the medium vibrating in phase in the direction of motion of the source, and increases the distances between them as the source travels in the opposite direction, i.e.

$$\lambda_1 = \lambda_0 - l \qquad \lambda_2 = \lambda_0 + l$$

or

$$\lambda_1 = uT_0 - vT_0 = (u - v)\,T_0 = (u - v)/\nu_0$$

$$\lambda_2 = (u + v)\,T_0 = (u + v)/\nu_0$$

These waves travelling with speed u arrive at the receiver at rest and excite in it sound vibrations with either the frequency $\nu_1 = u/\lambda_1$, or frequency $\nu_2 = u/\lambda_2$. Substituting for λ_1 and λ_2 gives

$$\nu_1 = \frac{u}{(u - v)/\nu_0} = \frac{\nu_0}{1 - v/u} \qquad \nu_2 = \frac{\nu_0}{1 + v/u}$$

or

$$\nu = \frac{\nu_0}{1 \mp v/u}$$

(The negative sign signifies the approaching source; the positive sign, the receding source.) Now, knowing ν_0, ν, and u, we find the velocity of the source, v:

$$v = u\,(\nu_1 - \nu_0)/\nu_1 = u\,(1 - \nu_0/\nu_1)$$

Let us look at the receiver approaching the source at rest. The source being stationary, the wave length does not vary and is equal to λ_0. Approaching the source at speed v, the receiver travels as regards the waves with velocity $u + v$

with the result that vibrations with the period $T_3 = \lambda_0/(u + v)$ or frequency $\nu_3 = (u + v)/\lambda_0$ are excited in it. As $\lambda_0 = u/\nu_0$ we have

$$\nu_3 = \nu_0 (u + v)/u = \nu_0 (1 + v/u)$$
$$\nu = \nu_0 (1 \pm v/u)$$

(The plus sign signifies the approaching receiver; the minus, the receding one.)

The dissimilar dependences of the received frequencies on the motion of the source or receiver are associated with the sound velocity being independent of these movements. What is important is not the relative motion of the source and receiver, but their motion relative to the elastic medium in which they are produced and propagate.

Using the numerical data of the problem we obtain

$$u = 20\ \mathrm{m/(s \cdot K^{1/2})} \cdot \sqrt{(273 + 16)\ \mathrm{K}} = 340\ \mathrm{m/s}$$
$$v = 340\ \mathrm{m/s} \cdot (1 - 360\ \mathrm{Hz}/400\ \mathrm{Hz}) = 34\ \mathrm{m/s}$$
$$\nu_2 = 360\ \mathrm{Hz}/(1 + 34\ \mathrm{m \cdot s^{-1}}/340\ \mathrm{m \cdot s^{-1}}) \approx 327\ \mathrm{Hz}$$
$$\nu_3 = 360\ \mathrm{Hz} \cdot (1 + 34\ \mathrm{m \cdot s^{-1}}/340\ \mathrm{m \cdot s^{-1}}) = 396\ \mathrm{Hz}$$

Answer. (1) The sound source travels at 34 m/s. (2) As the source recedes with the same speed the receiver measures the vibrations with frequency 327 Hz. (3) As the receiver approaches with the same speed (34 m/s) to the source at rest the receiver measures sound vibrations at frequency 396 Hz.

Oscillations

19.1. A material point performs 300 oscillations in 1.0 min. Determine the period and frequency of oscillations.

19.2. A material point vibrates with frequency $\nu = 10$ kHz. Calculate the period and the number of oscillations per minute.

19.3. Two spring pendulums oscillate vertically with equal periods. One of the pendulums starts oscillating with a delay of two periods; half a period. Discuss the directions of the relative velocities of these pendulums at any instant. How do the pendulums oscillate relative to each other?

19.4. What is the smallest phase difference for pendulum oscillations shown in Fig. 19.4? The displacement of each pendulum is equal to the amplitude. Will the phase difference be retained in case *a*? *b*?

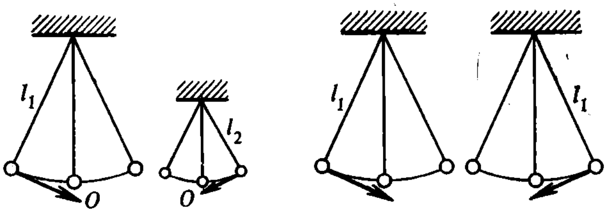


Fig. 19.4

19.5. A material point performs harmonic oscillations with the initial phase $\varphi_0 = 0$ and amplitude A. Determine the displacement from equilibrium at $t_1 = 0$, $t_2 = T/12$, $t_3 = T/4$, and $t_4 = T/2$.

19.6. What is the time needed for a material point performing harmonic oscillations to travel through the first half of the amplitude? Through the second half of the amplitude?

19.7. Write the equations of harmonic oscillations with the following parameters: (1) $A = 10$ cm, $\varphi_0 = \pi/4$ rad, $\omega = 2\pi$ rad/s; (2) $A = 5.0$ cm; $\varphi_0 = \pi/2$ rad, $T = 2.0$ s; (3) $A = 4.0$ cm, $\varphi_0 = \pi$ rad, $\nu = 2.0$ Hz.

19.8. A material point performs harmonic oscillations according to the law $x = 2 \sin\left(\frac{\pi}{4} t + \frac{\pi}{2}\right)$, where x is expressed in cm, t in seconds. Determine: the amplitude of oscillations, A; the initial phase, φ_0; the period of oscillations, T.

19.9. Derive the equation of harmonic oscillations for $A = 5.0 \cdot 10^{-2}$ m, $\varphi_0 = 0$, $T = 0.010$ s. Determine: the frequency of oscillations, ν; the angular frequency, ω; the amplitude of speed, v_{max}, and acceleration, a_{max}; the total energy of harmonic oscillations for a body of mass $m = 0.10$ kg.

19.10. A body of mass 0.10 kg oscillates harmonically after the law $x = 1.00 \cdot 10^{-1} \sin (314t + \pi/2)$ [m]. Determine: the amplitude of displacement, A; the initial phase, φ_0; the angular frequency, ω; the frequency of oscillations, ν; the period T; the amplitude of speed, v_{max}; and acceleration, a_{max}; the maximum kinetic energy, E_{max}.

19.11. A material point oscillating with frequency 10 Hz passes the equilibrium at a velocity of 6.28 m/s. Determine the maximum displacement and acceleration; write the equation of harmonic oscillations with zero initial phase.

19.12. The velocity of an oscillating body of mass $2.0 \cdot 10^{-1}$ kg is given by $v = 6.0 \cdot 10^{-2} \sin 100t$ [m/s]. Write the equation of the harmonic oscillations. Determine the maximum values of velocity and acceleration of the body, the energy of the harmonic oscillations of the body.

19.13. The velocity of a material point is $v = 2\pi \cdot 10^{-1} \cos 2\pi t$ [m/s]. Determine: the maximum acceleration; the displacement of the point in $t = 5/12$ s from the beginning of oscillations; the distance covered during the time.

19.14. Using the equation $x = 2.0 \cdot 10^{-1} \sin \pi t$ [m], determine: the displacement of the material point after 1.5 s from the onset of the oscillations; the distance travelled during the time; the restoring force acting on the point at the specified instant of time, if the mass of the material point is 0.20 kg.

19.15. For the data of problem 19.14 determine: the displacement, acceleration, restoring force, and potential energy after 1/6 s from the onset of oscillations.

19.16. A weight lies on a horizontal support performing harmonic oscillations vertically. What is the maximum acceleration of the support at which the weight will just stay on it? What will the amplitude of oscillation be here, if the period of oscillations is 0.50 s?

19.17. A horizontal board performs harmonic oscillations horizontally with a period of 2.0 s. It carries a body. At what amplitude will the body start to slide? The coefficient of static friction is 0.20.

19.18. A cylinder of mass m with base area S floats in a liquid of density ρ. It is sunk a bit deeper and let go.

Determine the period of harmonic oscillations of the cylinder. Ignore the resistance of the medium.

19.19. Two vertical communicating vessels contain a mass m of liquid. When disturbed the liquid performs oscillations. The liquid density is ρ, the cross-sectional area of each vessel is S. Determine the period of oscillations of the liquid.

19.20. For the data of problem 19.19 determine the period of oscillations of the liquid, if the cross-sectional areas of the vessels are S_1 and S_2.

19.21. A weight of mass 0.10 kg suspended from a spring performs vertical oscillations with amplitude 4.0 cm. Determine: the period of harmonic oscillations of the weight, if an elastic expansion of the spring by 1.0 cm requires a force of 0.10 N; the energy of harmonic oscillations of the pendulum. Ignore the mass of the spring.

19.22. A weight of mass m lying on a smooth stationary horizontal surface is drawn with the force $F = mg$ using a spring of stiffness k and then let go. Derive the equation of harmonic oscillations of the weight. Determine the energy of oscillations. How would the period of oscillations change if the system were transferred to the Moon? Ignore the mass of the spring.

19.23. A weight of mass 0.20 kg suspended from a spring performs 30 swings per minute with an amplitude of 0.10 m. Determine the stiffness of the spring and the kinetic energy of the weight after $^1/_6$ period past the equilibrium.

19.24. A weight of mass m is suspended from two massless springs with stiffness k_1 and k_2. Determine the period of harmonic oscillations of the weight with the spring connected in series; in parallel, if the weight is suspended in-between on a massless rod.

19.25. A simple pendulum of length 99.5 cm performs 30 full swings per minute. Calculate the period of oscillations of the pendulum and the free fall acceleration at the place where the pendulum is located.

19.26. Consider a simple pendulum of length 1.0 m. Assuming the free fall acceleration to be 9.81 m/s^2, compute the period of the oscillations. By how many times and how should the length of the pendulum be increased for the period to be doubled?

19.27. A simple pendulum completes one swing in 2 s. If the free fall acceleration is 9.81 m/s^2, determine the length of the pendulum. By how many times should the length of the pendulum be increased for its frequency to be doubled?

19.28. There are two simple pendulums, performing in the same period of time: one, 10 swings; the other, 20 swings. What is the ratio of the lengths of the pendulums?

19.29. Two identical pendulums oscillate on the Moon and the Earth ($g_E \approx g_M/6$). Find the ratio of the periods of oscillations of these two pendulums.

19.30. Two simple pendulums are suspended from the ceiling. In the same time one pendulum performs 5 swings, and the other 3 swings. What is the length of each pendulum, if the difference in their lengths is 48 cm?

19.31. Two simple pendulums of length 0.996 and 0.294 m simultaneously start oscillating in phase. What is the time required for their phases to be similar again? How often will it occur? ($g \approx 9.81$ m/s^2).

19.32. Two small balls are suspended from inextensible strings of equal length. One is raised up to the suspension point, the other, with a taut string, is displaced by a small angle from the vertical so that its oscillations may be regarded as harmonic. The balls are let go simultaneously. Which will come to the equilibrium position earlier?

19.33. Consider the simple pendulum of Fig. 19.33. The string bend point B lies on one vertical with the suspension point C at a distance of $l/2$ from the latter. Determine the period of oscillations.

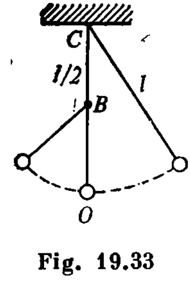


Fig. 19.33

19.34. A simple pendulum fastened at an end of an absolutely rigid massless rod oscillates with a period T deflecting by a small angle α from the vertical. A vertical wall is placed in the way of the pendulum that is perpendicular to the plane of oscillations so that the pendulum deflects in the direction of the wall by an angle $\beta = \alpha/2$. Assuming the collision absolutely elastic and ignoring the time of collision, determine the new period T_1 of oscillations of the pendulum.

19.35. Consider a simple pendulum on a planet whose mass and radius are four times as large as those of the Earth. What is the ratio of periods of the pendulum on the planet and that on the Earth?

19.36. A pendulum watch is right on the pole. It is transferred to the equator. If $g_p \approx 9.832$ m/s^2, $g_{eq} \approx$ ≈ 9.78 m/s^2, compute the lagging of the clock on the equator in 24 hours.

19.37. A pendulum clock is right at sea-level. Calculate the lagging of the clock in 24 hours if elevated at a height $h = 4.0$ km. The radius of the Earth is $R_E \approx 6.4 \cdot 10^3$ km.

19.38. A clock with a brass pendulum begins to lose on raising the temperature by 20°C. Assuming the pendulum to be simple and the coefficient of linear expansion for brass to be $\alpha = 2.0 \cdot 10^{-5}$ K^{-1}, compute the lagging of the clock in 24 hours.

19.39. A steel ball suspended from a long string executes vertical harmonic oscillations. A magnet is brought near from beneath. How will the tension in the string, the restoring force, and the period change?

19.40. The suspension point of a simple pendulum of length l travels vertically with an acceleration a. Determine the period of oscillations of the pendulum for $a < g$ when the latter is moving upward and downward.

19.41. The period of oscillations of a simple pendulum in a rocket raising up is one half of that on the Earth. Assuming the free fall acceleration to be constant and equal to g, determine the acceleration of the rocket.

19.42. There is a simple pendulum in a spaceship with the engines switched off. Determine the period of oscillations. Discuss the character of motion of the pendulum after the engines are switched off if at the instant of the switching-off (1) the pendulum is in its extreme position, (2) the pendulum moves.

19.43. The suspension point of a simple pendulum moves along a horizontal and straight trajectory with acceleration a. Determine the ratio of the period T_1 in this accelerated motion to the period T of the same pendulum (*a*) with fixed suspension point, (*b*) with uniform straight motion of the point.

19.44. The suspension point of a simple pendulum moves

in a vertical plane with a constant acceleration a directed at an angle α to the vertical. Determine the period of harmonic oscillations of the pendulum with length l. The free fall acceleration is g.

19.45. Consider a simple pendulum in a train car moving in a curve of radius R with a constant speed. The period of the pendulum is n times smaller than with uniform and straight motion of the train at the same speed. Determine the speed of the train. The free fall acceleration is g.

19.46. A simple pendulum that is essentially a ball of mass m carrying a charge $+q$ suspended from a string of length l is placed in the electric field of a flat capacitor charged to a voltage U. The capacitor plates are separated by a distance d. Determine the period of oscillations for the pendulum if the capacitor is oriented horizontally? Vertically?

19.47. A spaceship travels far from celestial bodies. A simple pendulum of length l suspended in the ship has a period of oscillations T. Determine the acceleration of the ship.

19.48. A simple pendulum that is a heavy ball of mass m suspended from a string of length l undergoes harmonic motion swinging by a small angle α from the vertical. Determine the energy of the motion and its maximum speed. The free fall acceleration is g.

19.49. A simple pendulum of mass m undergoing harmonic motion with an amplitude of A has an energy W. Determine the frequency of oscillations of the pendulum and the length of the string. Will the energy of the harmonic motion change if the amplitude is doubled, and the frequency is reduced by half?

19.50. A simple pendulum swinging from a small angle α from the vertical passes through the equilibrium position with a velocity v. Given that the oscillations are harmonic, determine the period of oscillations.

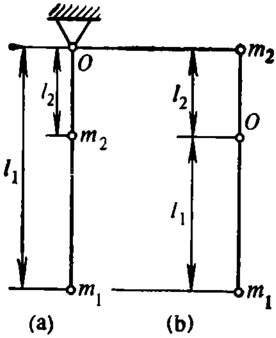
19.51. Determine the reduced length of a compound pendulum with a period of 4.0 s.

19.52. Determine the frequency of harmonic oscillations of a compound pendulum with a reduced length of 1.0 m.

19.53. Two small balls of masses m_1 and m_2 are suspended from a massless rod at distances l_1 and l_2 from a suspension

point O. For the cases indicated in Fig. 19.53 (a) and (b) determine the period of small oscillations of the rod.

19.54. A material point is simultaneously involved in two harmonic motions. What rule (scalar or vector addition) is used to determine the resulting displacement of the material point?



(a) (b)

Fig. 19.53

19.55. Using the curves of Fig. 19.55, write the equations of harmonic oscillations. Write the equation of the resulting oscillations and plot its curve. The oscillations are along one straight line.

19.56. Use the curves of Fig. 19.56 to answer the questions of problem 19.55. Determine the phase difference of the constituent oscillations

19.57. Using the curves of Fig. 19.57 answer the questions of problems 19.55 and 19.56

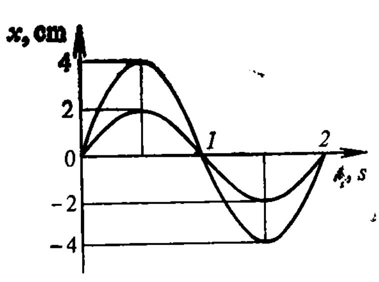


Fig. 19.55

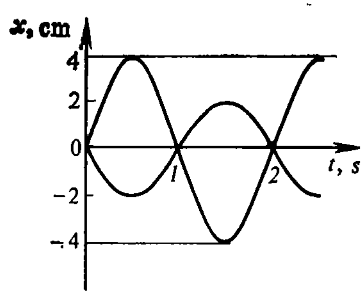


Fig. 19.56

19.58. Determine from Fig. 19.58: (1) amplitudes of oscillations, (2) frequencies, (3) maximum velocities, (4) maximum accelerations, (5) if it is possible to find the initial phase using the figure, (6) write the equations of the harmonic oscillations.

19.59. Two harmonic oscillations of similar period T with amplitudes A_1 and A_2 occur along the same straight

line. Determine the waveform of the resulting oscillation. Write its equation for oscillations in phase; in antiphase; at phase difference $\pi/2$.

19.60. Two harmonic oscillations with equal periods and amplitudes $A = 5.0$ cm and $A_2 = 2.0$ cm occur along the

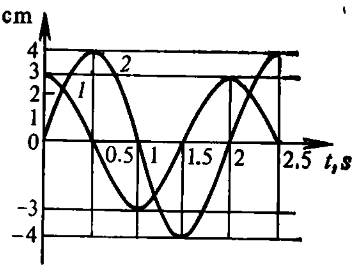


Fig. 19.57

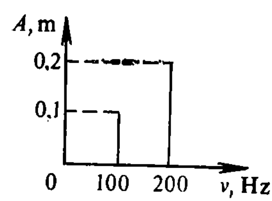


Fig. 19.58

same straight line. The period is $T = 1.2$ s. What is the period of the resulting oscillations? What are the maximum and minimum possible values of the amplitude of the resulting oscillations and what are the corresponding phase differences? What is the displacement of the resulting oscillations with equal phases in 0.10 s after the oscillations start?

19.61. Compare the free oscillations with the forced oscillations.

19.62. Figure 19.62 shows some pendulums. For which of the pendulums is a resonance possible? When will the resonance come sooner: with strong or weak damping of the natural oscillations?

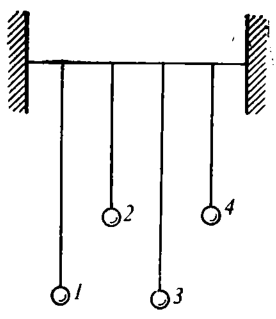


Fig. 19.62

19.63. In a resonance the energy of an oscillating system increases. Explain why.

19.64. In case of an unwanted resonance, what is to be done to stop it?

19.65. A train travels along a track made of rail pieces of length l. Given that the period of natural oscillations of train cars is T_0, determine the train speed required for the amplitude of vertical vibration of the car to be maximum.

Waves

19.66. If a wave having a frequency 200 Hz travels at a velocity of 340 m/s, what is the wavelength?

19.67. A source of vibration of period 0.0020 s excites in water waves with a length of 2.9 m. What is the velocity of wave propagation?

19.68. A running wave propagates with a velocity of 5 000 m/s at a frequency of 100 Hz. Determine the distance between the nearest points of the wave that lie on the same ray and are in phase.

19.69. A vibrator produces a wave with a period 0.010 s travelling at a speed of 340 m/s. There are two points located on the same ray so that the nearest point is 6.8 m away from the vibrator and the separation between the points is 3.4 m, 1.7 m, and 0.85 m. The amplitude of vibration of the points is the same and equal to 1.0 cm. Determine the phase difference at the points. Determine the displacement of the points when the vibrator displacement is zero.

19.70. A source vibrates with a period $T = 1.0 \cdot 10^{-3}$ s. Two points lying on a ray are separated from the s urce by $l_1 = 12$ m and $l_2 = 14$ m. The phase difference between the points is $3\pi/2$ rad. Determine the velocity of wave propagation.

19.71. A vibrator O in Fig. 19.71 produces a wave of length 4.0 m. Two points *1* and *2* are separated from the vibrator by distances $l_1 = 8.0$ m and $l_2 = 10$ m. What is the phase difference of the points?

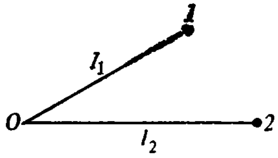


Fig. 19.71

19.72. Past a stationary observer standing on the shore of a lake, four wave crests passed in six seconds. The distance between the first and third crests is 12 m. Determine: the period of vibration of water particles, the travel speed and wave length.

19.73. A motor boat travels at 54 km/h. The distance between wave crests is 10 m, the period of vibration of water particles in the wave is 2 s. With what frequency do the waves strike the board of the boat if it travels with the waves? Into the waves?

19.74. The vibrator operation is described by the relation $x = 3.0 \cdot \sin 20\pi t$ [cm]. Assuming a plane wave, determine the displacement of a point located at a distance of 5.0 m from the vibration source 0.10 s after the onset of vibrations, if the wave propagates at 200 m/s.

19.75.* A plane wave of length λ propagates in a medium with a speed u. The amplitude of vibrations is A. Determine: the maximum velocity of vibrations of medium particles, v_{max}; the displacement of medium particles during a period. Write the equation of the plane wave.

19.76. The amplitude values of displacement and speed of a plane wave are respectively A and v_{max}. Assuming the velocity of wave travel in a given medium to be u, write the equations for the displacement and velocity. Determine the instantaneous values of displacement and velocity at a point located at a distance $l = \lambda/4$ from the vibrator in $t = (^3/_4)T$ after the vibration start.

19.77. A plane wave of length λ and frequency ν has an amplitude A. Determine: the velocity of wave travel, u; the maximum velocity of vibration of medium particles v_{max}; the displacement of particles during a period.

19.78. Two coherent sources produce transverse waves in phase. The periods of vibrations are $1.0 \cdot 10^{-1}$ s, the velocities of wave propagation in the medium are 1,000 m/s. What is the difference in the wave paths required for the intensification of vibrations to occur in overlapping?

19.79. Two coherent sources vibrate in phase with a frequency of 300 Hz. The velocity of propagation of the vibrations in the medium is 1 500 m/s. Determine the minimum difference of wave paths required for the vibrations to be maximally intensified, and maximally suppressed. What is the interference at a point separated from the first source by a distance of 20 m, and from the second, by 30 m?

---

* In problems 19.75-19.77 the initial phase of vibrations is taken to be zero.

19.80. Determine the length of a standing wave, if the first and third nodes are 0.20 m apart.

19.81. What is the phase difference at points of a standing wave vibrating between two neighbouring nodes? What are the phases of points of a standing wave on either side of a node (within $\lambda/2$ of it)?

19.82. Consider two points on a ray placed 5.0 cm and 15 cm apart and having equal amplitudes. What is the length of the standing wave?

19.83. A tuning fork produces in air a standing wave such that the distance between neighbouring nodes is 38.6 cm. If the sound velocity is 340 m/s, determine the frequency of vibrations of the tuning fork.

Sound

19.84. An observer located at a distance of 4 000 m from a cannon hears the sound of a shot 12.0 s after the flash. Determine the sound velocity in the air.

19.85. A man perceives sounds within the frequency range from 16 to 20,000 Hz. Given that the sound velocity in the air is 340 m/s, determine the corresponding wavelength interval.

19.86. The sound velocity in the air is related to the air temperature by the formula $u = 332\sqrt{1 + \alpha t}$ [m/s], where $\alpha = 1/273°C$ and t is the temperature in Celsius degrees. Determine the wavelength of sound produced in the air by a source vibrating at 100 Hz at air temperatures 0, 15, 20°C.

19.87. Determine the wavelength of sound in the water, if its wavelength in the air is 0.797 m. The sound velocity in the air is taken to be 343 m/s; in the water, 1 483 m/s.

19.88. From one ship to another two sound signals are transmitted simultaneously: one over the air, the other over the water. One signal is received 2.0 s after the other. Given that the sound velocity in the air is 340 m/s, and in the water 1 480 m/s, determine the distance between the two ships.

19.89. How does the sound velocity in a medium depend on the speed of travel of the source?

19.90. The sound velocity in the air is u, the wind speed is v. What is the sound velocity u' with wind for an observer who is stationary relative to air? To the Earth?

19.91. In man the aftersound lasts for about 0.10 s. If the sound velocity is 340 m/s, at what distance from a barrier must a man be to distinguish between the main and reflected sounds?

19.92. Consider the situation in Fig. 19.92. What is the length l of air column required for the amplitude of sound vibrations with wavelength λ and the loudness at the outlet to be maximum?

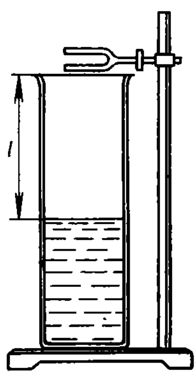


Fig. 19.92

19.93. Referring to Fig. 19.92, determine the minimum length of the air column in the vessel in resonance with the vibrations of the 440 Hz tuning fork. The sound velocity in the air is 344 m/s.

19.94. Determine the wavelength produced by an ultrasound generator in aluminium at a frequency of 10 MHz, if the sound velocity in aluminium is 5 100 m/s.

19.95. Consider two stationary ships. From one of them an ultrasound signal is transmitted through the water to be received by the sonic detector of the other ship twice: at times t_1 and t_2 ($t_2 > t_1$) after the generation. Given that the bottom is horizontal and sound velocity in the water is u, determine the depth of the sea H.

19.96. A sound source travelling at 17 m/s generates a signal during 2.0 s. What is the signal duration for a stationary observer, if the source approaches? Recedes? If the observer travels with the source, how long does he hear the signal? The sound velocity in the air is 340 m/s.

19.97. A 600 Hz sound source travels past a stationary observer with a speed of 40.0 m/s. The air temperature is 17°C. Compare the sound frequencies as perceived by the observer with an approaching and receding source.

SEC. 20. ALTERNATING CURRENT *

Example 58. A loop with 100 turns rotates steadily in a uniform magnetic field. The magnetic induction flux is given by $\Phi = 2.0 \cdot 10^{-3} \cos 314t$ [Wb]. Determine the time dependence of the e.m.f. induced in the loop; the maximum and effective values of the e.m.f.; the instantaneous value of the e.m.f. for $t = 0.0050$ s. How will the time variation of e.m.f. change if the angular velocity of the loop rotation is doubled?

Given: $N = 100$ is the number of turns of the loop; $\Phi = 2.0 \cdot 10^{-3} \cdot \cos 314t$ [Wb] is the time variation of the magnetic flux through the loop, $t = 0.0050$ s is the time at which the instantaneous value of the e.m.f. is to be determined.

Determine: $e = f(t)$—the time dependence of the e.m.f. induced in the loop;
$\mathscr{E}_0$—the maximum value of the e.m.f.;
$\mathscr{E}$—the effective value of the e.m.f.;
e—the instantaneous value of the e.m.f.; at $t = 0.0050$ s;
$e_1 = f(t)$—the time variation of the e.m.f. when the angular velocity is doubled.

Solution. The instantaneous value of the e.m.f. induced in each turn is equal to the negative of the time rate of change of the magnetic flux, i.e. $e = -\Phi'_t$; for N turns $e = -N\Phi'_t$. As $\Phi = \Phi_0 \cos \omega t$, we get

$$e = N\omega\Phi_0 \sin \omega t = \mathscr{E}_0 \sin \omega t$$

Now we find the time variation of the e.m.f.

$$e = 100 \cdot 314 \cdot 2.0 \cdot 10^{-3} \sin 314t = 62.8 \sin 314\, t \text{ [V]}$$

The maximum value of the e.m.f. will be

$$\mathscr{E}_0 = N\omega\Phi_0 = 62.8 \text{ V}$$

And the effective e.m.f. is

$$\mathscr{E} = \frac{\mathscr{E}_0}{\sqrt{2}} = \frac{N\omega\Phi_0}{\sqrt{2}} \approx 44.5 \text{ V}$$

---

* In Sections 20 and 21 symbols $\mathscr{E}$ and E, common in general physics, are retained for e.m.f. and electric field strength unlike electric engineering.

The instantaneous value of the e.m.f. is obtained by substituting in the relationship $e = \mathscr{E}_0 \sin \omega t$ for t. We thus have for $t = 0.0050$ s

$$e = 62.8 \sin 314 \cdot 0.0050 = 62.8 \sin \frac{3.14}{2} = 62.8 \text{ V}$$

It is seen from the relation $e = N\omega\Phi_0 \sin \omega t$ that if ω is doubled, the maximum value of the e.m.f. and the angular velocity of the e.m.f. variation are also doubled. The instantaneous value of the e.m.f. is given by

$$e_1 = N \cdot 2\omega\Phi_0 \sin 2\omega t$$

where ω is the initial angular velocity of the loop. We next find the dependence $e_1 = f(t)$ for the double angular velocity

$$\begin{aligned} e_1 = N \cdot 2\omega\Phi_0 \sin 2\omega t &= 2 \cdot 62.8 \cdot \sin 2 \cdot 314t \\ &= 125.6 \sin 628t [\text{V}] \end{aligned}$$

Answer. The time variation of the e.m.f. induced in the loop is given by $e = 62.8 \sin 314t$[V]; the maximum e.m.f. is $\mathscr{E}_0 = 62.8$ V; the effective e.m.f. is $\mathscr{E} \approx 44.5$ V; the instantaneous e.m.f. at $t = 0.0050$ s is 62.8 V. When the angular velocity is doubled, the instantaneous e.m.f. induced in the loop is given by $e_1 = 125.6 \sin 628t$.

Example 59. A water-wheel generator produces alternating current at standard frequency, its rotor rotating at 120 rpm. How many pairs of magnetic poles has the rotor?

Given: $n = 120$ rpm is the number of revolutions of the rotor per minute; $\nu = 50$ Hz is the standard frequency of alternating current.

Determine: p—the number of pairs of magnetic poles.

Solution. The alternating current frequency ν is related to the generator rotor frequency ν_{mec} by $\nu = p\nu_{mec}$.

The rotational speed of the rotor per second is $n/60$, then

$$\nu = p \frac{n}{60}$$

Hence

$$p = \frac{60\nu}{n} = \frac{60 \cdot 50 \text{ Hz}}{120 \text{ rpm}} = 25$$

Answer. The rotor of the water-wheel generator has 25 pairs of magnetic poles.

Example 60. As a coil is connected into a 12 V d.c. mains an ammeter reads 4.0 A. As the same coil is connected to a 12 V, 50 Hz a.c. mains the ammeter reads 2.4 A. Determine the inductance of the coil. What is the power in the circuit,

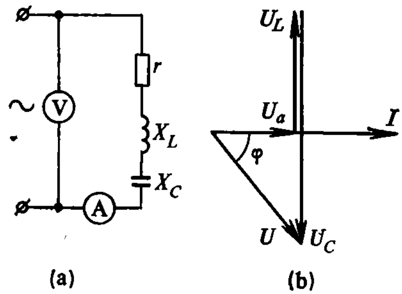


Fig. 60

if a 394 μF capacitor is connected in parallel with the coil (Fig. 60,*a*)? Draw the vector diagram for the case.

Given: $U_{d.c.} = 12$ V is the d.c. voltage, $I_{d.c.} = 4.0$ A is the direct current, $U_{a.c.} = 12$ V is the a.c. voltage, $I_{a.c.} = 2.4$ A is the alternating current, $\nu = 50$ Hz is the alternating current frequency, $C = 394 \cdot 10^{-6}$ F is the capacitor rating.

Determine: L—the inductance of the coil;

P—the active power in the circuit.

Solution. There being no reactance with a direct current, we have from Ohm's law the active resistance of the coil

$$R = U_{d.c.}/I_{d.c.}$$

For alternating current, according to the same law, the coil impedance is

$$Z_c = U_{a.c.}/I_{a.c.}$$

Then X_L is obtained from

$$Z_c^2 = R^2 + X_L^2$$

Knowing X_L and the frequency ν, and considering that $X_L = \omega L = 2\pi\nu L$, we arrive at L.

We find the active resistance of the coil

$$R = U_{d.c.}/I_{d.c.} = 12\,V/4.0\,A = 3.0\,\Omega$$

and the coil impedance

$$Z_c = U_{a.c.}/I_{a.c.} = 12\,V/2.4\,A = 5.0\,\Omega$$

Now we proceed to calculate X_L and L for the coil, and also the resistance of the capacitor, X_C

$$X_L = \sqrt{Z_c^2 - R^2} = \sqrt{25\,\Omega^2 - 9.00\,\Omega^2} = 4.0\,\Omega$$

$$L = \frac{X_L}{2\pi\nu} = \frac{4.0\,\Omega}{6.28\cdot 50\,\text{Hz}} = 0.0127\,\text{H} = 12.7\,\text{mH}$$

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{6.28\cdot 50\,\text{Hz}\cdot 394\cdot 10^{-6}\,\text{F}} = 8.0\,\Omega$$

The active power with a capacitor included in the circuit is given by

$$P = U_{a.c.} I'_{a.c.} \cos\varphi$$

where $I'_{a.c.}$ is

$$I'_{a.c.} = \frac{U_{a.c.}}{Z} = \frac{U_{a.c.}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

The required power factor is obtained as follows

$$\cos\varphi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Substituting the numerical values gives

$$\cos\varphi = \frac{3.0\,\Omega}{\sqrt{9.0\,\Omega^2 + (8.0 - 4.0)^2\,\Omega^2}} = 0.6$$

$$I'_{a.c.} = \frac{12\,\text{V}}{\sqrt{9.0\,\Omega^2 + 16\,\Omega^2}} = 2.4\,\text{A}$$

Hence

$$P = 12\,\text{V}\cdot 2.4\,\text{A}\cdot 0.6 = 17.3\,\text{W}$$

As X_C is larger than X_L, here the voltage vector will lag behind the current vector by the angle φ. Taking into account that U_C is twice as large as U_L, we construct the vector diagram (Fig. 60*b*).

Answer. The inductance of the coil is 12.7 mH, the active power in series connection of the coil and the capacitor is 17.3 W.

Time Variation of Induced EMF. Maximum and Effective Values of EMF and Current

20.1. A loop with 60 turns rotates steadily at 360 rpm in a uniform magnetic field of induction 0.025 T about a fixed axis normal to magnetic induction lines. Each loop side parallel to the rotation axis is 96 cm long, the separation from the rotation axis being 20 cm. Determine the time dependence of the induced e.m.f. Find the maximum and effective values of e.m.f. for the loop.

20.2. A wire loop of area 100 cm^2 with 100 turns rotates steadily in a uniform magnetic field of induction 0.30 T about an axis perpendicular to the magnetic induction lines. Initially the loop plane is normal to the magnetic induction vector. Determine the instantaneous induced e.m.f. in the loop 0.10 s from the rotation onset. Find the effective e.m.f. What is the average e.m.f. during the period? The amplitude value of e.m.f. is 1.2 V.

20.3. The magnetic flux on a loop rotating steadily in a uniform magnetic field is given by $\Phi = 2.0 \cdot 10^{-2} \cdot \cos 314t$ [Wb]. Find the time variation of the inductive e.m.f. in the loop. Determine the maximum and effective values of e.m.f.

20.4. On a loop with 100 turns rotating at a constant speed in a uniform magnetic field the magnetic flux varies as $\Phi = 1.0 \cdot 10^{-4} \cos 628t$ [Wb]. Determine the frequency of e.m.f. variation, its maximum and effective values.

20.5. The electromotive force in an a.c. circuit is given by $e = 120 \sin 628t$ [V]. Determine the effective e.m.f. and its period. How will the time dependence change if, other things being equal, the rotational speed is doubled?

20.6. A loop with 45 turns and area 3.6×10^2 cm^2 is placed in a uniform magnetic field with an induction of 0.032 T. Its ends are connected to slit-rings with brushes so that the axis of rotation of the loop is perpendicular to the magnetic induction lines, and the slit-rings shift from one brush to another when the e.m.f. is zero. Determine the

average potential difference between the brushes for the loop rotating steadily at 420 rpm.

20.7. A neon-filled lamp is connected in an a.c. circuit of industrial frequency and voltage 127 V. The lamp firing voltage is 84 V. Determine the duration of and time interval between flashes of the neon-filled lamp. The blanking voltage of the lamp is taken to be equal to the firing voltage.

20.8. The rotor of a four-polar generator rotates at 1 500 rpm. Determine the frequency of the alternating e.m.f. of the generator.

20.9. A water-wheel generator rotating at 125 rpm produces an alternating current of standard frequency. How many pairs of magnetic poles has the rotor of the generator?

20.10. From the data of problem 20.1, determine the time variation of the e.m.f. and effective e.m.f. for two mutually perpendicular pairs of magnetic poles, if each pair of magnetic poles produces a magnetic field with induction 0.025 T.

20.11. The current is given by $i = 8.5 \sin (314t + 0.651)$ [A]. Determine the amplitude value of the current, its initial phase and frequency. Find the current in the circuit at $t_1 = 0.080$ s and $t_2 = 0.042$ s. What is the reading of an ammeter included in the circuit?

20.12. A voltmeter connected to an a.c. circuit reads 220 V. What should the voltage rating of the isolation in this circuit be?

20.13. For a sub-circuit with an active resistance of 4.0 Ω the current varies as $i = 6.4 \sin 314t$ [A]. Determine the effective current, active power dissipated in the sub-circuit. What should the voltage rating of the wire insulation be?

20.14. From the data of problem 20.13, write the equation for instantaneous voltage. Determine the phase difference between the current and voltage.

20.15. A coil with a ferromagnetic core is alternatively connected to the same voltage in a.c. and d.c. circuits. Will the currents be equal? If not, which will be larger?

20.16. A coil of inductance 35.0 mH is connected into an a.c. circuit. Determine the inductive reactance of the coil at frequencies 60, 240, and 480 Hz.

20.17. A coil of inductance 0.020 H is connected to an a.c. source having a frequency of 50 Hz. The effective voltage is 100 V. Write the time variation of the instantaneous current. Determine the phase difference between the current and voltage. Construct the vector diagram. Ignore the active resistance of the coil.

20.18. The current flowing through a coil of inductance 0.50 H is given by $i = 0.10 \sin 628t$ [A]. Determine the time dependence of the voltage across the coil and the inductive reactance.

20.19. The instantaneous voltage and current for an inductor are 127 V and 0.50 A, respectively. If the alternating current frequency is 50 Hz, determine the coil inductance.

20.20. A 250 μF capacitor is connected to an a.c. circuit. Determine its resistance at frequencies 50, 200, and 400 Hz.

20.21. The current through the circuit varies as $i = 0.20 \cdot \sin 314t$ [A]. A capacitor with a capacitance $C = 2.0 \cdot 10^{-6}$ F is connected to the circuit. Determine its maximum permissible voltage.

20.22. The voltage across a $2.0 \cdot 10^{-5}$ F capacitor is given by $u = 220 \sin (314t - \pi/2)$ [V]. Write the equation for the instantaneous value of the current through the capacitor. Determine the phase difference between the current and voltage. Determine the time variation of the capacitor charge.

20.23. Consider a coil for which the voltage and current are given respectively by $u = 60 \sin (314t + 0.25)$ [V] and $i = 15 \sin 314t$ [A]. Determine: the phase difference between the voltage and current; the impedance of the coil Z; $\cos \varphi$; the active resistance of the coil, R; its inductive reactance, X_L; the total power S; the active power P.

20.24. Referring to the series circuit in Fig. 20.24, the resistances are: $r = 3\ \Omega$, $X_L = 6\ \Omega$, $X_C = 2\ \Omega$. Draw the vector diagram. Determine the impedance of the circuit, Z, and its power factor, $\cos \varphi$.

20.25. The lamp in Fig. 20.25 is connected to a constant a.c. voltage. How will its brightness change if (1) the capacitance of the capacitor is increased; (2) another capacitor is connected in parallel to the capacitor; (3) the frequency of the a.c. current is increased? The voltage U_{AB} is considered constant.

20.26. An incandescent lamp is put in series with an inductor. If the effective current is constant, how will the brightness of the lamp change if the inductance of the inductor is increased by a ferromagnetic core being moved within the coil and the a.c. frequency is increased?

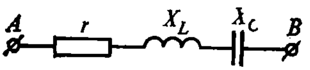


Fig. 20.24

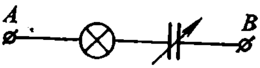


Fig. 20.25

20.27. A circuit supplied by an a.c. source of frequency ν includes in series a resistor R_1, capacitor C_1, inductor L_1, capacitor C_2, resistor R_2, and inductor L_2. Determine: the active resistance of the circuit, the reactance, the impedance.

20.28. In the series a.c. circuit of Fig. 20.24 the voltage across the sub-circuits are respectively $U_r = 40$ V, $U_L = 80$ V, and $U_C = 50$ V. Determine the impedance of the circuit, U_{AB}, and the power factor, $\cos \varphi$. Draw the vector diagram.

20.29. In the circuit of Fig. 20.24 the resistance is $r = 2.0\ \Omega$, the inductor rating is $L = 50$ mH, and the capacitor rating is $C = 25.0\ \mu$F. Determine: the impedance of the circuit, Z, at frequency $\nu = 50$ Hz, the phase difference between the current and voltage, φ. Determine the minimum resistance of the circuit and the corresponding frequency.

20.30. Referring to Fig. 20.24, can the resistance of the circuit decrease in simultaneously increasing all the resistances?

20.31. A coil having a resistance of 15 Ω and inductance of 52 mH is supplied by a 220 V, 50 Hz mains. The coil is placed in series with a 120 μF capacitor. Determine: the current in the circuit; the total, active, and reactive powers.

20.32. A coil of resistance 2.0 Ω and inductance 75 mH is connected in series with a capacitor and supplied by a 50 V, 50 Hz a.c. mains. What is the capacitance of the capacitor required for a voltage resonance to occur? What will the

voltages across the coil and capacitor be in this case? What is the danger of the voltage resonance?

20.33. In a series a.c. circuit of Fig. 20.24 a voltage resonance occurs at a frequency of 50 Hz. What is the maximum permissible voltage across the capacitor, if the effective voltage across the terminals is $U_{AB} = 220$ V, active resistance $R = 2.20\ \Omega$, inductance $L = \frac{1}{100\pi}$ H? In this case, what is the power dissipated at the resistance?

20.34. Figure 20.34 shows a vector diagram for a series a.c. circuit. Draw the circuit diagram. Determine the impedance and power factor of the circuit.

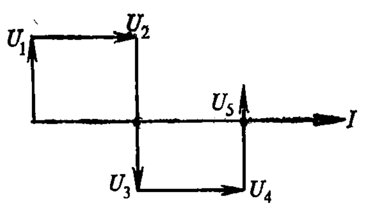


Fig. 20.34

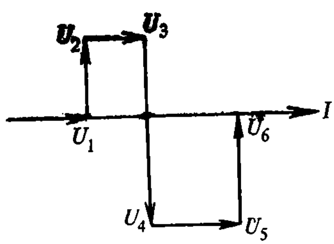


Fig. 20.35

20.35. From the vector diagram of Fig. 20.35 for a series circuit draw the circuit diagram. What phenomenon in the a.c. circuit is described by the vector diagram?

20.36. From the vector diagram of Fig. 20.35 determine the voltage, impedance, power factor, and power consumed. Voltages across the sub-circuits are respectively: $U_1 = 6.0$ V, $U_2 = 5.0$ V, $U_3 = 4.0$ V, $U_4 = 15$ V, $U_5 = 10$ V, and $U_6 = 10$ V. The resistance of the first sub-circuit is $r_1 = 1.5\ \Omega$.

20.37. Using the circuit diagrams shown in Fig. 20.37 (*a*), (*b*) and (*c*) determine the current in the series part of the circuit if $I_1 = 4$ A, $I_2 = 3$ A. Draw the vector diagrams.

20.38. From Fig. 20.37 (*c*) determine the current flowing through the series part of the circuit at $X_L = X_C = 10\ \Omega$, $U_{AB} = 100$ V.

20.39. Referring to the diagram in Fig. 20.37 (*c*), what is the frequency of the a.c. current required for the current resonance to occur, if the inductance is $L = 20$ mH, and the

capacitor rating is $C = 15\ \mu F$? Ignore the active resistance of the coil.

20.40. Is a current resonance possible in circuit diagrams shown in Fig. 20.37 (*a*), (*b*)?

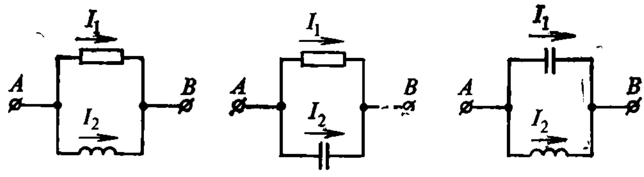


Fig. 20.37

20.41. Determine the current through a series part of the circuit (Fig. 20.41) using the following data: $R = 10\ \Omega$, $X_L = 10\ \Omega$, $X_C = 10\ \Omega$, $U_{AB} = 150$ V.

20.42. In the circuit shown in Fig. 20.42 $C = 106\ \mu F$, $L = 159$ mH, and $R = 56\ \Omega$. The active resistance of the

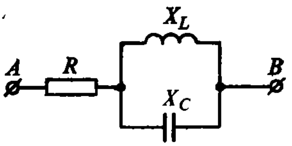


Fig. 20.41

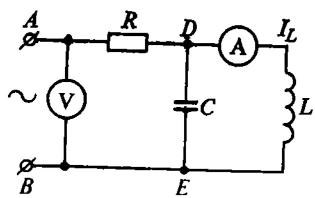


Fig. 20.42

coil is negligible so as to be ignored. The a.c. current frequency is 50 Hz. If the ammeter reads $I_L = 2.4$ A, determine the voltage U_{AB}. Draw the vector diagram.

20.43. The instantaneous, low-frequency current is given by $i = 0.564 \sin 12.56t$ [A]. What amount of heat will be liberated in the conductor with active resistance 15 Ω during the time of 10 periods?

20.44. At a sub-circuit with effective voltage U the phase difference between the current and voltage is φ. Determine the active power P dissipated by the resistance R of this sub-circuit.

20.45. Consider an a.c. generator such that at amplitude values of voltage and current $U_{max} = 200$ V and $I_{max} = 100$ A the active power produced by the generator is $P = 9.0$ kW. Determine the power factor.

20.46. A series circuit includes an incandescent lamp, inductor, and variable capacitor. Assuming that initially the capacitor resistance X_C is lower than the inductance of the inductor, X_L, $(X_C < X_L)$, determine how the lamp brightness will change as the capacitance of the capacitor is decreased. The effective voltage and frequency of the a.c. current are taken to be constant.

Transformer

20.47. A transformer working under open-circuit conditions consumes low power from the mains. How is it consumed? What is the phase shift in the primary under open-circuit conditions?

20.48. What will happen if a transformer with the primary rated at 127 V is connected to a d.c. mains of the same voltage? Why is the transformer efficiency much higher than that of an electric motor?

20.49. A step-up transformer[7] is supplied by a voltage $U_p = 120$ V. The primary has $N_p = 90$ turns. Determine the transformation ratio k and the number of turns in the secondary N_s, if under open-circuit condition the voltage across the secondary is $U_s = 3\,000$ V.

20.50. To determine the number of turns in the primary and secondary of the transformer without exposure, $N_n = 80$ turns of wire were wound over the secondary, and after the primary was connected to a mains of $U_p = 220$ V, voltages were determined with a voltmeter across the measuring winding $(U_n = 20$ V) and the secondary $(U_s = 36$ V). What is the number of turns in the primary, N_p, and the secondary, N_s? What is the transformation ratio?

20.51. If to the primary of a transformer a voltage of 220 V is applied, then under open-circuit conditions the secondary produces 130 V. The primary has 400 turns. Determine the number of turns in the secondary, if the leakage flux accounts for 3.8%.

20.52. A transformation ratio is 20. Which of the windings, primary or secondary, must have thicker wire? Explain.

20.53. Consider a lamp connected to the primary of a transformer. How will its brightness change if the load (current) in the secondary increases? Check experimentally. Account for the phenomenon using the energy conservation law.

20.54. The primary of a transformer carries a current of 4.8 A, the voltages across it being 127 V. The current through the secondary is 2.5 A at a voltage of 220 V. Determine the efficiency of the transformer at $\cos \varphi = 1$.

20.55. The primary of transformer connected to an a.c. line of 220 V has 1 500 turns. Determine the number of turns in the secondary if it supplies a 6.3 V, 0.50 A circuit with an active load. The resistance of the secondary is 0.20 Ω. The resistance of the primary is to be ignored.

20.56. To the primary of a transformer a voltage of 3 500 V is supplied. The secondary is connected with conducting wires to the user with voltage 220 V, the power consumed being 25 kW at $\cos \varphi = 1$. Determine the resistance of the conducting wires, if the transformation ratio is 15. What is the current through the primary of the transformer. Ignore the resistance of the secondary.

20.57. Consider a step-up transformer with a transformation ratio of 0.50. The secondary having a resistance of 0.20 Ω supplies an active load of 10.80 Ω. The voltage across the load is 216 V. Ignoring the resistance of conducting wires, determine the voltage and current in the primary, and the efficiency of the transformer.

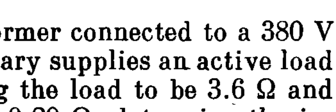


Fig. 20.59

20.58. The primary of a transformer connected to a 380 V mains has 1,320 turns. The secondary supplies an active load with a power of 360 W. Assuming the load to be 3.6 Ω and the resistance of the secondary 0.20 Ω, determine the induced e.m.f. in the secondary and the number of turns in it, the current through the primary, and the efficiency of the transformer. The power factor is taken to be $\cos \varphi = 1$.

20.59. The transformer of Fig. 20.59 has a core whose

cross-sectional area is equal throughout. Its primary ($n_p = 100$ turns) is supplied by a 200 V source. What is the voltage produced in the secondary ($n_s = 200$ turns)?

20.60. At the end of a 7 200 V a.c. line with resistance $R_1 = 12\ \Omega$ there is a step-down transformer with a transformation ratio $k = 20$. The secondary supplies $P_p = 21$ kW at a current $I_p = 60$ A to an active load. Ignoring the power loss in the primary, determine the transformer efficiency. In the secondary circuit the load is active.

20.61. A generator produces constant power. By how many times must its voltage be increased for the line losses to decrease n-fold?

SEC. 21. THREE-PHASE CURRENT

Example 61. Each line of a three-phase, four-wire circuit with a neutral includes resistances as shown in Fig. 61 (a) (Y-connection). Resistances of all the phases are similar and

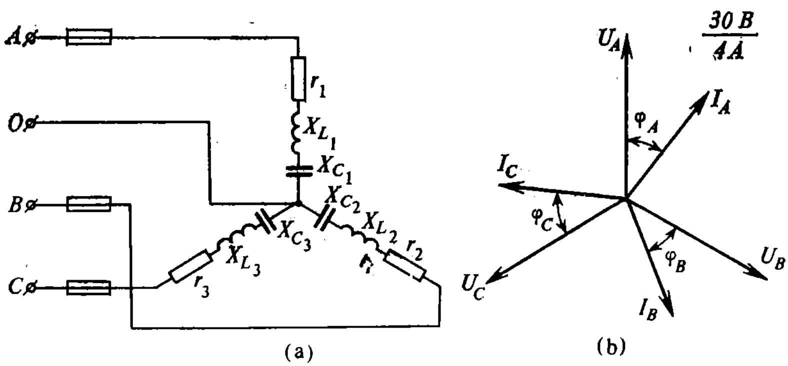


Fig. 61

equal: active 8.0 Ω, inductive 12 Ω, capacitive 6.0 Ω. The line voltage is 220 V. For each phase determine: (1) impedance, power factor, phase shift between current and voltage; phase currents; active, reactive, and total power for each phase; (2) line currents, neutral current; active, reactive, and total powers of the consumer. Draw the vector diagram.

Given: $r_1 = r_2 = r_3 = R_{ph} = 8.0\ \Omega$ — resistances of phases; $X_{L_1} = X_{L_2} = X_{L_3} = X_{L_{ph}} = 12\ \Omega$ — inductive reactances of phases; $X_{C_1} = X_{C_2} = X_{C_3} = X_{C_{ph}} = 6.0\ \Omega$ — capacitive reactances of phases; $U_l = 220$ V — the line voltage.

Determine: (1) Z_{ph} — the impedance of each phase;
$\cos\varphi_{ph}$ — the power factor;
φ_{ph} — the phase shift between current and voltage;
I_{ph} — phase currents;
P_{ph}, Q_{ph}, S_{ph} — the active, reactive, and total powers, respectively;
(2) I_l — line currents;
I_0 — the neutral current;
P, Q, S — the active, reactive, and total powers of the consumer.

Solution. (1) As the resistances and reactances in all the phases are equal (symmetric loads), it suffices to perform calculation for one phase only. The impedance is given by

$$Z_{ph} = \sqrt{R_{ph}^2 + (X_{L_{ph}} - X_{C_{ph}})^2}$$

Substituting the data of the problem gives

$$Z_{ph} = \sqrt{[8.0^2 + (12 - 6.0)^2]\ \Omega^2} = 10\ \ \Omega$$

The power factor for the phase is

$$\cos\varphi_{ph} = \frac{R_{ph}}{Z_{ph}}$$

Now we proceed to find the phase shift φ_{ph} between the current and voltage. To determine the direction of the phase shift we need $\sin\varphi_{ph}$:

$$\sin\varphi_{ph} = \frac{X_{L_{ph}} - X_{C_{ph}}}{Z_{ph}}$$

If $\sin\varphi_{ph} > 0$ (the load is mainly inductive) the current lags behind the voltage by an angle φ_{ph}; if $\sin\varphi_{ph} < 0$ (the load is mainly capacitive) the current is ahead of the voltage in phase.

We now find the power factor for the phase

$$\cos\varphi_{ph} = \frac{8.0\,\Omega}{10\,\Omega} = 0.80$$

Hence

$$\varphi_{ph} = 36°52'$$

$$\sin\varphi_{ph} = \frac{(12-6)\,\Omega}{10\,\Omega} = 0.6$$

as $\sin\varphi_{ph} > 0$, the current lags behind the voltage in phase, and in the vector diagram the current vector is shifted by 36°52′ back (clockwise) as related to the voltage vector. With symmetric loading the power factor for the total load is equal to the power factor for the phase: $\cos\varphi = \cos\varphi_{ph} = 0.80$.

The phase currents are obtained from Ohm's law

$$I_{ph} = \frac{U_{ph}}{Z_{ph}}$$

In Y-connection with a neutral each phase voltage, regardless of the kind and resistance of the phase, is always similar and equal to $U_{ph} = U_l/\sqrt{3}$, then $I_{ph} = U_l/\sqrt{3}Z_{ph}$. Also, in Y-connection the line currents are equal to phase ones:

$$I_l = I_{ph}$$

We thus determine the phase and line currents

$$I_p = I_l = \frac{220\text{ V}}{\sqrt{3}\cdot 10\,\Omega} = 12.7\text{ A}$$

The active, reactive and total powers of the phases are given by the relations

$$P_{ph} = I_{ph}^2 R_{ph} = I_{ph}U_{ph}\cos\varphi = \frac{I_l U_l}{\sqrt{3}}\cos\varphi_{ph}$$

$$Q_{ph} = I_{ph}^2 X_{r,\,ph} = I_{ph}^2 (X_L - X_C) = I_{ph}U_{ph}\sin\varphi_{ph}$$

$$S_{ph} = I_{ph}U_{ph} = \frac{I_{ph}U_l}{\sqrt{3}} = \frac{I_l U_l}{\sqrt{3}}$$

Substituting the data, we arrive at the active, reactive and total powers of the phase

$$P_{ph} = (12.7\ A)^2 \cdot 8.0\ \Omega = 1\,200\ W = 1.29\ kW$$
$$Q_{ph} = (12.7\ A)^2 \cdot (12 - 6.0)\,\Omega = 968\ var$$
$$S_{ph} = \frac{12.7\ A \cdot 220\ V}{\sqrt{3}} = 1\,613\ V \cdot A = 1.61\ kV \cdot A$$

(2) The neutral current is determined using the vector diagram

$$\mathbf{I}_0 = \mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C$$

where

$$\mathbf{I}_A = \mathbf{I}_{l_1} \quad \mathbf{I}_B = \mathbf{I}_{l_2} \quad \mathbf{I}_C = \mathbf{I}_{l_3}$$

With symmetrical load, the neutral current is $I_0 = 0$.

The active power for the total load is the sum of active powers of the phases

$$P = P_{ph_1} + P_{ph_2} + P_{ph_3}$$

With symmetrical load $P = 3P_{ph} = \sqrt{3}\, I_l U_l \cos \varphi_{ph}$.

The reactive power of the load is equal to the algebraic sum of reactive powers of the phases

$$Q = Q_{ph_1} + Q_{ph_2} + Q_{ph_3}$$

(The positive sign signifies the prevailing inductive load, the negative sign, the capacitive load.) In this case

$$Q_{ph_1} = Q_{ph_2} = Q_{ph_3} = Q_{ph} \quad \text{and} \quad Q = 3Q_{ph}$$

And the total power for all the loads is obtained as follows:

$$S = \sqrt{P^2 + Q^2}$$

Here $S = 3S_{ph}$.

We now obtain the active, reactive, and total powers for the loads

$$P = 3 \cdot 1\,290\ W = 3\,870\ W = 3.87\ kW$$
$$Q = 3 \cdot 968\ var = 2\,904\ var = 2.90\ kvar$$
$$S = 3 \cdot 1\,613\ kV \cdot A = 4.84\ kV \cdot A$$

Finally, we construct the vector diagram (see Fig. 61, *b*). We begin with the phase voltages arranging these at an angle of 120° to each other. At angles φ_A, φ_B, φ_C (in this problem 36°52′) to respective vectors of phase voltages we draw the phase current vectors: $I_{ph} = 12.7$ A, $U_{ph} = 127$ V.

Answer. (1) The phase impedance is 10 Ω; the power factor is 0.80; the phase shift between current and voltage is 36°52′; the phase and line currents are 12.7 A; the phase powers are: active 1.29 kW, reactive 0.96 var, total 1.61 kV·A; (2) the neutral current is zero; the load powers: active 3.87 kW, reactive 2.90 kvar, total 4.84 kV·A.

Example 62. A consumer with a symmetrical load whose phases are delta-connected is supplied by a three-phase circuit with a line voltage of 220 V (Fig. 62). The corresponding resistances and reactances in the phases are similar and equal: resistances 6.0 Ω, inductive reactances 4.0 Ω, capacitive reactances 12 Ω. Determine: the impedance of each phase, power factor for each phase, phase and line currents; active, reactive, and total powers for each phase; active, reactive, and total powers of the load.

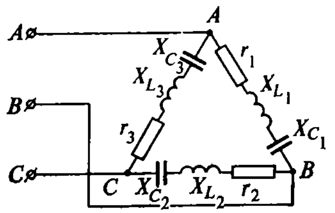


Fig. 62

Given: $U_l = 220$ V is the line voltage; $r_1 = r_2 = r_3 = R = 6.0\ \Omega$ are the phase resistances; $X_{L_1} = X_{L_2} = X_{L_3} = X_L = 4.0\ \Omega$ are the inductive reactances of phases; $X_{C_1} = X_{C_2} = X_{C_3} = X_C = 12\ \Omega$ are capacitive reactances of phases.

Determine: Z_{ph} — the impedance of each phase;
$\cos\varphi_{ph}$ — the power factor of each phase;
I_{ph}, I_l — the phase and line currents;
P_{ph}, Q_{ph}, S_{ph} — the active, reactive, and total powers of each phase;
P, Q, S — the active, reactive, and total powers of the load.

Solution. With symmetrical load, it is only necessary to determine the required quantities for one phase.

The impedance of the phase is obtainable from the formula

$$Z_{ph} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z_{ph} = \sqrt{[6.0^2 + (4.0 - 12)^2]\,\Omega^2} = 10\,\Omega$$

The phase power factor is

$$\cos\varphi_{ph} = \frac{R}{Z_{ph}}$$

in this problem the power factor for the total load is

$$\cos\varphi = \cos\varphi_{ph}$$

$$\sin\varphi_{ph} = \frac{X_L - X_C}{Z_{ph}}$$

$$\cos\varphi = 6.0\,\Omega/10\,\Omega = 0.60$$

$$\varphi = 53°8'$$

The phase current is to be found from Ohm's law

$$I_{ph} = \frac{U_{ph}}{Z_{ph}}$$

In delta-connection the phase voltage is equal to the line voltage, $U_{ph} = U_l$, therefore $I_{ph} = U_l/Z_{ph}$.

To find the line current we take into account that with symmetrical load

$$I_l = I_{ph}\sqrt{3}$$

Substitution gives

$$I_{ph} = \frac{220\ \text{V}}{10\,\Omega} = 22\ \text{A}$$

$$I_l = I_{ph}\sqrt{3} \qquad I_l = 22\sqrt{3}\ \text{A} = 38.1\ \text{A}$$

The appropriate phase powers are obtained by the relationships

$$P_{ph} = I_{ph}^2 R = I_{ph} U_{ph} \cos\varphi_{ph} = (22\ \text{A})^2 \cdot 6\,\Omega = 2\,904\ \text{W}$$

$$Q_{ph} = I_{ph}^2 X_{r,\,ph} = I_{ph}^2 (X_L - X_C) = I_{ph} U_{ph} \sin\varphi_{ph}$$

$$= 22\ \text{A} \cdot 220\ \text{V} \cdot (-0.80) = -3\,872\ \text{var}$$

$$S_{ph} = I_{ph} U_{ph} = 22\ \text{A} \cdot 220\ \text{V} = 4\,840\ \text{V} \cdot \text{A}$$

The active power of the load is given by

$$P = 3P_{ph} = \sqrt{3}\, I_l U_l \cos\varphi = 3\cdot 2\,904 \text{ W} = 8\,712 \text{ W} = 8.7 \text{ kW}$$

The reactive power of the load will be

$$Q = 3Q_{ph} = \sqrt{3}\, I_l U_l \sin\varphi = 3\cdot(-3\,872 \text{ var}) = -11{,}616 \text{ var} = -11.6 \text{ kvar}$$

And finally we determine the total power of the load

$$S = \sqrt{P^2 + Q^2}$$

$$S = 3S_{ph} = \sqrt{3}\, I_l U_l = 3\cdot 4\,840 \text{ V}\cdot\text{A} = 14{,}520 \text{ V}\cdot\text{A} = 14.5 \text{ kV}\cdot\text{A}$$

Answer. The impedance of the phase is 10 Ω, the power factor of the phase is 0.60, the phase currents are 22 A, the line currents are 38.1 A; the powers of the phase: active 2.9 kW, reactive 3.87 kvar, total 4.84 kV·A; the load powers: active 8.7 kW, reactive—11.6 kvar; total 14.5 kV·A.

21.1. What is a three-phase a.c. circuit?

21.2. Describe a symmetrical system of three-phase e.m.f.'s.

21.3. Consider a three-phase generator with identical windings. In one of the windings the electromotive force varies as $e = \mathscr{E}_0 \sin 314t$. Write the equations of instantaneous values of e.m.f. in other windings.

21.4. What is a three-phase disconnected system?

21.5. Name the connections of phase windings of a three-phase generator. Compare them.

21.6. What is the phase voltage? Line voltage? How are these related in Y-connection of generator windings? Delta-connection?

21.7. The windings of a three-phase generator are normally Y-connected. Why?

21.8. Determine the phase voltage of a generator, if the line voltage is $U_l = 380$ V, when its windings are Y-connected and delta-connected.

21.9. The windings of a three-phase symmetrical generator are delta-connected. Draw the vector diagram for e.m.f.'s in generator windings. What is the sum of the three symmetrical e.m.f.'s?

21.10. What will happen if with delta-connected generator the end of the first phase, X, is connected not with the beginning of the second phase, B, but with its end Y? Draw the connection diagram. Draw the vector diagram.

21.11. A generator may be Y- or delta-connected. What connections are possible here for the consumers — Y or delta?

21.12. Can a load be regarded as symmetrical if (1) when delta-connected the phase currents are equal; (2) when Y-connected in a four-wire system with a neutral the line currents are equal; (3) active powers of the three phases are equal?

21.13. Can a load be regarded as symmetrical if the phase currents are equal and phase-shifted relative to appropriate phase voltages by similar angles φ in the same direction?

21.14. The consumer uses Y-connection with a neutral. Under symmetrical load the phase currents are $I_{\text{ph}} = 10$ A. What are the line currents I_{l}? What is the current in the neutral?

21.15. In a four-wire system the neutral is not supplied with a fuse. Explain.

21.16. Is a neutral required for a circuit with symmetrical load?

21.17. For a three-phase load the phase powers are given: active P_1, P_2, P_3, and reactive Q_1, Q_2, Q_3. Determine the active power P, reactive power Q, and total power S of the load. What is the meaning of the sign of the reactive load?

21.18. In a three-phase symmetrical load the active power of the phase is $P_{\text{ph}} = 400$ W, the reactive power of the phase is $Q_{\text{ph}} = -300$ var. Determine: the total phase power S_{ph}, the power factor for each phase, $\cos \varphi_{\text{ph}}$; the active (P), reactive (Q) and total (S) powers of the load. The power factor for all the load is $\cos \varphi$.

21.19. A three-phase motor (symmetrical load) with Y-connected windings is supplied by a 380 V mains. Determine the power consumed by the motor at a line current of 19 A and power factor 0.8.

21.20. A symmetric load connected to a mains with a line voltage of 380 V dissipates an active power of 3.3 kW. Determine the line current in the circuit, if the phase shift between the current and voltage is 60°.

21.21. Incandescent lamps of power $P = 100$ W each are connected as shown in Fig. 21.21: Y-connection with a neutral to form a three-phase four-wire circuit with line voltage $U_l = 380$ V. Into phase A $n_1 = 20$ lamps are connected; into phase B, $n_2 = 16$ lamps; into phase C, $n_3 = 26$ lamps.

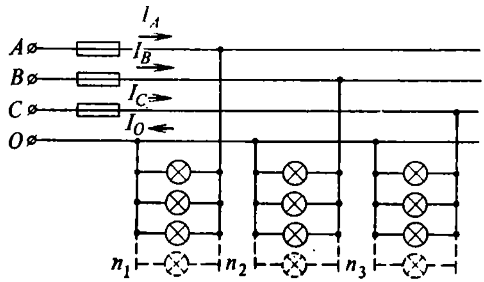


Fig. 21.21

Determine: the phase voltage U_{ph}, the current flowing through each lamp, I_1; the line currents I_A, I_B, I_C; the powers P_A, P_B, P_C consumed by each phase; the power P consumed by the whole load.

21.22. An illuminating network consisting of three branches with 10 lamps of power 150 W in each is Y-connected to a three-phase four-wire circuit with a neutral. The line voltage is 380 V. Determine the phase (I_{ph}) and line (I_l) currents, the energy consumed by the network in 8 hours of operation.

21.23. A load is Y-connected to a three-phase four-wire line with a neutral. Using the vector diagram of Fig. 21.23 determine the character of the load and the current flowing through the neutral, if the phase currents are similar and equal to 10 A.

21.24. Use the vector diagram of Fig. 21.24 to determine the character of the load in each phase. Draw the circuit diagram. The connection is as in problem 21.23. Will there be any current in the neutral?

21.25. Each phase of a three-phase four-wire system with a neutral includes resistances (see Fig. 61, a). In all the phases the resistances and reactances are equal: $r_1 = r_2 = r_3 =$

$= 15\,\Omega$, $X_{L_1} = X_{L_2} = X_{L_3} = 25\,\Omega$; $X_{C_1} = X_{C_2} = X_{C_3} = 5\,\Omega$. The line voltage in the circuit is $U_l = 380$ V. Determine: the phase (I_{ph}) and line (I_l) currents; the active (P_{ph}), reactive (Q_{ph}), and total (S_{ph}) powers of each phase; the

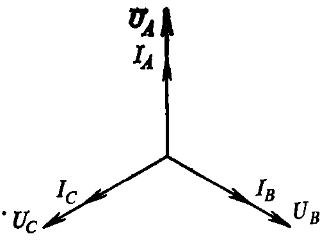


Fig. 21.23

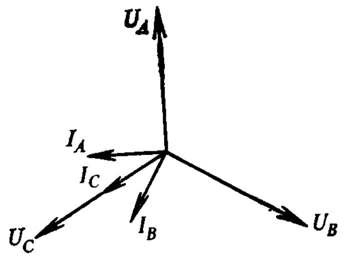


Fig. 21.24

power factor for the phase; the active (P), reactive (Q), and total (S) powers of the load.

21.26. The line voltage in a three-phase three-wire mains is 220 V. What is the connection used by the consumer with voltage rating 220 V? 127 V?

21.27. A three-phase, delta-connected motor is rated at 220 V with a power factor of 0.85. If the power of the motor is 3.4 kW, determine the line and phase currents.

21.28. A three-phase, delta-connected electric motor is rated at 220 V. The phase current through the windings is 30 A. Determine the line current and power of the motor. The power factor is 0.8.

21.29. A three-phase electric motor with shaft power 10 kW, power factor 0.85, and efficiency 0.8 is supplied by a line voltage of 220 V. For delta-connection determine: the line currents through conducting wires; the active and total resistances of each phase. If the system is Y-connected and the motor power is constant, what is the maximum permissible voltage across the motor windings?

21.30. A motor is Y-connected. The line voltage and power factor being constant, how will the line currents and motor power change with delta-connection?

21.31. Thirty lamps are divided into three equal groups and delta-connected into a three-phase, three-wire mains with a line voltage of 220 V. The power of each lamp is

100 W. Determine the currents flowing through the line wires.

21.32. Referring to Fig. 21.32, the consumers with active loads are delta-connected into a three-phase, three-wire network with a line voltage 220 V. The phase resistances are: $r_{ph_1} = 22\ \Omega$, $r_{ph_2} = 44\ \Omega$, $r_{ph_3} = 11\ \Omega$. Determine: (1) the phase currents I_{AB}, I_{BC}, I_{CA}; (2) the phase powers P_{AB}, P_{BC}, P_{CA}, (3) the total power of the load, P; (4) Draw the vector diagram and from it determine the line currents I_A, I_B, I_C. Check the results obtained by computation using the cosine law.

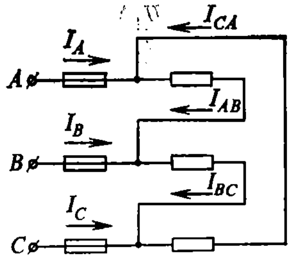


Fig. 21.32

21.33. To a three-wire network with a line voltage of 220 V a symmetrical load is delta-connected such that the resistance of each phase is 3 Ω, the inductive reactance 4 Ω. Determine: the phase and line currents; the power factor; the phase active power; the total power of the load.

21.34. A three-wire mains supplies a delta-connected symmetrical load (Fig. 62). The resistance of each phase is 8.0 Ω, the inductive reactance 12 Ω, the capacitive reactance 6.0 Ω. The active power consumed by each phase is 3 872 W. Determine: the phase impedance; the power factor; the phase and line currents; the phase and line voltages; the total phase power; the active and total power of the load.

21.35. Determine the algebraic sum of instantaneous values of line currents in a three-wire cable connecting the generator with the three-phase load that is symmetrically and asymmetrically connected using Y-connection without a neutral. Delta-connection.

21.36. Is it possible to detect the magnetic field near a three-phase cable with a load switched on? Are there any eddy currents in steel armouring of the cable with the load switched on?

21.37. An induction motor is connected to a three-phase 50 Hz mains. For two-pole ($p = 1$), four-pole ($p = 2$), and

six-pole ($p = 3$) stator windings, what is the rotational speed (in rpm) of the motor's rotating magnetic field?

21.38. Think of a way of reversing the rotation of magnetic field in a three-phase induction motor.

SEC. 22. ELECTRICAL OSCILLATIONS AND ELECTROMAGNETIC WAVES

Example 63. An oscillatory circuit consists of a 48 μF capacitor and a 24 MH coil having a resistance of 20 Ω. Determine the frequency of free electromagnetic oscillations in this circuit. How will the frequency change if we ignore the resistance of the coil?

Given: $C = 4.8\cdot10^{-5}$ F is the capacitor rating, $L = 2.4\cdot10^{-2}$ H is the coil inductance, $R = 20\ \Omega$ is the coil resistance.

Determine: ν_1 — the frequency of the free electromagnetic oscillations in the circuit;

$\Delta\nu = (\nu_2 - \nu_1)$ — the change in the frequency with zero resistance.

Solution. The frequency is given by

$$\nu_1 = \frac{1}{T_1}$$

where

$$T_1 = \frac{2\pi}{\sqrt{1/LC - (R/2L)^2}}$$

And the frequency ν_1 will be

$$\nu_1 = \frac{\sqrt{1/LC - (R/2L)^2}}{2\pi}$$

$$= \frac{\left[\dfrac{1}{2.4\cdot10^{-2}\ \text{H}\cdot4.8\cdot10^{-5}\ \text{F}} - \left(\dfrac{20\ \Omega}{2\cdot2.4\cdot10^{-2}\ \text{H}}\right)^2\right]^{1/2}}{6.28} = 132\ \text{Hz}$$

If the resistance R is zero, the relationship for the period of oscillations takes the form

$$T_2 = 2\pi\sqrt{LC}$$

Hence we obtain the period of oscillations at $R = 0$ and the frequency ν_2, and the $\Delta\nu$.

Now we proceed to determine ν_2

$$\nu_2 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{2.4\cdot 10^{-2}\ \text{H}\cdot 4.8\cdot 10^{-5}\ \text{F}}} = 148\ \text{Hz}$$

The frequency change will be

$$\Delta\nu = \nu_2 - \nu_1 = 148\ \text{Hz} - 132\ \text{Hz} = 16\ \text{Hz}$$

Answer. The frequency of free oscillations in the circuit is 132 Hz, ideally, i.e. when $R = 0$, the frequency of free oscillations in the circuit is by 16 Hz higher.

Example 64. Determine the wavelength of electromagnetic oscillations in a vacuum to which an oscillatory circuit is tuned, if the maximum charge on the capacitor is $2.0\cdot 10^{-8}$ C and the maximum current through the circuit is 1.0 A. What is the capacitor rating, if the circuit inductance is $2.0\cdot 10^{-7}$ H? What is the energy of the electric field within the capacitor at the time when the energy of the magnetic field is 3/4 of its maximum? Determine the voltage across the capacitor at that moment. Ignore the circuit resistance.

Given: $q_m = 2.0\cdot 10^{-8}$ C is the maximum charge on the capacitor, $I_m = 1.0$ A is the maximum current through the circuit, $L = 2.0\cdot 10^{-7}$ H is the circuit inductance, $R = 0$ is the circuit resistance, $c = 3\cdot 10^8$ m/s is the velocity of propagation of electromagnetic waves in vacuum.

Determine: λ — the wavelength of electromagnetic oscillations to which the circuit is tuned;
C — the capacitor rating;
W_{el} — the energy of electric field when the energy of magnetic field is 3/4 of its maximum;
U — the voltage across the capacitor at that time.

Solution. The wavelength is simply

$$\lambda = cT$$

where $T = 2\pi\sqrt{LC}$. To determine the period of oscillations we make use of the energy conservation law. With continuous oscillations the maximum energy of magnetic field is equal to that of electric field and to the total energy of

electromagnetic oscillations in the circuit, i.e. $W_{\mathrm{el.\,m}} = W_{\mathrm{mag.\,m}} = W$. Hence

$$q_{\mathrm{m}}^2/2C = LI_{\mathrm{m}}^2/2 \quad \text{and} \quad LC = q_{\mathrm{m}}^2/I_{\mathrm{m}}^2$$

Then

$$T = 2\pi \frac{q_{\mathrm{m}}}{I_{\mathrm{m}}}$$

Now we find the wavelength

$$\lambda = c \cdot 2\pi \frac{q_{\mathrm{m}}}{I_{\mathrm{m}}} = 3 \cdot 10^8 \text{ m/s} \cdot 2\pi \frac{2.0 \cdot 10^{-8} \text{ C}}{1.0 \text{ A}} \approx 38 \text{ m}$$

Knowing the circuit inductance, we arrive at the capacitor rating

$$C = \frac{q_{\mathrm{m}}^2}{LI_{\mathrm{m}}^2} = \frac{(2.0 \cdot 10^{-8} \text{ C})^2}{2.0 \cdot 10^{-7} \text{ H} \cdot 1.0 \text{ A}^2} = 2.0 \cdot 10^{-9} \text{ F}$$

The total energy of electromagnetic oscillations in the circuit is equal to the sum of instantaneous values of energies of electric and magnetic fields, and for continuous oscillations is a constant

$$W = W_{\mathrm{mag.m}} = W_{\mathrm{el}} + W_{\mathrm{mag}}$$

where

$$W_{\mathrm{mag}} = \frac{3}{4} W_{\mathrm{mag.m}}$$

Consequently,

$$W_{\mathrm{mag.m}} = \frac{3}{4} W_{\mathrm{mag.m}} + W_{\mathrm{el}}$$

hence

$$W_{\mathrm{el}} = \frac{1}{4} W_{\mathrm{mag.m}} = \frac{1}{4} \frac{IL_{\mathrm{m}}^2}{2}$$

Substituting the numerical values, we determine the energy of electric field for this instant

$$W_{\mathrm{el}} = \frac{1}{4} \frac{2.0 \cdot 10^{-7} \text{ H} \cdot 1.0 \text{ A}^2}{2} = 2.5 \cdot 10^{-7} \text{ J}$$

The energy of electric field is obtainable from the formula $W_{el} = CU^2/2$. We thus obtain $\frac{CU^2}{2} = \frac{1}{4}\frac{LI_m^2}{2}$. Hence the instantaneous value of the voltage U across the capacitor is

$$U = \frac{I_m}{2}\sqrt{\frac{L}{C}} = \frac{1.0\ \mathrm{A}}{2}\sqrt{\frac{2.0\cdot 10^{-7}\ \mathrm{H}}{2.0\cdot 10^{-9}\ \mathrm{F}}} = 5.0\ \mathrm{V}$$

Answer. The wavelength of the electromagnetic oscillations is 38 m; the capacitor rating is $2.0\cdot 10^{-9}$ F; the instantaneous value of the energy of electric field is $2.5\cdot 10^{-7}$ J; the instantaneous voltage is 5.0 V.

Example 65. Determine the wavelength of electromagnetic radiation in vacuum, if the frequency is $4.5\cdot 10^{11}$ Hz. What is the velocity of propagation and the wavelength of the same wave in benzene, if its dielectric constant is 2.28? (*Hint*: Use the Maxwell theory).

Given: $\nu = 4.5\cdot 10^{11}$ Hz is the frequency of the oscillations; $\varepsilon = 2.28$ is the dielectric constant for benzene, $\varepsilon_0 = 8.85\cdot 10^{-12}$ F/m is the electric constant, $\mu_0 = 4\pi\cdot 10^{-7}$ H/m is the magnetic constant.

Determine: λ_0 — the wavelength in vacuum;
v — the wave velocity in benzene;
λ — the wavelength in benzene.

Solution. The velocity of the electromagnetic waves in vacuum is

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = \frac{c}{\nu} = \frac{1}{\sqrt{8.85\cdot 10^{-12}\ \mathrm{F/m}\cdot 4\pi\cdot 10^{-7}\ \mathrm{H/m}}} = 3\cdot 10^{8}\ \mathrm{m/s}$$

The wavelength in vacuum is

$$\lambda_0 = \frac{c}{\nu} = \frac{3\cdot 10^{8}\ \mathrm{m/s}}{4.5\cdot 10^{11}\ \mathrm{Hz}} = 0.67\cdot 10^{-3}\ \mathrm{m}$$

Now we calculate the velocity of the wave in benzene*

* Substances transparent to electromagnetic waves are dielectrics whose magnetic properties are but slightly dependent on their type, therefore their relative permeability can be taken to be unity. Because

$$\frac{c}{v} = \sqrt{\varepsilon\mu} = \sqrt{\varepsilon}$$

we get

$$v = \frac{c}{\sqrt{\varepsilon}}$$

and the wavelength

$$v = \frac{c}{\sqrt{\varepsilon}} = \frac{3 \cdot 10^8 \text{ m/s}}{\sqrt{2.28}} = 2 \cdot 10^8 \text{ m/s}$$

$$\lambda = \frac{v}{\nu} = \frac{2 \cdot 10^8 \text{ m/s}}{4.5 \cdot 11^{11} \text{ Hz}} = 0.44 \cdot 10^{-3} \text{ m}$$

Answer. The wavelength in vacuum is 0.67 mm; the velocity of the wave in benzene is $2 \cdot 10^8$ m/s; the wavelength in benzene is 0.44 mm.

22.1. Discuss the role of the inductance and capacitance in the oscillatory circuit.

22.2. Describe the influence of the resistance of the coil on electromagnetic oscillations in the oscillatory circuit.

22.3. Other things being equal, what will the influence on the free electromagnetic oscillations in the oscillatory circuit of the increased coil resistance be?

22.4. Under what conditions will continuous electromagnetic oscillations be produced in the oscillatory circuit?

22.5. What are the energy losses in the oscillatory circuit?

22.6. Sometimes an oscillatory circuit is equipped with a variable capacitor or variable inductor. Why?

22.7. Propose some way of increasing the energy that goes into electromagnetic radiation in the oscillatory circuit.

22.8. What are the postulates of Maxwell's theory of electromagnetic field?

22.9. What characteristics of the field vary periodically in a travelling electromagnetic wave?

22.10. When will an electromagnetic wave transfer the maximum energy to an oscillatory circuit located along its path?

22.11. What is to happen to the natural oscillations in the oscillatory circuit with a negligible resistance, if its capacitance is trebled, and inductance is reduced threefold?

22.12. How will the period and frequency of free oscilla-

The last formula may be used to find the wavelength in benzene, considering that $\lambda = v/\nu$.

Remember in using these relations that the dielectric constant is generally strongly frequency dependent, and tables list electrostatic values of ε, i.e. the value of ε at $\nu = 0$. But in benzene and most of gases ε is almost independent of frequency, therefore in this case it may be taken from tables.

tions change in an oscillatory circuit with $R = 0$, if its inductance is doubled, and capacitance is increased fourfold?

22.13. Determine the period and frequency of natural oscillations in an oscillatory circuit with capacitance 2.2 μF and inductance 0.65 mH.

22.14. Compute the frequency of natural oscillations in a circuit with $R = 0$, if its inductance is 12 mH, and capacitance 0.88 μF. How will the frequency change, if another three identical capacitors are placed in series into the oscillatory circuit?

22.15. Determine the period of natural oscillations in an oscillatory circuit with $L = 2.5$ mH and $C = 1.5$ μF. How will the period change, if another three identical capacitors are connected in parallel to the capacitor?

22.16. A resonance in an oscillatory circuit is achieved at a frequency of 4.20 kHz. Ignoring the resistance, determine the inductance of the coil, if the capacitor rating is 2.2 μF. What is the phase shift between the current and voltage in the oscillatory circuit?

22.17. The electric charge on the capacitor plates in an oscillatory circuit is given by $q = 10^{-2} \cos(2\pi t + \pi)$ [C]. Determine: the angular velocity, the frequency, period and initial phase of charge oscillations and the maximum current.

22.18. In an oscillatory circuit the capacitor C is charged to a maximum voltage U_m. Determine the resonance frequency of oscillations in the circuit, if the maximum current in it is I_m. Ignore the resistance.

22.19. An oscillatory circuit consists of an inductor of inductance $L = 1.0$ mH and a capacitor with $C = 10.0$ μF. The capacitor is charged to the maximum voltage $U_m = 100$ V. Determine the maximum charge of the capacitor, q_m, the maximum current in the circuit, I_m. Regarding the oscillations as continuous, write the equation to determine the instantaneous current flowing through the circuit.

22.20. An oscillatory circuit includes an inductor ($L = 0.20$ mH) and two capacitors ($C_1 = C_2 = 4$ μF) placed in parallel. The maximum current in the circuit is 0.10 A. Determine the period of free oscillations in the circuit, the maximum charge of the capacitors, and the maximum voltage across each capacitor.

22.21. In an oscillatory circuit of inductance L and capacitance C the capacitor is charged to a maximum voltage U_m. If the oscillations are considered continuous, what will the current be at the time when the voltage across the capacitor is reduced by half?

22.22. Consider an oscillatory circuit with inductance 0.40 H, capacitance $2.0 \cdot 10^{-5}$ F, and maximum current $1.0 \cdot 10^{-1}$ A. If the oscillations are continuous, what will be the voltage across the capacitor at the instant when the energies of the electric and magnetic fields are equal?

22.23. In an oscillatory circuit a 10 μF capacitor received a charge of 10^{-3} C, thus giving rise to damped electromagnetic oscillations. Determine the amount of heat liberated by the time when the maximum voltage across the capacitor is four times smaller than the initial maximum voltage.

22.24. What is the wavelength produced by a radio station operating at a frequency of 1.5 MHz?

22.25. An oscillatory circuit (2.6 pF, 0.012 mH) oscillates at natural frequency. Determine the wavelength in vacuum of the electromagnetic radiation produced in the process.

22.26. An oscillatory circuit produces in the air electromagnetic radiation of wavelength 150 m. Neglecting the resistance, what is the capacitance of the circuit, if its inductance is 0.25 mH?

22.27. An oscillatory circuit in a radio set has inductance 0.32 mH and a variable capacitor. The receiver can receive waves in the range 188-545 m. Ignoring the resistance of the oscillatory circuit, what is the range of variation of the capacitance?

22.28. What is the working range of a radio receiver, if the inductance of the receiving circuit is 1.5 mH, and the capacitance varies from 75 to 650 pF? Ignore the resistance of the circuit.

22.29. The input circuit of a radio receiver consists of a coil of inductance 2.0 mH and a flat capacitor with plates of area 10.0 cm^2 separated by 2.0 mm. The interplate space is filled with mica having a dielectric constant of 7.5. At what wavelength is the receiver tuned?

22.30. Consider an oscillatory circuit with inductance L, maximum current I_m and maximum voltage across the ca-

pacitor U_m. If the velocity of propagation of the waves is v, at what wavelength is the circuit tuned?

22.31. There is an oscillatory circuit such that the maximum capacitor charge is q_m, the maximum current flowing through the circuit is I_m. If the waves propagate at a velocity v, determine the wavelength at which the circuit is tuned.

22.32. In radio communication the high-frequency oscillations are said to be carrier frequency. Why?

22.33. Consider a vacuum valve in a radio receiver. In the lamp between the grid and the cathode a device is fitted to produce at the grid a negative potential relative to the cathode (grid bias). Explain why?

22.34. Discuss the amplification of a signal received by the radio receiver.

22.35. The operation of an automobile radio receiver is impaired when passing under a trestle bridge or a passover. Explain.

22.36. What electromagnetic waves are to be produced by an oscillatory circuit for the radiation to be directional?

22.37. A radar station must send radio signals in the form of short pulses that follow each other with an interruption. Why?

22.38. What element of a radar station measures the time between the production of a signal and detection of the reflected pulse?

22.39. By how many times should the power of a radar be increased for its range to be doubled?

22.40. Consider an oscillatory circuit consisting of a 0.064 μF capacitor, 0.18 mH inductor and 50 Ω resistance. What is the period of free oscillations of the circuit?

22.41. What is the frequency of free oscillations of an oscillatory circuit featuring a capacitance of 2.2 μF, inductance of 0.12 mH, and resistance of 15 Ω?

22.42. There are free oscillations in an oscillatory circuit with a capacitance of 2.400 pF, inductance 0.054 mH, and resistance of 76 Ω. What is the wavelength of the radiation produced in vacuum?

22.43. The frequency of free oscillations in a circuit is 250 kHz. If the inductance is 0.024 mH and resistance is 34 Ω, what is the capacitance of the circuit?

CHAPTER IV

OPTICS AND SPECIAL RELATIVITY

SEC. 23. VELOCITY OF LIGHT. NATURE OF LIGHT

Example 66. Determine the optical density (absolute refractive index) of glycerin, if the wavelength of greenlight in it is 407 nm at a photon energy of $3.31 \cdot 10^{-19}$ J.

Given: $\lambda = 4.07 \cdot 10^{-7}$ m is the wavelength in glycerin, $\varepsilon = 3.31 \cdot 10^{-19}$ J is the photon energy, $h = 6.62 \cdot 10^{-34}$ J·s is Planck's constant.

Determine: n — the absolute refractive index for glycerin.

Solution. The optical density of a medium, n, is the ratio of the velocity of light in a vacuum to the velocity of light in the medium

$$n = c/v$$

As the wavelength varies directly with the velocity of radiation, we get

$$\lambda/\lambda_0 = c/v$$

$$n = \lambda_0/\lambda$$

where λ_0 is the wavelength of green light in a vacuum. To solve the problem we also need the wavelength in a vacuum which is obtainable from the relationship. Hence,

$$\lambda_0 = hc/\varepsilon$$

$$n = hc/\varepsilon\lambda$$

Substituting the data, we arrive at the absolute refractive index, n, for glycerin

$$n = \frac{6.62 \cdot 10^{-34}\ \text{J}\cdot\text{s}\cdot 3 \cdot 10^{8}\ \text{m}\cdot\text{s}}{3.31 \cdot 10^{-19}\ \text{J}\cdot 4.07 \cdot 10^{-7}\ \text{m}} = 1.47$$

Answer. The absolute refractive index for glycerin is 1.47.

Example 67. Monochromatic light of wavelength $\lambda = 0.44\ \mu m$ travels from glass of absolute refractive index 1.5 to a vacuum. Determine the change in the wavelength and the velocity of light in the glass.

Given: $\lambda = 4.4 \cdot 10^{-7}$ m is the wavelength of light in the glass, $n = 1.5$ is the absolute refractive index of the glass.

Determine: $\Delta\lambda$ — the change in the wavelength in transition from glass to vacuum;
v — the velocity of light in the glass.

Solution. Recalling that the frequency of light is independent of the medium, we write $\lambda = v\nu$ and $\lambda_0 = c\nu$, where λ_0 is the wavelength in a vacuum. Hence the wavelength is proportional to the velocity of light

$$\lambda_0/\lambda = c/v$$

We then form the derivative proportion

$$\frac{\lambda_0 - \lambda}{\lambda} = \frac{c - v}{v}$$

hence

$$\Delta\lambda = (n - 1)\,\lambda = (1.5 - 1)\cdot 4.4\cdot 10^{-7}\ \text{m} = 2.2\cdot 10^{-7}\ \text{m}$$

By definition the refractive index is $n = c/v$, whence the light velocity in the glass is

$$v = \frac{c}{n} = \frac{3\cdot 10^8\ \text{m/s}}{1.5} = 2\cdot 10^8\ \text{m/s}$$

Answer. At the interface of glass and vacuum the wavelength increases by $2.2\cdot 10^{-7}$ m; the light velocity in the glass is $2\cdot 10^8$ m/s.

23.1. Light covers the distance from the Earth to the Moon in 1.28 s. What is the velocity of light?

23.2. How long will it take the light to travel from the Sun to the Earth, if the separation between them is $150\cdot 10^6$ km?

23.3. Determine the radius of the Earth, if light travels in a vacuum the distance equal to the Earth's equator length in 0 139 s.

23.4. It is common knowledge that man perceives as light the radiation within frequency range $4 \cdot 10^{14}$-$7.5 \cdot 10^{14}$ Hz. Determine the corresponding wavelength range.

23.5. The wavelength of yellow light in a vacuum is 0.589 μm. What is the corresponding frequency?

23.6. Electromagnetic radiation of frequency $9.5 \cdot 10^{14}$ Hz is incident on the eye of a man. Can the man see this radiation? What is the wavelength in a vacuum of this radiation?

23.7. Discuss the phases of oscillation at two points on a light ray, if the separation between them is 3λ, $2n \cdot \lambda/2$, where n is an integer.

23.8. If light travels from vacuum to a medium, do its wavelength and frequency change in the process?

23.9. Can the wavelength change from 0.6 to 0.4 μm as light passes from a medium to a vacuum?

23.10. What is the change in the wavelength of violet light at the water-vacuum face, if the velocity of light in water is $223 \cdot 10^3$ km/s?

23.11. Is the velocity of light dependent on frequency? Wavelength?

23.12. A radiation consists of photons of energy $6.4 \cdot 10^{-19}$ J. Determine the frequency and wavelength in a vacuum. Can man see this radiation?

23.13. What is the energy of a photon of red light with a wavelength in a vacumm of 0.72 μm.

23.14. What is the difference between the energy of a photon of violet light of frequency $7.5 \cdot 10^{14}$ Hz, and that of red light of frequency $4 \cdot 10^{14}$ Hz?

23.15. Does the energy of a photon change as it travels from one medium to another?

23.16. By how many times is the energy of a quantum of X-rays of wavelength 10 nm higher than that of yellow light of wavelength 590 nm?

23.17. How many photons of yellow light of wavelength 520 nm in a vacuum have a total energy of 0.001 J?

SEC. 24. GEOMETRICAL OPTICS

Example 68. In Fig. 68 the rays would pass through point A, but a flat mirror is placed in the way. Having reflected at the mirror, these rays converged at point A_1. If the dis-

tance from point A to the mirror is 35 cm, determine the position of point A_1 relative to the mirror.

Given: $AO = 35$ cm is the distance from point A to the mirror.

Determine: A_1O — the distance from the image of point A to the mirror.

Solution. As the ray going to point A normal to the mirror is reversed upon reflection, the image is bound to lie on the continuation of perpendicular AO dropped from point A on the mirror. The location of point A_1 may be found as an intersection of some other reflected ray (e.g. from point B) with the continuation of perpendicular AO. Distance A_1O is found from triangle ABA_1.

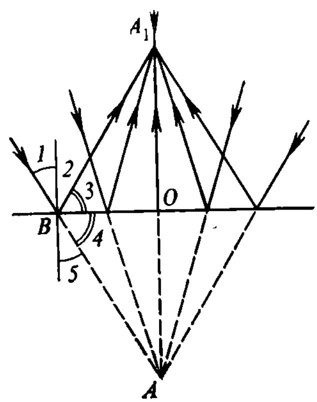


Fig. 68

It follows from reflection laws that $\angle 1 = \angle 2$, $\angle 1 = \angle 5$ (see Fig. 68). Hence, $\angle 2 = \angle 5$ and $\angle 3 = \angle 4$. It follows that in triangle ABA_1 line BO, that is the height, is also the bisector, and the median as well, i.e. $AO = A_1O$. As this reasoning is valid for any other ray reflected from the mirror, all of them will pass through point A_1 which will be the actual image of point A. As, in fact, at point A there are no rays, it is said to be the virtual image.

Answer. The flat mirror produces the real image of luminous point that is symmetrical to the virtual image relative to the mirror ($AO = A_1O$), i.e. at a distance of 35 cm from it.

Example 69. A stone lies in 100 cm of water (Fig. 69 on p. 277). An observer looks at the stone so that his line of vision makes an angle of 30° with the normal to the water surface. Eyes are assumed to be disposed so that their pertinent lines of vision lie in one vertical plane. Determine the apparent depth and horizontal displacement of the stone.

Given: $H = 100$ cm is the depth of the stone, $\angle \beta = 30°$

is the angle of refraction, $n = 4/3$ is the absolute refractive index for water.

Determine: h — the apparent depth;
l — the horizontal displacement of the image.

Solution. Owing to refraction at water-air interface the observer will see the stone not at A, but at a point B. As the image of the stone is formed by the rays entering the eye, then the angles of incidence at the interface differ by a small amount Δi. The same is also true of the angles of refraction, β. The suitable equations may be otbained from triangles BOC and BOC_1

$$l_1 - l = h \tan \beta \qquad l_2 - l = h \tan (\beta + \Delta\beta)$$

The segments l_1 and l_2 can be calculated from triangles ADC and ADC_1

$$l_1 = H \tan i \qquad l_2 = H \tan (i + \Delta i)$$

The angles of ray incidence, i and $i + \Delta i$, can be found using the refraction laws

$$\frac{\sin i}{\sin \beta} = \frac{1}{n} \qquad \frac{\sin (i + \Delta i)}{\sin (\beta + \Delta\beta)} = \frac{1}{n}$$

Considering that Δi and $\Delta\beta$ are small and therefore $\sin \Delta i \approx \Delta i$, $\sin \Delta\beta \approx \Delta\beta$, thus

$$\cos \Delta i \approx 1 \quad \text{and} \quad \cos \Delta\beta \approx 1$$

We next eliminate l from the first two relations:

$$l = l_1 - h \tan \beta \qquad l = l_2 - h \tan (\beta + \Delta\beta)$$

hence

$$l_1 - h \tan \beta = l_2 - h \tan (\beta + \Delta\beta)$$

Substituting for l_1 and l_2 gives

$$H \tan i - h \tan \beta = H \tan (i + \Delta i) - h \tan (\beta + \Delta\beta)$$

or

$$h = H \frac{\tan (i + \Delta i) - \tan i}{\tan (\beta + \Delta\beta) - \tan \beta}$$

Making use of the formula for the difference of tangents we obtain

$$h = H\frac{\sin \Delta i \cos \beta \cos (\beta + \Delta\beta)}{\cos i \cos (i + \Delta i) \sin \Delta\beta}$$

hence, again recalling that Δi and $\Delta\beta$ are small, we get

$$h = H\frac{\cos^2 \beta}{\cos^2 i}\,\frac{\Delta i}{\Delta\beta}$$

The ratio of angles Δi and $\Delta\beta$ is to be found from the refraction law

$$\frac{\sin (i + \Delta i)}{\sin (\beta + \Delta\beta)} = \frac{\sin i}{\sin \beta} \qquad \frac{\sin i \cos \Delta i + \cos i \sin \Delta i}{\sin \beta \cos \Delta\beta + \cos \beta \sin \Delta\beta} = \frac{\sin i}{\sin \beta}$$

$$\sin i \sin \beta \cos \Delta i + \cos i \sin \beta \sin \Delta i = \sin i \sin \beta \cos \Delta\beta + \sin i \cos \beta \sin \Delta\beta$$

Using the smallness of angles Δi and $\Delta\beta$ we have thus

$$\sin i \sin \beta + \Delta i \cos i \sin \beta = \sin i \sin \beta + \Delta\beta \sin i \cos \beta$$

or

$$\frac{\Delta i}{\Delta\beta} = \frac{\sin i \cos \beta}{\sin \beta \cos i} = \frac{\cos \beta}{n \cos i}$$

Finally, we arrive at

$$h = \frac{H}{n}\left(\frac{\cos \beta}{\cos i}\right)^3$$

Now we proceed to calculate angle i and $\cos i$

$$\frac{\sin i}{\sin 30^\circ} = \frac{3}{4} \qquad \sin i = 0.375$$

We take from the tables that $i = 22^\circ$, $\cos i = 0.927$, $\cos \beta = 0.866$.

Next we determine h

$$h = \frac{100\,\text{cm} \cdot 3}{4}\left(\frac{0.866}{0.927}\right)^3 = 75\,\text{cm} \cdot 0.815 \approx 61\,\text{cm}$$

The value of l is obtained from the relation $l = H \tan i - h \tan \beta$.

$$l = 100\,\text{cm} \cdot 0.404 - 61\,\text{cm} \cdot 0.577 \approx 5.2\,\text{cm}$$

Answer. The apparent depth of the stone is about 0.61 m, the horizontal displacement is about $5.2 \cdot 10^{-2}$ m.

Example 70. An object is placed before a concave spherical mirror normal to its principal axis so that the ratio of the image size to the object size is $M_1 = 1.5$. After the object has been removed by $l = 16$ cm from the mirror the ratio

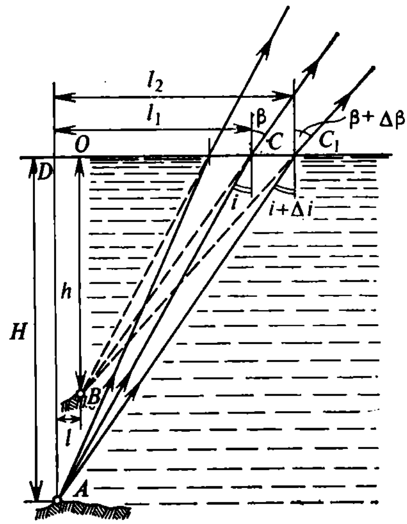


Fig. 69

became equal to $M_2 = 0.5$. Find the curvature radius of the concave mirror.

Given: $M_1 = 1.5$ is the linear magnification in the first case, $l = 0.16$ m is the separation between the two object locations, $M_2 = 0.5$ is the linear magnification in the second case.

Determine: R—the curvature radius of the concave mirror.

Solution. The radius of curvature, R, and the focal length, F, of the concave spherical mirror are related by $R = 2F$; consequently, the problem reduces to determining the focal length of the mirror. Using the relationships for the concave mirror:

$$\frac{1}{F} = \frac{1}{d} + \frac{1}{f} \quad \text{and} \quad k = \frac{f}{d}$$

we write for the first location of the object

$$F = \frac{d_1 f_1}{d_1 + f_1} \qquad f_1 = k_1 d_1$$

or, after substitution for f_1,

$$F = \frac{k_1}{1+k_1} d_1 \tag{1}$$

Similarly, for the second location

$$F = \frac{d_2 f_2}{d_2 + f_2} \qquad f_2 = k_2 d_2$$

or

$$F = \frac{k_2}{1+k_2} d_2$$

But, as $d_2 = d_1 + l$, we get

$$F = \frac{k_2}{1+k_2} (d_1 + l) \tag{2}$$

Using relation (1) we determine d_1 and the value obtained we substitute in equation (2)

$$d_1 = \frac{1+k_1}{k_1} F$$

Then

$$F = \frac{k_2}{1+k_2} \left(\frac{1+k_1}{k_1} F + l \right)$$

$$F = \frac{k_1 k_2}{k_1 - k_2} l$$

Substituting gives

$$F = \frac{1.5 \cdot 0.5}{1.5 - 0.5} 0.16 \text{ m} = 0.12 \text{ m} \qquad R = 0.24 \text{ m}$$

Answer. The radius of curvature of the concave mirror is 0.24 m.

Example 71. A converging lens of power 2.0 diopters and a diverging lens of optical power 1.5 diopters are spaced 40 cm apart and have a common axis (see Fig. 71). An object AB 20 cm high is located at a distance of 4.0 m from the converging lens as shown. Find the location and nature of the image.

Given: $D_1 = 2.0$ diopters is the optical power of the converging lens, $D_2 = -1.5$ diopters is the power of the diverging lens, $a = 0.40$ m is the separation of the lenses, $d_1 =$

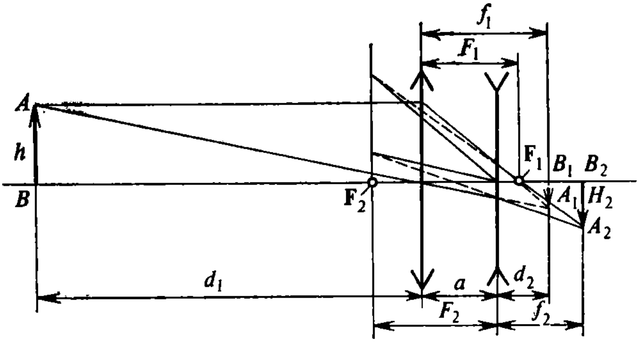


Fig. 71

4.0 m is the distance from object AB to the converging lens, $h = 0.20$ m is the height of the object.

Determine: f_2—the distance from the diverging lens to the image A_2B_2;

H_2—the height of image A_2B_2.

Solution. Using the lens equation $1/d + 1/f = D$ we may find the location and size of the image of object AB to be produced by the converging lens. This image (A_1B_1 in Fig. 71) will in turn be the object for the diverging lens that will give the final image, A_2B_2. Its location is obtained from the above equation, and height from the relation $h/H = d/f$.

Now we may find the location and height of the image of subject AB to be obtained using the converging lens

$$\frac{1}{4.0\,\text{m}} + \frac{1}{f_1} = 2.0 \text{ diopters} \quad f_1 = 0.57\,\text{m}$$

$$\frac{0.2\,\text{m}}{H_1} = \frac{4.0\,\text{m}}{0.57\,\text{m}} \quad H_1 = 0.0285\,\text{m}$$

It is seen that the rays going from point A are incident on the diverging lens as a converging pencil, and this will not actually give image A_1B_1, but the diverging lens will

give image A_2B_2. We will find the distance to A_2B_2 considering that d_2 for the diverging lens is $(f_1 - 0.4)$ m, i.e. 0.17 m, and this value is to be substitute in the equation with the negative sign:

$$-\frac{1}{0.17\text{ m}}+\frac{1}{f_2}=-1.5 \text{ diopters} \qquad f_2=0.223\text{ m}$$

Next we determine the height of the image, H_2, remembering that in the relation $h/H = d/f$, we should substitute H_1 for h:

$$\frac{0.0285\text{ m}}{H_2}=\frac{0.17\text{ m}}{0.223\text{ m}} \qquad H_2=0.035\text{ m}$$

Answer. The real inverted image A_2B_2 of object AB of height 3.5 cm is behind the diverging lens at a distance of about 22 cm.

Example 72. Determine the magnification provided by a magniglass with focal length 1.25 cm.

Given: $F = 1.25$ cm is the focal length of the magniglass, $f_{n.p.} = 25$ cm is the distance to the near point.

Determine: β—the magnification of the glass.

Solution. Textbooks give two equations for the lens magnification. The first one is $\beta_1 = 0.25/F$ and is valid for the case, where the object is at the principal focus of the lens, and the eye is accommodated at infinity. The second formula has the form $\beta_2 = 0.25/F + 1$ and is good for the case where the eye sees the image at the near point. This results in a large magnification, but the eye will be tired when viewing a subject for a long time. This is accounted for by the fact that in its natural condition, i.e. fully relaxed, the eye is accommodated at infinity. As the problem contains no indications to the effect we shall calculate the magnification for both cases.

For the first case:

$$\beta_1=\frac{25\text{ cm}}{1.25\text{ cm}}=20$$

For the second case:

$$\beta_2=\frac{25\text{ cm}}{1.25\text{ cm}}+1=21$$

Answer. When a normal eye is accommodated at infinity, the lens gives 20X magnification, and with the accommodation at near point 21X magnification.

Reflection and Refraction of Light

24.1. A light ray strikes a flat mirror at an angle of 20°. How will the angle between the incident and reflected rays change, if the ray makes an angle of 35° with the mirror?

24.2. A flat mirror was turned about an axis passing through a point at which a ray strikes it, normal to the plane in which both incident and reflected rays lie. Through what angle was the mirror turned, if the reflected ray turned through 42°? By how many degrees will the angle between the incident and reflected rays change in the process?

24.3. A small round flat mirror may rotate about its vertical axis. On a wall 1.2 m away from the mirror there hangs a flat screen parallel to the mirror plane. A horizontal ray strikes at the centre of the mirror making an angle of 12° with it, and is reflected onto the screen. Determine the displacement of the lightspot on the screen as the mirror is turned through 15°

24.4. In a room a mirror hangs vertically so that its upper edge is at the level of the upper part of the head of a man 182 cm high. What is the shortest length of the mirror required for the man to see himself in the mirror in full length?

24.5. To light the bottom of a well with sun rays use was made of a flat mirror positioned at an angle of 25° to normal. Determine the terrain clearance altitude of the Sun.

24.6. As a ray crosses the interface between two transparent media, its direction changes. Explain why.

24.7. On a glass a light ray is incident with an intensity of 25 W/m^2. Determine the intensity of light penetrating the glass, if the reflection factor is 0.18. How will the intensity change as the incidence angle decreases?

24.8. As a glass plate is being lighted 18 W/m^2 penetrates inside the glass, and 2.0 W/m^2 reflects. Determine the reflection factor.

24.9. When we seat at a camp fire objects beyond the fire seem swaying. Account for the phenomenon.

24.10. Why is it so difficult to hit a fish with a gun in several centimetres of water?

24.11. In what cases is there no refraction at an interface of two transparent media?

24.12. A glass contains two transparent liquids separated by a sharp horizontal boundary. Using a light ray, how can you establish in which liquid the velocity of light is lower?

24.13. By how many times is the velocity of light in diamond lower than in sugar?

24.14. The velocity of light in a liquid is 240,000 km/s. On the surface of this liquid a light ray is incident from air at an angle of 25°. Determine the angle of refraction of the ray.

24.15. The velocity of light in one transparent medium is 225,000 km/s, in the other 200,000 km/s. A ray strikes the interface between these media making an angle of 30° with it and goes to the second medium. Determine the angle of refraction.

24.16. A ray strikes an interface between two transparent media at an angle of 35° and refracts at an angle of 25°. What will refraction angle be, if the ray strikes at an angle of 50°?

24.17. A ray is incident from air on the surface of a liquid at an angle of 40° and refracts at 24°. At what angle of incidence will the angle of refraction be 20°?

24.18. A ray travels from glycerin to air. If the ray makes an angle of 22° with the surface, what will the angle of refraction be?

24.19. A ray entering air from ice is incident on the ice surface at an angle of 15°. Determine the angle of refraction of the ray in the air.

24.20. A ray travels from water to glass with refractive index 1.7. If the angle of refraction is 28°, determine the angle of incidence.

24.21. A ray goes from glycerin to water. Determine the angle of refraction, if the angle of incidence is 30°.

24.22. If the angle between the refracted and reflected rays is 90°, determine the angle of incidence from air on the water surface.

24.23. Determine the angle of refraction of a ray passing from air in ethyl alcohol, if the angle between the reflected and refracted rays is 120°.

24.24. Sun light enters the water surface when the terrain clearance altitude of the Sun is 30°. Calculate the path of the rays in the water after the refraction.

24.25. Two observers, one on the ground and the other under the water, try to sight the Sun. For which will the Sun seem higher?

24.26. Into the bottom of a pond a pole 1.25 m high is driven. Determine the length of the shadow of the pole on the bottom, if the sun rays make an angle of 38° with the surface of the pond, and the pole is fully under the water.

24.27. A pile is driven into the bottom of a pond 1.5 m deep so that it projects out of water by 30 cm. Find the length of the shadow on the bottom if the sun rays make an angle of 45° with the surface of the pond.

24.28. A diver measured the angle of refraction of a ray in the water. It appeared to be 32°. What angle are the rays making with the water surface?

24.29. A diver under water found that the direction of the Sun makes with the normal an angle of 28°. When out of water he saw that the Sun was lower. Determine the change in the angle for the diver.

24.30. A ray enters glass from air, the refractive index of the glass being 1.5. Determine the angular deviation of the ray if the angle of incidence is 25°; 65°.

24.31. A stone lies on the bottom of a stream. A boy wants to hit it with a stick. Taking aim the boy holds the stick in the air at an angle of 45°. At what distance from the stone will the stick hit the bottom, if the depth is 32 cm?

24.32. Why does the depth of a river determined visually appear actually much less than the true depth?

24.33. A maximum angle of incidence for a ray travelling from turpentine to air is 41°50′. What is the velocity of light in turpentine?

24.34. Compute the critical angles for water, sugar, and diamond.

24.35. A ray enters air from water. The angle of incidence is 52°. Calculate the angle of refraction.

24.36. Determine the critical angle for a diamond-sugar system.

24.37. A ray travels from methyl alcohol to air. Will

the ray leave the liquid if it strikes the surface at an angle of 45°?

24.38. Glass is known to be a transparent material. But ground glass is opaque and white in colour. Account for the phenomenon.

24.39. If a white sheet of paper is stained with vegetable oil, the paper turns transparent. Explain.

24.40. Is the total internal reflection possible if a ray travels from water to glass?

24.41. What are the critical angle and angle of polarization as a ray travels from a glass with refractive index 1.7 to water?

24.42. Determine the critical angle for a ray travelling from a liquid to air, if the angle of polarization here is 36°19′.

24.43. Consider a crystal for which the critical angle is 44°12′. Determine the angle of incidence inside the crystal at which the reflected ray will be fully polarized.

24.44. A point light source is 1.5 m away from the water surface in air. For an observer under water, at what distance from the surface will the image of the source be seen?

24.45. A point light source is located in the air above water. For an observer under the water below the light source the distance from the source to the water surface seems to be 2.5 m. Determine the true distance.

24.46. An observer is under water at a depth of 40 cm. He sees a lamp above, the distance to which for him is 2.4 m. Calculate the true distance from the lamp to the water surface.

24.47. In the air the distance from a lamp to water surface is 1.2 m. At a depth of 60 cm in the water there is an observer. Determine the distance at which he will see the lamp.

24.48. At the bottom of a glass bath there lies a mirror covered by 20 cm of water. In the air, 30 cm above the water there hangs a lamp. Find the distance from the mirror at which an observer looking into the water will see the image of the lamp in the mirror.

24.49. Consider a glass bath with a 15 cm layer of carbon tetrachloride. A mirror lies on the bottom of it. A point light source is located 25 cm above the liquid. Calculate

the distance from the light source to its virtual image in the mirror.

24.50. There is a plate 4.0 cm thick made of glass with refractive index 1.6. A ray is incident on its surface at an angle of 55°. Determine the displacement of the ray after it has passed through the plate.

24.51. A plane-parallel plate has a refractive index of 1.7. A ray is incident on the plate at an angle of 50°, and having passed through the plate, is displaced by 2.0 cm. Compute the thickness of the plate.

24.52. There are two contacting plane-parallel plates of thickness 16 and 24 mm. One is made of crown glass with refractive index 1.5, the other of flint glass with refractive index 1.8. A ray making an angle of 48° with the normal enters one of them. Calculate the displacement of the ray after it travels from the plates into the air.

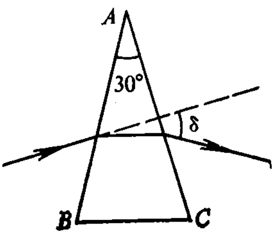


Fig. 24.55

24.53. A ray making an angle of 30° with the normal is incident on a prism with an angle of 40°. The refractive index of the material of the prism is 1.5. Determine the angle of deviation of the ray after it leaves the prism.

24.54. Consider a prism with an angle of 36° made of glass with refractive index 1.6. A ray strikes it at an angle of 15°. What is the change in the angle of deviation if the angle of incidence increases to 30°.

24.55. Figure 24.55 shows glass prism ABC with angle 30°. On the prism a ray is incident so that inside the prism it goes parallel to BC. Determine the angle of deviation, δ, if $AB = AC$, and the refractive index of the material is 1.6.

Spherical Lenses and Mirrors

24.56. A concave mirror has a radius of curvature of 1.2 m. To obtain a search light, where relative to the mirror must a powerful light source be located?

24.57. A point light source is located 2.8 m away from a concave spherical mirror with a radius of curvature of 90 cm on its principal axis. Calculate the position of the image.

24.58. A point light source lies on the principal axis of a concave spherical mirror with radius of curvature 1.6 m. Its virtual image appears to be at a distance of 70 cm from it. Determine the location of the light source.

24.59. A concave spherical mirror with radius of curvature 80 cm produces a real image of an object 50 cm away from the mirror. Calculate the distance between the object and the mirror.

24.60. A concave spherical mirror gives on a screen a 12X magnification when the object is separated by 45 cm from the mirror. Determine the focal length and the radius of curvature of the mirror.

24.61. When an object is spaced 2.0 m apart from the concave spherical mirror, its real image occurs at a distance of 50 cm from the mirror. Find the position, size, and nature of the image, if it is shifted from the mirror another 1.2 m.

24.62. Determine the magnification of a concave spherical mirror with radius of curvature of 64 cm, if an object is placed 16 cm apart from the mirror.

24.63. A light spot is 150 cm away from a convex spherical mirror with radius of curvature 72 cm. Compute the image distance.

24.64. A convex spherical mirror produces the tenfold demagnified image of an object located at a distance of 180 cm from the mirror. Determine the radius of curvature of the mirror.

24.65. Converging rays fall on a convex spherical mirror with radius of curvature 56 cm so that the reflected rays converge on the principal axis of the mirror. The distance from the intersection of these rays to the mirror is 20 cm. Find the intersection point of the rays without the mirror.

24.66. Two identical concave spherical mirrors facing each other are disposed so that their principal axes coincide, the separation between the mirrors being much larger than their radii of curvature. If in the principal focus of one of them a motion picture film is placed, and in the

focus of the other a lamp, then on lighting the lamp the film will ignite. Explain.

24.67. On buses, trolley-buses, and trams convex mirrors are used as outside rear-view mirrors. Why?

24.68. An object is placed before a concave spherical mirror normal to its principal axis so that its magnification is 1.2. After the object has been removed further from the mirror by 25 cm, the ratio of linear dimensions of the image and object becomes 0.4. Calculate the focal length and radius of curvature of the mirror.

24.69. A thin double convex lens has a focal length of 75 cm. Find its power.

24.70. A thin double concave lens has a focal length of —50 cm. What is its power?

24.71. The powers of four lenses are 2, 16, —4, and —12 diopters. Determine their respective focal lengths.

24.72. A double convex lens with identical radii of curvature 20 cm is made of glass of refractive index 1.5. Determine the focal length.

24.73. A double concave lens with identical radii of curvature 25 cm is made of glass of refractive index 1.6. Find its power.

24.74. A planar convex quartz lens has a power of 8.2 diopters. What is the radius for curvature of the convex surface of the lens?

24.75. A hollow with radius of curvature 12 cm contains water. After the water has frozen, an ice planar convex lens is produced. At what distance from the lens will solar rays converge if these fall parallel to the principal axis?

24.76. Determine the power of a double convex lens of stone salt with radii of curvature 40 cm in air, and in carbon disulfide.

24.77. What is the principal focal length of a planar concave lens of sylvite with radius of curvature 25 cm in the air? In acetone? In aniline?

24.78. How on a sunny day can you determine the principal focal length of a converging lens with a ruler only?

24.79. A thin lens has a power of 5.0 diopters. An object is placed at 60 cm from the lens. Find the location and nature of the image.

24.80. There is a ball of diameter 1.24 cm made of glass

with refractive index 1.5. Determine the location of the image of the Sun produced by the ball.

24.81. A double convex lens has a principal focal length of 50 cm. A 1.2 cm high object is placed at a distance of 60 cm from the lens. Determine the location and size of the image.

24.82. A picture on a slide is 2.0 cm high, on a screen 80 cm. If the distance from the objective to the slide is 20.5 cm, calculate the power of the objective.

24.83. A projector has a principal focal length of 15 cm. A slide is 15.6 cm away from the objective. Find the linear magnification of the projector.

24.84. A diverging lens has a principal focal length of 12 cm. The image of an object is at a distance of 9.0 cm from the lens. Determine the object distance.

24.85. The distance between a candle and a wall is 200 cm. When a converging lens is placed in between at a distance of 40 cm from the candle, on the wall a sharp image of the candle is produced. Determine the principal focal length of the lens. What image is produced?

24.86. What is the magnification of a projector if its objective with a principal focal length of 18 cm is placed at a distance of 6.0 m from the screen?

24.87. A projector is equipped with a 24X magnification objective. If a slide is placed 20.8 cm away from the objective, determine the power of the objective.

24.88. Given that the real image of an object placed at a distance of 30 cm from a lens is obtained at the same distance from it, find the focal length of the lens.

24.89. A luminous object is placed a distance of 12.5 m from a lens, and its real image is 85 cm away from it. Find the location of the image, if the object is moved 2.5 cm nearer to the lens.

24.90. An object is placed a distance of 40 cm from a lens of power 2 diopters. How will the image distance change, if the object is moved closer to the lens by 15 cm?

24.91. An object is placed a distance of 1.6 F from a lens. The object is brought closer to the lens by 0.8 F. Determine the displacement of the image, if the power of the lens is 2.5 diopters?

24.92. An object is at a distance of 1.5 F from a lens.

The object is brought closer to the lens by 0.7 F. Determine the displacement of the image, if the power of the lens is —2.4 diopters.

24.93. The separation between an object and screen is 120 cm. Where should a converging lens with focal length 25 cm be located, so that a distinct image be produced on the screen?

24.94. At what distance from a lens with power —4.5 diopters must an object be placed for its image to be reduced sixfold?

24.95. Given that the image of an object placed 50 cm in front of a diverging lens is fivefold reduced, compute the principal focal length of the lens.

24.96. A 15 cm high object is placed at a distance of 1.5 F from a lens. Find the height of the image on a screen, if it is normal to the principal axis.

24.97. Consider a projector 6.4 cm wide and 6.4 cm high producing on a screen that is 3.6 m away from the objective an image of area 1.96 m^2. Find the power of the objective.

24.98. The objective of a projector has a power of 5.4 diopters. A screen is 4.0 m away from the objective. Determine the dimensions of the screen just large enough for a 6×9 cm^2 slide to be projected on it.

24.99. There is a converging lens of focal length 15 cm. Determine the object distance such that the real image be 2.5 times larger than the object.

24.100. The virtual image of an object obtained with a lens is 4.5 times larger than an object. If the object is at a distance of 3.8 cm from the lens, what is the power of the lens?

24.101. Rays pass through point A in Fig. 24.101. If a converging lens is placed 40 cm in front of point A, the rays will pass through point A_1 at a distance of 30 cm from the lens. Determine the principal focal length of the lens.

24.102. Converging rays fall on a lens of power 2.5 diopters, and pass through a point at a distance of 20 cm from the optical center of the lens lying on its principal optical axis. If the lens is removed, where will the rays converge?

24.103. Converging rays pass through point A (Fig. 24.103). If a diverging lens is placed in their way at a distance of 30 cm from point A, the rays will pass through

point A_1 at a distance of 60 cm from the lens. Determine the principal focal length of the lens.

24.104. Converging rays strike a lens of power —2.5 diopters. Thereafter the rays diverge so that their continuations converge on the principal axis on either side of

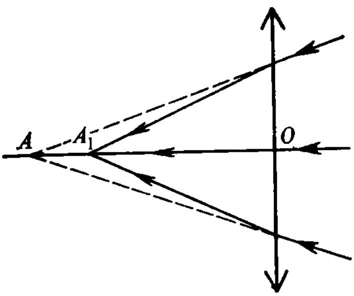


Fig. 24.101

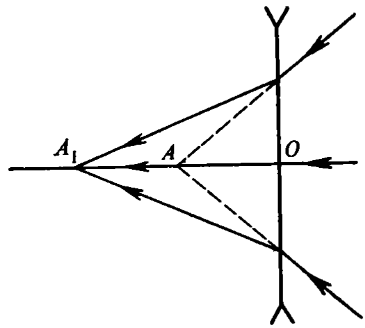


Fig. 24.103

the lens at a distance of 150 cm from it. If the lens is removed, where will the rays converge?

24.105. Figure 24.105 (*a*) and (*b*) shows the direction of principal axes of lenses OO_1, light points A and their

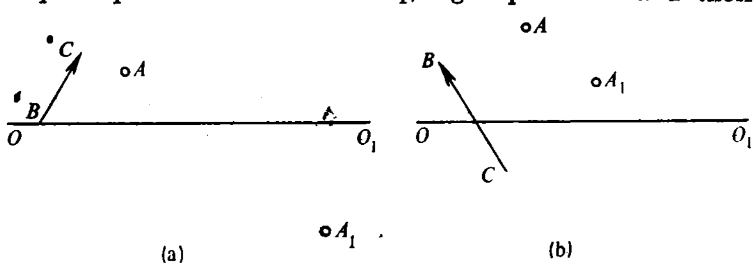


Fig. 24.105

images A_1. Find the positions and nature of the lenses, and draw the images of objects BC.

24.106. Figure 24.106 shows the position of the principal axis of lens OO_1, light point A and its image A_1. Find the position of the lens and draw the image of object BC.

24.107. Figure 24.107 gives the location of the principal axis of lens OO_1, luminous point A and its image A_1. Find the lens location and draw the images of points B and C.

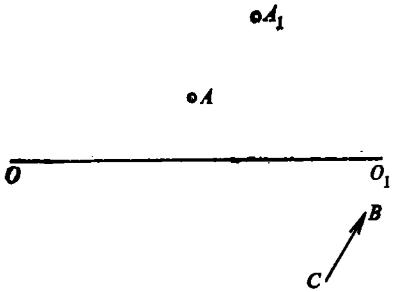


Fig. 24.106

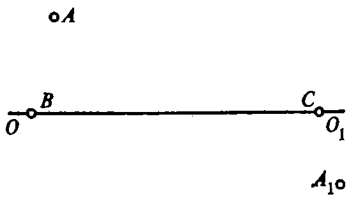


Fig. 24.107

24.108. A 12 cm high object is placed at a distance of 40 cm from a lens of power 3.5 diopters normal to the principal axis. If the object is placed at 50 cm from the lens, how will the size of the image change?

24.109. An object 15 cm high is placed 125 cm away from a lens with power 2.0 diopters normal to the principal axis. If the object is brought closer to the lens by 50 cm, how will the height of the image change?

24.110. A 16 cm high object is located at a distance of 80 cm from a lens with power —2.5 diopters. If the object is moved closer to the lens by 40 cm, how will the height of the image change?

24.111. Distances from the object to the lens and from the lens to the real image are identical and equal to 60 cm. By how many times will the image increase, if the object is placed by 20 cm closer to the lens?

24.112. The image of an object on the clouded glass of a photographic camera when taking picture at a distance of 8.5 m has a height of 13.5 mm; and at a distance of 2.0 m, a height of 60 mm. Find the focal length of the objective.

24.113. An object 20 cm high produces using a lens a real image of height 80 cm. When the object is shifted by 5.0 cm, a real image 40 cm high is produced. Find the focal length and power of the lens.

24.114. For an object of height 3.0 cm a lens gives a real image of height 18 cm. When the object is moved by 6.0 cm, a virtual image 9.0 cm high is produced. Calculate the focal length and power of the lens.

24.115. A candle is located at a distance $l_1 = 1$ m from a screen. A double convex lens placed in between gives a sharp image on the srceen at two positions of the lens separated by a distance $l_2 = 0.2$ m. Compute the focal length of the lens.

24.116. If an object is located 10 cm in front of the focus of a converging lens, the image will be at a distance of 2.5 m behind the other focus. Determine the power of the lens.

24.117. Consider two thin lenses in contact with each other. What is the power of the two-lens system, if the powers of the lenses are 4.8 and 12 diopters? 3.5 and —8.2 diopters? —5 and —2.6 diopters?

24.118. Two converging thin lenses are in contact with each other. Determine the focal length of the two-lens system, if their focal lengths are 40 cm and 0.80 m. How will the power of the system change, if the lenses are separated by 20 cm so that their axes coincide?

24.119. (1) An illuminated slit of height 3.5 cm is projected on a screen using a converging lens with a focal length of 15 cm. What will the height of the slit be on the screen, if the screen is placed at a distance of 45 cm from the lens? (2) To this lens another lens of power 4.0 diopters is brought in contact. In what direction and how should the screen be shifted so that a sharp image of the slit be obtained on it? What will the height of the image be?

24.120. An object is placed at a distance of 40 cm from the lens so that a virtual image of the object is produced at a distance of 1.2 m from the lens. Another lens of power 2.0 diopters is brought in contact to the first one. Find the location and nature of the image.

24.121. How on a sunny day can you determine with a ruler the powers of a converging and a diverging lenses, if the absolute value of the power of the converging lens is larger than that of the diverging one?

24.122. A point light source is placed at the principal focus of a thin converging lens. Find the location and nature

of the image of the source, if a flat mirror is brought up against the back side of the lens normal to the principal axis of the lens?

24.123. On a flat mirror there lies a thin converging lens with principal focal length 84 cm. A point light source is on the principal axis of the lens. Determine the location and nature of the source, if the distance from the source to the mirror is 60 cm; 96 cm.

24.124. On a flat mirror there lies a thin diverging lens with principal focal length 64 cm, and a point light source lies on the principal axis of the lens. Determine the location and nature of the source, if the distance from the source to the mirror is 72 cm; 48 cm.

Angle of View and Optical Instruments

24.125. When looking at a row of telegraph poles, we perceive them as becoming ever increasingly smaller. Explain.

24.126. At what distance from an observer will a luminous object 5.0 cm high be perceived as a luminous point, if the angle of view here is taken to be less than 1′?

24.127. A 5.4 m high pole is 120 m away from an observer. Determine the angle of view for the pole.

24.128. An observer sees a building with an angle of view of 9°30′. Calculate the distance from the observer to the building, if the height of the building is 10.5 m.

24.129. The diameter of a clock's dial is 12 cm. The clock is 1.7 m away from eyes with principal focal length 1.7 cm. Find the diameter of the dial image on the retina of the eye.

24.130. If the power of a normal eye changes from 58.6 to 70.6 diopters, how can its focal length change?

24.131. If an eye is accommodated at infinity, what magnification is given by a lens with focal length 7.5 cm?

24.132. A lens has a magnification of 12X for a normal eye. Determine the focal length, if the eye is accommodated at infinity. How will the magnification change, if the lens is used by a near-sighted person without spectacles?

24.133. A magnifying glass provides a 8X magnification for an eye accommodated at the near point. Find the focal length of the lens and its power.

24.134. A lens of power 50 diopters is used as a magnifying glass. If the eye is accommodated at the near point, what is its linear magnification?

24.135. Find the focal length of a microscope, if the principal focal length of the objective is 4.0 mm, the principal focal length of the eyepiece is 15 mm, and the drawtube is 12 cm long.

24.136. Calculate the focal length of the eyepiece of a microscope, if the instrument has 900X magnification, and the magnification of the objective is 90X.

24.137. A 600X magnification microscope uses an eyepiece with focal length 16.7 mm. If the microscope has a power of 20 diopters, what magnification will the microscope give?

24.138. Compute the magnification of a telescope, if the focal length of its objective is 140 cm, and the principal focal length of the eyepiece is 28 mm.

24.139. Consider a telescope whose objective has a focal length of 20 m, and the eyepiece provides a magnification of 5X. What is the magnification of the instrument?

24.140. A Keplerian telescope 15 m long has a magnifying power of 249X. Determine the focal lengths of the eyepiece and objective.

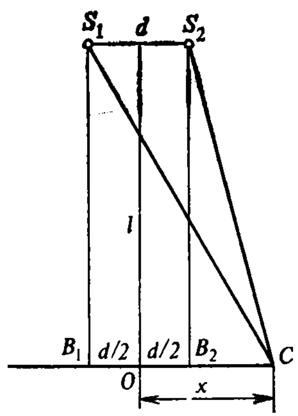


Fig. 73

SEC. 25. PHENOMENA ARISING FROM WAVE NATURE OF LIGHT

Example 73. The distance between two coherent light sources S_1 and S_2 in air is $d =$ $= 0.15$ mm (Fig. 73). The distance from these sources to the screen is $l = 4.8$ m. Determine the optical path difference for the rays coming from sources S_1 and S_2 at point C on the screen, if $OC =$ $= 16$ mm.

Given: $d = 1.5 \cdot 10^{-4}$ m is the separation between the

light sources, $l = 4.8$ m is the distance from the light sources to the screen, $x = 1.6 \cdot 10^{-2}$ m is the distance from point C to the centre of screen O.

Determine: $n\,(S_1C\text{-}S_2C)$—the optical path difference.

Solution. As the rays go in the air, the optical path difference is equal to the geometrical one. We drop perpendiculars S_1B_1, S_2B_2 on the screen and connect points S_1 and S_2 with point C. Then according to the Pythagoras theorem, we obtain for triangles S_1B_1C and S_2B_2C

$$(S_1C)^2 = (S_1B_1)^2 + (B_1C)^2$$
$$(S_2C)^2 = (S_2B_2)^2 + (B_2C)^2$$

As $S_1B_1 = S_2B_2 = l$, $OC = x$ and $B_1O = OB_2 = d/2$, we have

$$(S_1C)^2 = l^2 + \left(x + \frac{1}{2}\,d\right)^2 \quad \text{and} \quad (S_2C)^2 = l^2 + \left(x - \frac{1}{2}\,d\right)^2$$

Subtracting the appropriate expressions gives

$$(S_1C)^2 - (S_2C)^2 = l^2 + \left(x + \frac{1}{2}\,d\right)^2 - l^2 - \left(x - \frac{1}{2}\,d\right)^2$$

or

$$(S_1C + S_2C)\,(S_1C - S_2C)$$
$$= \left(x + \frac{1}{2}\,d + x - \frac{1}{2}\,d\right)\left(x + \frac{1}{2}\,d - x + \frac{1}{2}\,d\right)$$

As d and x are small compared to l (which is always the case in observations of light interference), the sum $(S_1C + S_2C)$ may be approximated by $2l$, and $n\,(S_1C - S_2C)$ is the required path difference. We denote it by δ and obtain

$$\frac{2l\delta}{n} = 2x \cdot 2\,\frac{d}{2} \qquad \delta = \frac{dx}{l}\,n$$

And we arrive at

$$\delta = \frac{1.5 \cdot 10^{-4}\ \text{m} \cdot 1.6 \cdot 10^{-2}\ \text{m} \cdot 1}{4.8\ \text{m}} = 0.5 \cdot 10^{-6}\ \text{m}$$

Answer. The optical path difference is $0.5 \cdot 10^{-6}$ m.

Example 74. A planar convex lens of radius of curvature $R = 12$ m is placed on a plane parallel plate with its convex side down (Fig. 74). On the plane side of the lens parallel to its principal axis a beam of monochromatic light of wavelength 600 nm is incident. On the lens a series of dark and bright fringes is seen in the reflected light, and at the centre a dark fringe. Determine the radius r_{3d} of the third dark fringe.

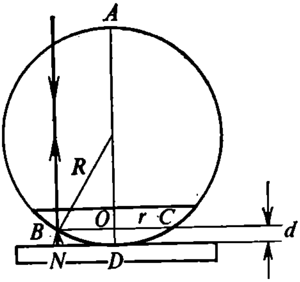


Fig. 74

Given: $R = 12$ m is the radius of curvature of the lens, $\lambda = 600 \cdot 10^{-9}$ m is the wavelength of light, $k = 3$ is the serial number of the fringe.

Determine: r_{3d}—the radius of the third dark fringe.

Solution. From the theorem of chord segments intersecting at a point, we have

$$AO \cdot OD = BO \cdot OC \text{ or } (2R - d)\, d = r^2$$

As the waves reflected from the convex surface of the lens and from the plate interfere the optical path difference for these waves is $2dn$, where n is the refractive index for the substance in the gap between the lens and the plate. Thus, the relation $2dn = m\lambda/2$, depending on the numerical value of m, expresses the condition of the constructive and destructive interference of light. If the interference is observed in reflected light, then the reflection from the plate at point N produces a path difference of $\lambda/2$; therefore for even m the relation $2dn = m\lambda/2$ expresses the condition of the destructive interference, i.e. it corresponds to dark Newton's fringes. For the odd m the same relation is the condition for the constructive interference, i.e. it corresponds to bright Newton's fringes.

As d is small as compared to $2R$, the formula $(2R - d)\, d = r^2$ can be simplified so that d in brackets will be eliminat-

ed: $2Rd = r^2$. As $d = m\lambda/4n$, we have

$$r_m = \sqrt{\frac{mR\lambda}{2n}}$$

where m is the serial number of the fringe, if the central dark fringe is considered zero. As the third dark fringe is the sixth counting all the fringes, then here $m = 6$, and $n = 1$, because the gap between the lens and the plate is filled with air. Therefore

$$r_6 = \sqrt{\frac{6\cdot 12\ \text{m}\cdot 600\cdot 10^{-9}\ \text{m}}{2\cdot 1}} = 4.65\cdot 10^{-3}\ \text{m}$$

Answer. The radius of the third dark Newton's fringe is $4.65\cdot 10^{-3}$ m.

Example 75. A diffraction grating with 75 slits per mm is exposed to light used of a wavelength $\lambda = 500$ nm (Fig. 75). On the screen separated from the grating by a distance l bright equidistant fringes are seen. The distance from the central bright fringe on the screen to the second fringe is $h = 11.25$ cm. Find l.

Given: $d = 1/75$ mm is the diffraction element, $\lambda = 500\cdot 10^{-6}$ mm is the wavelength of the light, $k = 2$ is the number of the fringe, $h = 112.5$ mm is the spacing between the second and the zeroth fringe.

Determine: l—the distance from the diffraction grating to the screen.

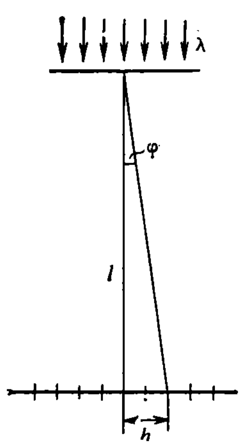


Fig. 75

Solution. The required distance l (see Fig. 75) can be found from the relation $h/l = \tan\varphi$. To determine φ we will use the relation for the diffraction grating

$$k\lambda = d\sin\varphi$$

In this case the angle φ is small, as $\sin\varphi$ is smaller than 0.1, which is seen from the following calculation

$$\sin\varphi = k\lambda/d \quad \sin\varphi = \frac{2\cdot 500\cdot 10^{-6}\ \text{mm}\cdot 75}{1\ \text{mm}} = 0.075$$

For these small angles it may be assumed that $\sin\varphi \approx \approx \tan\varphi$. Therefore

$$\frac{k\lambda}{d} = \frac{h}{l} \qquad l = \frac{hd}{k\lambda}$$

Thus

$$l = \frac{112.5\ \text{mm}\cdot 1\ \text{mm}}{75\cdot 2\cdot 500\cdot 10^{-6}\ \text{mm}} = 1.5\cdot 10^{3}\ \text{mm}$$

Answer. The distance from the diffraction grating to the screen is 1.5 m.

25.1. Beams of coherent light arrive at a point with the optical path difference 2.0 μm. Determine the nature of the interference at the point for a light with wavelength 760 nm, 600 nm, and 400 nm.

25.2. Beams of coherent light arrive at a point with the geometrical path difference 1.2 μm, the wavelength of the light in a vacuum being 600 nm. Determine the nature of interference at the point in the air; in the water, in glass with refractive index 1.5.

25.3. Radiation with wavelength 480 nm from two coherent sources spaced 120 μm apart falls on a screen. The screen is 3.6 m away from the light sources. The interference produces on the screen alternating dark and bright fringes. Determine the separation between the centres of two neighbouring dark fringes on the screen. What will this distance be, if the interfering light has a wavelength of 650 nm?

25.4. When observing in the air the interference of light from two coherent sources alternating dark and bright fringes are seen on a screen. What is to happen to the width of the fringes, if the observation is to occur in water, other things being equal?

25.5. Using the Fresnel biprism two virtual sources S_1 and S_2 of monochromatic light with wavelength 560 nm are obtained. Their separation from the screen is $l = 3.2$ m

(Fig. 25.5). Through point B at a distance $x = 28$ mm from the centre of the screen, O (for demonstration purposes the picture is not to scale) a third dark fringe passes (numeration beginning from the central dark fringe) that also passes through point O. Determine the distance d between virtual light sources.

25.6. A screen is illuminated with a light of wavelength 590 nm from two coherent sources S_1 and S_2 separated by 200 μm. Also at a distance of 15 mm from the centre of the screen O (see Fig. 25.5) the centre of the second interference dark fringe passes through point B, counting from point O. Determine l, i.e. the distance from virtual light sources to the screen.

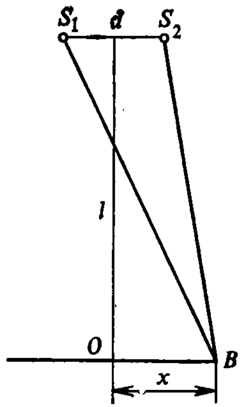


Fig. 25.5

25.7. Two coherent light sources separated by 0.24 mm are 2.5 m away from a screen, the screen showing dark and bright fringes. It is found that a grating 5.0 cm long has 10.5 fringes. What is the length of the incident light?

25.8. Monochromatic light from two virtual sources of $\lambda = 520$ nm interferes so that on a screen 4.0 cm long 8.5 fringes are seen. Determine the spacing between the light sources, if the screen is 2.75 m away.

25.9. What interference pattern is observed on a screen (see Fig. 25.5), if the coherent sources (slits) pass white light? Where will the pattern be brighter? Why?

25.10. Coherent light sources separated by 0.32 mm are narrow slits. A screen on which the interference of light from these sources is observed is at a distance of 3.2 m from them. Find the distance between the red ($\lambda_r = 760$ nm) and violet ($\lambda_v = 400$ nm) fringes of the second interference pattern on the screen.

25.11. Determine the separation between coherent sources of white light, if on the screen the distance between the red and violet lines (see problem 25.10) of the first interference pattern is 5.6 mm. The distance from the sources to the screen is 2.6 m.

25.12. A plate is made of a material with refractive index 1.54, it is illuminated with light of wavelength $\lambda =$ $= 750$ nm normal to the plate surface. What is the minimum thickness of the plate at which it appears in reflected light as red? Black?

25.13. A thin film 0.50 μm is illuminated with light of wavelength 590 nm. What colour will the film show in the transmitted light, if the refractive index of the film substance is 1.48, and the rays are perpendicular to the film surface? What will happen to the film coloration, if it is inclined relative to the rays?

25.14. A thin film illuminated with white light appears green in reflected light if viewed normal to the surface. What will happen with the colour of the film, if the latter is inclined as related to the light ray?

25.15. When a thin film is illuminated with parallel monochromatic beams, in some places the film shows bright spots, and in others dark. Explain.

25.16. When a thin film is illuminated with parallel beams of white light the film shows a coloured pattern. Account for the phenomenon.

25.17. Oil spots on the surface of water exhibit iridescent coloration. Why?

25.18. When illuminating two thin films of the same material with white light incident normal to the surface of the film, one of them appears red, and the other blue. From this, is it possible to determine which film is thicker?

25.19. When illuminating two films of the same transparent material with white light incident normal to their surface, both films in the reflected light appear green. Can the thickness of these films be considered the same?

25.20. When illuminating a thin film of transparent material with monochromatic light incident normal to the surface of the film, it shows parallel alternating dark and bright equidistant fringes. Is the thickness of individual areas of the film the same?

25.21. When illuminating a wedge with a very small angle made of glass with refractive index 1.5, with a beam of light of wavelength $\lambda = 650$ nm incident normal to its surface, the latter shows alternating dark and bright fringes. Determine the angle α, if the separation between two neigh-

bouring dark fringes on the surface of the wedge is 12 mm.

25.22. When a quartz wedge with angle $\alpha = 5.0''$ is illuminated with monochromatic light of wavelength $\lambda = 600$ nm incident normal to its surface, an interference pattern is observed. Determine the width of these fringes.

25.23. To measure the thickness of a hair the latter is placed on a glass plate and covered with another plate. The distance from the hair to the line of contact of the plates, which is parallel to the hair, is 20 cm. In illuminating the plates with red light ($\lambda = 750$ nm) an interference pattern is obtained such that it shows 8 fringes per cm. Determine the thickness of the hair.

25.24. Between two glass plates a thin metal wire of 0.085 mm diameter is squeezed. The distance from the wire to the line of contact of the plates forming the air wedge is 25 cm. When the plates are illuminated with monochromatic light with wavelength 700 nm, interference fringes are seen parallel to the line of contact of plates with the wire. Determine the number of fringes per 1 cm of length.

25.25. The interference colour pattern of the same place on the surface of a soap bubble changes continually. Why?

25.26. Referring to Fig. 74, the radius of curvature of the lens lying on the flat plate is 6.4 m, and the beam is parallel to the principal axis of the lens. Determine the diameter of the second bright Newton's fringe observed in reflected light with wavelength 640 nm. What will the radius of the same fringe be, if the lens with the plate are put in water?

25.27. A lens lies on a flat plate. It is illuminated with light of wavelength 520 nm incident normal to the principal axis of the lens. In the interference pattern produced the fourth bright Newton's fringe observed in reflected light is 4.5 mm. Determine the radius of curvature of the lens.

25.28. In observing Newton's fringes in reflected monochromatic light the diameter of the fourth dark Newton's fringe is found to be 14.4 mm. The pattern is obtained using a planar convex lens lying on a flat plate with radius of curvature of 22 m. If the beam is parallel to the principal axis of the lens, determine the wavelength of the light.

25.29. A lens of radius of curvature 18 m is illuminated

with light of wavelength 450 nm incident parallel to the principal axis of the lens. Determine the nature of Newton's fringe (bright or dark) that in reflected light has a radius of 5.3 mm. What is the radius of the same fringe, if the gap between the lens and plate on which the lens lies, is filled with ethyl alcohol?

25.30*. A diffraction grating with diffraction element 0.004 mm is illuminated with light of wavelength 687 nm. At what angle to the grating should the observation be performed, so that the image of the second spectrum could be seen?

25.31. Determine the diffraction element of a grating, if when illuminated with light of wavelength 656 nm the second spectrum is seen at an angle of 15°.

25.32. When illuminating a diffraction grating with light of wavelength 590 nm the third-order spectrum is seen at an angle of 10°12′. Determine the wavelength at which the second-order spectrum is seen at an angle of 6°18′.

25.33. Determine the wavelength for a fringe in the third-order spectrum coinciding with the image of the line in the fourth-order spectrum with wavelength 490 nm.

25.34. What is the maximum order of spectrum obtainable with a diffraction grating having 500 lines per 1 mm, when illuminated with light of wavelength 720 nm?

25.35. Spectra of diffraction grating with 100 lines per 1 mm are projected on a screen disposed parallel to the grating at a distance of 1.8 m from it. Determine the wavelength of monochromatic light incident on the grating, if the distance from the second spectrum to the central bright fringe is 21.4 cm.

25.36. The distance between a screen and diffraction grating with 125 lines per 1 mm is 2.5 m. When the grating is illuminated with light of wavelength 420 nm blue fringes are seen on the screen. Determine the distance from the central fringe to the first fringe on the screen.

25.37. A diffraction grating has 400 lines per 1 mm. It is located at a distance of 25 cm from a screen. The distance on the screen between the third fringes to the left and

---

* In all the problems on diffraction grating it is to be assumed that the light falls normal to the surface of the grating.

to the right relative to the zeroth fringe is found to be 27.4 cm. Determine the wavelength of incident light.

25.38. The illumination of a diffraction grating with light of wavelength $\lambda = 627$ nm produced on the screen fringes such that the separation between the central and the first fringes is 39.6 cm. Knowing that the screen is 120 cm away from the grating, find the diffraction element.

25.39. The polarizing angle refers to the angle of incidence of light on the interface, for which the reflected ray appears completely plane-polarized. Determine the polarizing angle for a ray travelling from air to water.

25.40. The polarizing angle for rays incident on the surface of a liquid is 57°46′. What liquid is it?

SEC. 26. PHOTOMETRY *

Example 76. A 250 cd lamp hangs at a height of 3.0 m from the floor, and a 150 cd lamp at 4.0 m (Fig. 76). The distance between the lamps is 2.5 m. Compare the illuminances on the floor under each of the lamps.

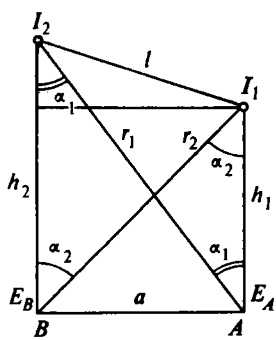


Fig. 76

Given: $I_1 = 250$ cd is the luminous intensity of the first lamp, $I_2 = 150$ cd is the luminous intensity of the second lamp, $h_1 = 3.0$ m is the height of the first lamp, $h_2 = 4.0$ m is the height of the second lamp, $l = 2.5$ m is the separation between the lamps.

Determine: E_A/E_B—the ratio of illuminances due to each lamp.

Solution. The illuminance under each lamp is the sum of illuminances due to the first and the second lamps. Therefore, from the illuminance laws we have (see Fig. 76).

$$E_A = \frac{I_1}{h_1^2} + \frac{I_2}{r_1^2}\cos\alpha_1 = \frac{I_1}{h_1^2} + \frac{I_2 h_2}{r_1^3}$$

$$E_B = \frac{I_2}{h_2^2} + \frac{I_1}{r_2^2}\cos\alpha_2 = \frac{I_2}{h_2^2} + \frac{I_1 h_1}{r_2^3}$$

* In all the problems of this section it is implied that the lamps have no reflectors.

The required quantities r_1 and r_2 may be obtained from the Pythagorean theorem, having found preliminarily the distance a between points A and B from the relation $a = \sqrt{l^2 - (h_2 - h_1)^2}$.

Determine r_1 and r_2:

$$r_1 = \sqrt{h_2^2 + l^2 (h_2 - h_1)^2} = \sqrt{16\ \mathrm{m}^2 + 6.25\ \mathrm{m}^2 - 1\ \mathrm{m}^2}$$

$$= \sqrt{21.25\ \mathrm{m}^2} \approx 4.6\ \mathrm{m}$$

$$r_2 = \sqrt{h_1^2 + l^2 - (h_2 - h_1)^2} = \sqrt{9\ \mathrm{m}^2 + 6.25\ \mathrm{m}^2 - 1\ \mathrm{m}^2}$$

$$= \sqrt{14.25\ \mathrm{m}^2} \approx 3.8\ \mathrm{m}$$

We now find E_A and E_B

$$E_A \approx \frac{250\ \mathrm{cd}}{9\ \mathrm{m}^2} + \frac{150\ \mathrm{cd} \cdot 4.0\ \mathrm{m}}{(4.6)^3\ \mathrm{m}^3} \approx 27.8\ \mathrm{lx} + 6.2\ \mathrm{lx} \approx 34\ \mathrm{lx}$$

$$E_B \approx \frac{150\ \mathrm{cd}}{16\ \mathrm{m}^2} + \frac{250\ \mathrm{cd} \cdot 3.0\ \mathrm{m}}{(3.8)^3 \cdot \mathrm{m}^3} \approx 9.4\ \mathrm{lx} + 13.7\ \mathrm{lx} \approx 23\ \mathrm{lx}$$

Determine the illuminance ratio

$$E_A/E_B \approx 34\ \mathrm{lx}/23\ \mathrm{lx} \approx 1.48$$

Answer. **Under the first lamp the illuminance is higher, than under the second one by about 1.5 times.**

26.1. A central solid angle of 0.75 sr cuts on a sphere an area of 468 cm². Determine the radius of the sphere.

26.2. A central solid angle cuts on a sphere of radius 1.4 m an area of 2 350 cm². What is the area cut out by the angle from the sphere, if the radius of the sphere is increased by 60 cm?

26.3. What is the luminous flux produced by a point light source of 25 cd inside a solid angle of 0.64 sr?

26.4. Determine the luminous intensity of a point source, located at the centre of a sphere of radius 85 cm, if on the sphere of radius 1.50 m² a luminous flux of 360 lm is produced. What is the total luminous flux of the source?

26.5. Determine the luminous flux incident on the surface of a table, if its average illuminance is 9 500 lx (in shooting a film), and the area is 1.6 m².

26.6. On a round opaque glass of 0.45 m diameter a luminous flux of 120 lm is incident. What is the illuminance of the glass?

26.7. Determine the average luminous intensity of a 120 W lamp, if its luminous efficiency is 13 lm/W.

26.8. The average luminous intensity of a 100 W lamp is 80 cd. Determine the luminous efficiency of the lamp.

26.9. The luminous intensity of the surface of melted platinum of area 0.50 cm^2 normal to the surface is 30 cd. Determine the luminance of the surface.

26.10. Determine the luminous intensity of a stearin candle, if the luminance of its flame is $5 \cdot 10^3$ cd/m^2, and the cross-sectional area is 2 cm^2.

26.11. A light source constitutes a uniformly luminous spherical surface. How will the luminance of the source change if we approach it? Move away from it?

26.12. A 150 cd lamp hangs at a height of 150 cm over a horizontal surface of the table. Determine the illuminance of a surface under the lamp. What is the illuminance if the lamp is raised another 0.25 m?

26.13. A small surface is illuminated with a lamp of 90 cd. It is substituted by a lamp of 30 cd. By how many times should the distance from the lamp to the surface be increased for the illuminance of the surface to remain the same?

26.14. Consider a 200 cd point source. At what distance from the source will a luminous flux of 0.025 lm pass through an area of 500 cm^2 on the surface normal to the light rays?

26.15. A book is illuminated by the sun so that a luminous flux of 36 lm is incident normal to the surface of the book. What will the luminous flux be, if it is turned by 30°?

26.16. The illuminance of the Earth's surface for a terrain clearance altitude of the Sun of 45° is 80,000 lx. Determine the illuminance for a terrain clearance altitude of 25°.

26.17. A lamp illuminates a wall of a building 10 m away from the lamp so that the illuminance is 1.2 lx and the angle of incidence is 42°. What is the luminous intensity of the lamp?

26.18. Light from a 150 cd lamp is incident on a desk at an angle of 30°, the illuminance being 25 lx. At what distance from the desk is the lamp?

26.19. A small screen is illuminated by three closely spaced candles, which are 1.2 m away from the screen. One

of the candles is put out. By what distance must the screen be brought nearer to the candles for its illuminance to remain the same?

26.20. A 120 cd lamp hangs 1.5 m above the middle of a table. If the length of the table is 1.5 m, the width is 1.0 m, find the maximum and minimum illuminance of the surface of the table.

26.21. A 300 cd lamp hangs on a post at a height of 3.0 m above the ground. Determine the illuminance of a point on the ground 4.0 m away from the point on the ground that is directly under the lamp.

26.22. A table of 1.2 m diameter is illuminated with a lamp hanging at a height of 1.2 m above the middle of the table. What is the illuminance at the edge of the table, if the total luminous flux of the lamp is 750 lm?

26.23. Two 250 cd lamps hang one above the other at heights 2.5 and 3.5 m from the ground. Find the illuminance of the ground at a distance of 2.5 m from the point on the ground above which the lamps hang.

26.24. In a yard, at the same height of 3.0 m three 200 cd lamps hang. All the lamps are arranged at a distance of 2.5 m from each other. Find the illuminance under each lamp.

26.25. Two lamps of luminous intensity 100 and 50 cd are separated by a distance of 2.4 m. Where should an opaque screen be located between them for both sides of it to be equally illuminated?

26.26. To the left of a photometer a 50 cd lamp is placed at a distance of 20 cm, to the right—a lamp to be tested. The illuminance of both sides of the instrument appears to be the same, when the lamp tested is 60 cm away from the photometer. Determine the luminous intensity of the lamp tested.

26.27. One side of an opaque screen is illuminated with three identical candles arranged close to each other at a distance of 2.4 m from the screen centre. At what distance from the other side must a similar candle be placed for the illuminance at the centre of the screen to be the same on either side?

26.28. A 90 cd lamp is 1.0 m away from a screen. A flat mirror parallel to the screen is placed 1.0 m behind the

lamp. Find the illuminance at the screen centre. How will it change, if the mirror is removed?

26.29. A 400 cd lamp is separated by a distance of 1.0 m from a screen. At what distance behind the lamp should a flat mirror parallel to the screen be placed for the illuminance at the screen centre to increase by 100 lx?

26.30. Figure 26.30 shows a lamp of luminous intensity 72 cd hanging at a height of 120 cm over a table and at a distance of 80 cm from a wall. Below the lamp on the wall there hangs a mirror so that the distance of the mirror from its centre to the lamp is 100 cm. Determine the illuminance on the table under the lamp. How will the illuminance change, if the mirror is removed?

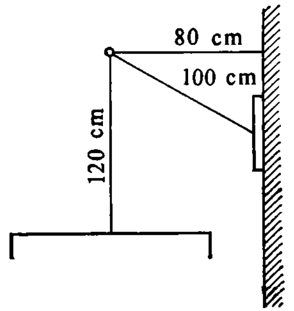


Fig. 26.30

SEC. 27. RADIATION AND SPECTRA

Example 77. The surface of some melted platinum is 1.0 cm^2. Assume the platinum to be an absolutely black body and ignore heat losses. Determine the power required to maintain the temperature of the platinum at 1 773°C. In the radiation spectrum what is the wavelength that corresponds to the maximum energy?

Given: $T = 2\,046$ K is the temperature of the platinum, $S = 1.0 \cdot 10^{-4}$ m^2 is the surface area of the platinum, $b = 0.0029$ m·K is Wien's constant, $\sigma = 5.67 \cdot 10^{-8}$ W/($m^2 \cdot K^4$) is the Stefan-Boltzmann constant.

Determine: P—the power required to maintain the platinum temperature constant;
λ'—the wavelength corresponding to the most energetic radiation.

Solution. The required power P can be obtained from the relation $E = Pt$, where E is the energy of radiation from the surface of platinum during the time t, i.e. $E = ISt$.

The total intensity of the radiation of platinum is determined by the Stefan-Boltzmann law

$$I = \sigma T^4$$

As $P = E/t$, we have

$$P = ISt/t = IS$$

Substituting I by its value obtained from the Stefan-Boltzmann law we get

$$P = \sigma T^4 S = 5.67 \cdot 10^{-8}\ \text{W}/(\text{m}^2 \cdot \text{K}^4) \cdot (2\,046\ \text{K})^4 \cdot 10^{-4}\ \text{m}^2 = 99.4\ \text{W}$$

The wavelength λ' corresponding to the energy maximum in the spectrum can be found from the Wien law

$$\lambda' T = b$$

$$\lambda' = \frac{b}{T} = \frac{0.0029 \cdot 10^9\ \text{nm} \cdot \text{K}}{2\,046\ \text{K}} = 1\,400\ \text{nm}$$

Answer. To keep the melted platinum at constant temperature a power of about 100 W is required; the maximum energy in platinum spectrum will correspond to wavelength 1.4 μm.

27.1. It has been found experimentally that the refractive index of water for red boundary light in the visible spectrum is 1.329, and for violet boundary light 1.344. Determine the velocity of red and violet light in water. By how many times is the velocity of red light in water higher than that of violet light?

27.2. The velocity in glass (light crown) of red boundary light in the visible spectrum is $199 \cdot 10^3$ km/s, and of the violet boundary light $196 \cdot 10^3$ km/s. Determine the refractive indices of red and violet rays.

27.3. Dispersion of a medium may be evaluated by the variation of refractive index within a chosen frequency interval (wavelength interval). Using the data of the preceding problems establish which of the two media, water or glass, has the higher dispersion. Compare the visible spectra in refraction in water and glass under the same conditions.

27.4. Figure 27.4 shows the variation of refractive index with wavelength of light incident on glass. Are the dispersions of red and blue light in glass equal? Where does the refractive index vary faster with wavelength? How does it tell on the spectrum obtained using a glass prism?

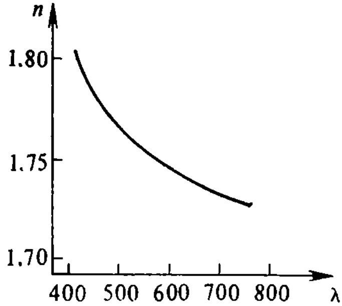


Fig. 27.4

27.5. Prism spectrum is more often used to study the short-wave range of the spectrum, whereas for the long-wave radiation the diffraction spectrum is more suitable. Why?

27.6. A glass prism is not suitable for production of infra-red and ultra-violet spectra. Why? What prisms are necessary for the two cases?

27.7. Why is the medical lamp with much ultra-violet radiation called "mountain sun"?

27.8. Why is the colour of certain materials under daylight and electric light different?

27.9. Which lamps are to be recommended to light a department of a store that sells fabrics?

27.10. In green-houses the temperature is markedly higher than that of ambient air, even without heating and fertilizers. Explain.

27.11. To perform a spectral analysis a material to be studied is placed in the flame of a burner or in an electric arc. Why?

27.12. What can be said of the composition of an alloy from the brightness of spectral lines in its spectrum?

27.13. Why do X-ray tubes use high voltage of the order of tens and hundreds of kilovolts?

27.14. The continuous X-ray spectrum produced by the tube exhibits a sharp boundary at the short-wave end. Account for it.

27.15. Are any electromagnetic waves radiated by the chair on which you are sitting? The book you are reading?

27.16. When does a kettle produce more radiation: when it contains boiling water, or water at room temperature?

27.17. In a room there are two identical aluminium kettles in which there are equal quantities of water at 90°C. One of them becomes thick with soot whereas the other is clean. Which of the kettles will cool faster? Why?

27.18. On a light-coloured piece of earthen ware a dark design is applied. If the job is placed into a high-temperature furnace, a light design is seen against a dark background. Why?

27.19. If a room is not heated, the temperature of all the bodies in it becomes the same. Account for it.

27.20. By how many times is the energy of radiation of an absolutely black body higher at 100°C than at 0°C?

27.21. In a furnace with the open door a temperature of 800°C is maintained. The door has dimensions 22 $\times$ $\times$ 15 (cm^2). How much energy per second does the room obtain from the furnace through the open door?

27.22. Assuming the filament temperature of a lamp to be 2 000°C, determine the wavelength corresponding to the maximum energy of its radiation. To what part of the spectrum does the wavelength belong?

27.23. Given that in the solar radiation spectrum the maximum corresponds to a wavelength of 550 nm, determine the temperature of the Sun's surface.

27.24. Determine the wavelength corresponding to the maximum energy in the spectrum of a star with temperature 30,000 K. What is the intensity of the radiation of the star?

SEC. 28. PHENOMENA ARISING FROM QUANTUM NATURE OF LIGHT

Example 78. The surface of an absolutely black body of area 2.4 cm^2 is exposed to X-ray radiation with wavelength 1.5 nm. How many photons per second are required to create a pressure on the surface equal to that exerted on an absolutely black surface on the Earth's orbit? What will the result be, if solar radiation strikes a mirror surface that reflects it completely?

Given: $J_S = 1\ 370\ W/m^2$ is the solar constant, $\lambda = 1.5 \times \times 10^{-9}$ m is the wavelength of X-ray radiation, $\Delta t = 1.0$ s is the time during which the photons are incident on the surface, $S = 2.4 \cdot 10^{-4}\ m^2$ is the area of the surface,

$c = 3 \cdot 10^8$ m/s is the velocity of photons, $h = 6.62 \cdot 10^{-34}$ J·s is Planck's constant.

Determine: N_1—the number of X-ray photons absorbed by the surface per second;
N_2—the number of photons reflected by the surface in the second case.

Solution. As the pressures exerted by X-ray and solar radiations are to be equal, we can write

$$J_S/c = J_X/c$$

because the pressure of light is given by the relation $p = J/c$, where J is the intensity of the radiation given by the relationship

$$J = \frac{E}{S\,\Delta t}$$

Knowing the solar constant J_S, the last equation gives the energy of radiation, E, and then we find the number of photons N, as $E = \varepsilon N$, where ε is the energy of one X-ray photon to be obtained from Planck's formula $\varepsilon = hc/\lambda$.

The intensities of radiations being equal, we write

$$J_S = \frac{E}{S\,\Delta t} \quad \text{or} \quad J_S = \frac{\varepsilon N_1}{S\,\Delta t}$$

Substituting ε from Planck's formula gives

$$J_S = \frac{hcN_1}{\lambda S\,\Delta t} \qquad N_1 = \frac{J_S \lambda S\,\Delta t}{hc}$$

$$N_1 = \frac{1\,370\ \text{W/m}^2 \cdot 1.5 \cdot 10^{-9}\ \text{m} \cdot 2.4 \cdot 10^{-4}\ \text{m}^2 \cdot 1.0\ \text{s}}{6.62 \cdot 10^{-34}\ (\text{J} \cdot \text{s}) \cdot 3 \cdot 10^8\ \text{m/s}} = 2.5 \cdot 10^{15}$$

As the pressure on the mirror surface is twice that on the black one, then in the second case the number of photons should be twice as large as in the first case. This is accounted for by the fact that in reflecting the photons the surface obtains additional momentum.

The light pressure on the mirror surface is given by $p = 2J/c$; therefore

$$N_2 = 2N_1 = 2 \cdot 2.5 \cdot 10^{15} = 5.0 \cdot 10^{15}$$

Answer. In the first case the surface receives $2.5 \cdot 10^{15}$ X-ray photons per second, and in the second case $5.0 \cdot 10^{15}$.

Example 79. Onto the surface of caesium ultra-violet radiation of wavelength 75 nm is incident. Determine the wavelength corresponding to the electrons ejected from the caesium with maximum speed, if the work function for caesium is 1.97 eV.

Given: $\lambda_{u.v.} = 75\cdot10^{-9}$ m is the wavelength of incident radiation, $W = 1.97$ eV $= 3.15\cdot10^{-19}$ J is the work function, $h = 6.62\cdot10^{-34}$ J·s is Planck's constant, $m_e = 9.11\cdot10^{-31}$ kg is the electron mass.

Determine: λ_e—the wavelength corresponding to electrons ejected with maximum speed.

Solution. According to de Broglie's theory, to each body travelling at a speed v there corresponds a wave whose wavelength is determined by the relation $\lambda = h/mv$. The required velocity v may be found using Einstein's photoelectric equation

$$hc/\lambda_{u.v.} = W + \frac{1}{2} m_e v_{max}^2$$

From this relation we obtain v_{max} considering that $\lambda_{u.v.}$ is the wavelength of radiation incident on the metal, and m_e is the mass of electron:

$$v_{max} = \sqrt{\frac{2}{m}\left(\frac{hc}{\lambda_{u.v.}} - W\right)}$$

$$= \sqrt{\frac{2}{9.11\cdot10^{-31}\ \text{kg}}\left(\frac{6.62\cdot10^{-34}\cdot3\cdot10^{8}}{75\cdot10^{-9}} - 3.15\cdot10^{-19}\right)\ \text{J}}$$

$$= 2.26\cdot10^{6}\ \text{m/s}$$

We now calculate the wavelength corresponding to the electrons with maximum speed

$$\lambda_e = \frac{h}{mv_{max}} = \frac{6.62\cdot10^{-34}\ \text{J·s}}{9.11\cdot10^{-31}\ \text{kg}\cdot2.26\cdot10^{6}\ \text{m/s}} = 3.2\cdot10^{-10}\ \text{m}$$

Answer. The wavelength corresponding to electrons of maximum speed is 0.32 nm.

28.1. Assuming the Earth to be an absolutely black body, calculate the pressure exerted by solar radiation on the Earth. The Earth's radius is taken to be 6 400 km.

28.2. Determine the increase that would occur in the mass of the Earth due to the total absorption of the solar radiation. Why is there actually no such an increase?

28.3. Is the magnitude of the solar constant J_S dependent on the distance to the Sun, r? If so, how?

28.4. Given that the distance from the Mars to the Sun is 1.52 times larger than that from the Earth, determine the solar constant on the Mars.

28.5. Determine the solar constant on the Venus, if the Venus-Sun distance is $108 \cdot 10^6$ km, and the Earth-Sun distance is $150 \cdot 10^6$ km.

28.6. A meteorite of diameter 1.2 mm is on the Earth's orbit. By how many times is the force of its attraction to the Sun larger than the force of light pressure, if the density of meteorite material is $7.0 \cdot 10^3$ kg/m^3? Assume that the meteorite completely absorbs the incident radiation. How will the answer vary as the diameter of the meteorite decreases?

28.7. A light cross-head with four lobes rotates on a vertical axis inside a glass evacuated bulb (Fig. 28.7). One surface of each lobe is mirror, the other black. If the bulb is exposed to light, the cross-head begins to rotate so that the mirror surface will move against the light, and the black one in the direction of light. Can this experiment be accounted for by the pressure of light?

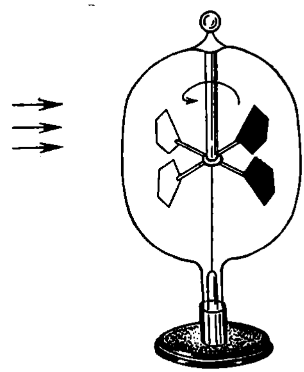


Fig. 28.7

28.8. What must the light energy input be to each square millimetre of the absolutely black surface per second, for the pressure of light on it to be 1 Pa? If the pressure is produced by green light of wavelength 550 nm, what is the number of quanta required to fall every second on 1 mm^2 of the surface?

28.9. Each square centimetre of an absolutely black surface receives every second $2.8 \cdot 10^{17}$ quanta of 400 nm radiation. What is the pressure exerted by the radiation?

28.10. What is the role played in nature by photosynthesis?

28.11. What radiation should be selected to produce a chemical effect? Thermal effect?

28.12. Compare the external and internal photoelectric effects.

28.13. There are separate electrically neutral plates of metal and semiconductor. In illuminating the metal the external photoelectric effect occurs, in illuminating the semiconductor, the internal photoelectric effect. Will the plates remain neutral? If no, what will the sign of the charge be?

28.14. The wavelength corresponding to the photoelectric threshold for sodium is 530 nm. Determine the work function for sodium.

28.15. The work function for silver is $7.85 \cdot 10^{-19}$ J. Find the photoelectric threshold for silver.

28.16. The work function for gold is 4.59 eV. What is the photoelectric threshold for gold?

28.17. The stopping potential for aluminium is 4.25 V. Determine the photoelectric threshold for aluminium.

28.18. The stopping potential for magnesium is 3.69 V, for caesium 1.93 V. The materials are exposed to 590 nm light. Is the photoelectric effect produced here? In both metals?

28.19. The work function for mercury is 4.53 eV. Will there be any photoelectric effect, if the mercury surface is exposed to visible light?

28.20. If the work function for potassium is 2.26 eV, determine the maximum kinetic energy of electrons ejected from the surface of it when exposed to 345 nm light.

28.21. The maximum kinetic energy of electrons liberated by rubidium under ultra-violet light with $\lambda = 317$ nm, is $2.84 \cdot 10^{-19}$ J. Determine the work function and photoelectric threshold for rubidium.

28.22. Radiation with wavelength 220 nm is incident on the surface of tungsten. Determine the maximum velocity of ejected electrons, if the stopping potential for tungsten is 4.56 V.

28.23. Strontium, when exposed to some radiation, releases electrons with maximum kinetic energy $1.8 \cdot 10^{-19}$ J. If the photoelectric threshold for strontium is 550 nm, what is the wavelength required here?

28.24. The work function for cadmium is 4.08 eV. What must the wavelength of radiation incident on cadmium be

for the maximum velocity of photoelectrons to be $7.2 \cdot 10^5$ m/s?

28.25. In the iconoscope an array of tiny photocells based on the external photoelectric effect are used. Explain why? How are they discharged?

28.26. Solar cells use silicon, and not some other semiconducting material. Why?

28.27. Explain how in the theatre the set-scene is made to glow when required imperceptibly for the audience.

28.28. A proton travels at a spead of $4.6 \cdot 10^4$ m/s. Find the wavelength corresponding to the proton.

28.29. A wavelength of 0.18 nm corresponds to a travelling electron. What are the velocity and momentum of the electron?

SEC. 29. FUNDAMENTALS OF SPECIAL RELATIVITY THEORY

Example 80. A spaceship travels as related to a stationary observer with velocity $v = 0.99c$ (c is the velocity of light in a vacuum). What time has elapsed by the observer's clock, if by the clock on board the ship one year has passed? For the stationary observer, how will the linear dimensions of bodies on the ship change (in the direction of motion)? For the same observer, how will the density of matter of the ship change?

Given: $v = 0.99c$ is the velocity of the spaceship in relation to a stationary reference system, $t_0 = 1.00$ year is the time elapsed by the clock on the ship (proper time).

Determine: t—the time elapsed by the clock of the stationary observer;
l—linear dimensions of bodies in the spaceship (in the direction of motion) for the stationary observer;
ρ—the density of matter of the ship for the observer.

Solution. The time by the observer's clock is given by

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}} = \frac{1.00 \text{ year}}{\sqrt{1 - (0.99c/c)^2}} \approx 7.1 \text{ years}$$

The dimensions of bodies (in the direction of motion) are obtainable from the relation

$$l = l_0 \sqrt{1 - v^2/c^2}$$

where l_0 is the proper length of the bodies. Then

$$l = l_0 \sqrt{1 - (0.99c/c)^2} \approx 0.14 l_0$$

The density of matter of the spaceship for the observer will be

$$\rho = m/V \quad \text{where} \quad m = m_0/\sqrt{1 - v^2/c^2} \quad \text{and} \quad V = lS$$

As the transverse (in reference to the direction of motion) dimensions of bodies do not change, then

$$V = l_0 S \sqrt{1 - v^2/c^2}$$

$$\rho = \frac{m_0}{\sqrt{1 - v^2/c^2}\, l_0 S \sqrt{1 - v^2/c^2}} = \frac{m_0}{V_0 (1 - v^2/c^2)} = \frac{\rho_0}{1 - v^2/c^2}$$

We now calculate the density of matter of the spaceship for the stationary observer

$$\rho = \frac{\rho_0}{1 - (0.99c/c)^2} = \frac{\rho_0}{0.0199} \approx 50.2 \rho_0$$

Answer. It will take about 7.1 years by the observer's clock; the transverse dimensions of bodies in the direction of motion will contract and account for about $0.14 l_0$; for the stationary observer the density of matter of the spaceship increases by about 50 times.

Example 81. Two spaceships move towards each other with velocities $v_1 = v_2 = \frac{3}{4} c$ with respect to a stationary observer. Determine the approach velocity of the spaceships using the classical and relativistic formulas of composition of velocities.

Given:

$v_1 = \frac{3}{4} c$—the velocity of the first ship | with respect to the stationary observer
$v_2 = \frac{3}{4} c$—the velocity of the second ship | with respect to the stationary observer

Determine: u—the velocity of the spaceships relative to each other.

Solution. We find the relative velocity of the spaceships:

(1) Using the classical formula of composition of velocities

$$u_{cl} = v_1 + v_2 = \frac{3}{4}c + \frac{3}{4}c = 1.5c$$

(2) Using the relativistic formula

$$u_{rel} = \frac{v_1 + v_2}{1 + v_1 v_2/c^2} = \frac{{}^3/_4 c + {}^3/_4 c}{1 + ({}^3/_4 c \cdot {}^3/_4 c)c^2} = 0.96c$$

Answer. By the classical formula the approach velocity of the spaceships is $1.5c$, by relativistic $0.96c$.

Example 82. An electron travels with a velocity $v = 0.80c$. The rest mass of the electron is $m_0 \approx 9.1 \cdot 10^{-31}$ kg. Determine: the rest energy of the electron (in terms of joules and electron-volts); the mass of the electron; the total energy of the electron; the kinetic energy of the electron.

Given: $m_0 = 9.1 \cdot 10^{-31}$ kg is the rest mass of the electron, $v = 0.80c$ is the velocity of the electron.

Determine: E_0—the rest energy of the electron;
m_e—the mass of the electron;
E—the total energy of the electron;
E_k—the kinetic energy of the electron.

Solution. The rest energy of the electron is given by

$$E_0 = m_0 c^2 \approx 9.1 \cdot 10^{-31}\ \text{kg} \cdot 9.00 \cdot 10^{16}\ \text{m}^2/\text{s}^2 \approx 8.2 \cdot 10^{-14}\ \text{J}$$

Now we express the electron energy in terms of electron-volts using the relation $1\ \text{eV} = 1.6 \cdot 10^{-19}$ J

$$E_0 = \frac{8.2 \cdot 10^{-14}\ \text{J}}{1.6 \cdot 10^{-19}\ \text{J/eV}} \approx 5.12 \cdot 10^5\ \text{eV} \approx 0.51\ \text{MeV}$$

The mass of the moving electron is found from the relationship

$$m_e = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{9.1 \cdot 10^{-31}\ \text{kg}}{\sqrt{1 - (0.80c/c)^2}} = \frac{9.1 \cdot 10^{-31}\ \text{kg}}{0.60} \approx 1.52 \cdot 10^{-30}\ \text{kg}$$

The total energy is

$$E = mc^2 \approx 1.52 \cdot 10^{-30}\ \text{kg} \cdot 9.00 \cdot 10^{16}\ \text{m}^2/\text{s}^2$$
$$\approx 13.65 \cdot 10^{-14}\ \text{J} \approx 13.7 \cdot 10^{-14}\ \text{J}$$

Using the relation $E = E_0 + E_k$ we obtain the kinetic energy of the electron

$$E_k = E - E_0 \approx 13.65 \cdot 10^{-14}\ \text{J} - 8.2 \cdot 10^{-14}\ \text{J} \approx 5.5 \cdot 10^{-14}\ \text{J}$$

Answer. The rest energy of the electron is about $8.2 \cdot 10^{-14}$ J, or 0.51 MeV; the mass of the moving electron in this reference system is about $1.52 \cdot 10^{-30}$ kg; the total energy of the electron in this reference system is about $13.7 \cdot 10^{-14}$ J; the kinetic energy of the electron is about $5.5 \cdot 10^{-14}$ J.

29.1. The location of a particle at rest in an inertial frame of reference K is determined by the coordinates $x = 200$ m, $y = 5.0$ m, $z = 15$ m. Frame K' travels uniformly and rectilinearly with respect to frame K in x-direction at a speed $v_x = 20$ m/s. Axis X' coincides with axis X, and axes Y' and Z' are parallel to appropriate axes of frame K. At time $t = 0$ the origins of the frames, O and O', coincide. Determine the coordinates of the material point in the K' frame at $t_1 = 0$, $t_2 = 5.0$ s, $t_3 = 15$ s.

29.2. Frames K and K' are as in problem 29.1. Write Galilean transformations for $x = 2.0 \cdot 10^3$ m, $y = 0$, $z = 0$, $v_x = 100$ m/s. Find x' at $t_1 = 0$, $t_2 = 10$ s, $t_3 = 20$ s, $t_4 = 30$ s.

29.3. Inertial frames K and K' are as in problem 29.1. Frame K' travels at a speed $v_x = 10$ m/s in x-direction. In frame K a particle is at rest. Determine the coordinates of the particle in the K frame, if its position in the K' frame in 20 s from zero time reference is $x'_1 = 100$ m, $y'_1 = 0$, $z'_1 = 0$; $x'_2 = -200$ m, $y'_2 = 0$, $z'_2 = 0$; $x'_3 = -300$ m, $y'_3 = 20$ m, $z'_3 = 15$ m.

29.4. The position of a particle at rest in the K frame is given by the coordinates $x = 400$ m, $y = 25$ m, $z = 12$ m. At $t = 0$ the origins of inertial frames K and K' coincide. At what speed is inertial frame K' travelling, if at $t = 50$ s the position of the particle with reference to the K' frame is determined by the coordinates $x' = 150$ m, $y' = 25$ m, $z' = 12$ m?

29.5. Write the formula of composition of velocities in classical mechanics. Using the formula determine the approach speed of two automobiles whose velocities with respect to the Earth are $v_1 = 30$ m/s and $v_2 = 20$ m/s.

What is their relative velocity when they travel in the same direction with the above velocities?

29.6. A boat moves normal to the current direction at 4.0 m/s. If the current velocity is 3.0 m/s, what is the velocity of the boat in relation to the bank?

29.7. A rowing boat travels at a speed $v_1 = 4.0$ m/s so that the direction of its travel makes an angle $\alpha = 60°$ with the direction of the current. Determine the velocity of the rowing boat with reference to the bank, if the current velocity is $v_2 = 3.0$ m/s?

29.8. Are the velocity of a body and the velocity of light dependent on the velocity of a reference frame?

29.9. An inertial frame K' and inertial frame K, taken to be stationary, are oriented as in problem 29.1. Using the Lorentz transformations, determine the coordinates of an event in frame K', if in the K frame the coordinates of the event are (1) $x = 0$, $y = 0$, $z = 0$; (2) $x = 3.0 \cdot 10^8$ m, $y = 0$, $z = 0$, $t = 1$ s, $v_x = 0.8c$ (v_x is the velocity of frame K' in relation to frame K); (3) $x = 3.0 \cdot 10^8$ m, $y = 0$, $z = 0$, $t = 5$ s, $v_x = 0.8c$.

29.10. A particle is at rest in a frame K. An inertial frame K' travels relative to frame K at a velocity $v_x = 0.8c$ in x-direction so that axes X and X' coincide, and axes Y and Y', Z and Z' are parallel. At $t = 0$ the origins O and O' coincide. Using the Lorentz transformations, determine the coordinates of the particle in the K frame, if its coordinates in the K' frame are as follows: $x' = 6.0 \cdot 10^3$ m, $y' = 2.0 \cdot 10^2$ m, $z' = 15$ m, $t' = 2.0 \cdot 10^{-5}$ s.

29.11. A stationary observer I located midway between points A and B in Fig. 29.11 sees two lightnings hitting

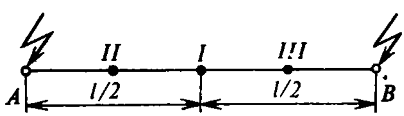


Fig. 29.11

simultaneously these points. Are these events simultaneous for two stationary observers II and III? Besides I, for what other observers, stationary as related to A and B, will the events at A and B be simultaneous?

29.12. A train in Fig. 29.12 travels rectilinearly and uniformly so that when the middle of the train moves past an observer *II* standing in the middle of a platform, lights *A* and *B* are switched on at the ends of the platform

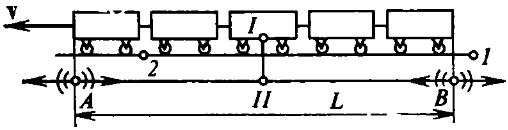


Fig. 29.12

simultaneously for the observer. Are these events simultaneous for an observer *I* standing in the middle of the travelling train?

29.13. In problem 29.12, what light will be switched on first, for observer *I*, if the lights were switched on at the instant when the observer was at point *1* (see Fig. 29.12); at point *B*; at point *2*. Where should observer *I* be at the instant when the lights are switched on for the light from points *A* and *B* to reach him simultaneously?

29.14. In your opinion, are events simultaneous in one inertial frame also simultaneous in all the other inertial frames?

29.15. What is the proper length of a rod? Is the length of a rod the same in various inertial frames of reference? Does it make sense to speak of the "length of a rod" without reference to a frame?

29.16. The proper length of a rod is 1.0 m. Find its length for an observer in respect of which the rod travels at a velocity $v = 0.60c$ in the direction along the rod's axis.

29.17. By what fraction of the proper length does the rod length change for a stationary observer relative to which the rod travels at a velocity of $\frac{4}{5}c$ directed along the rod's axis?

29.18. Find the velocity at which the relativistic contraction of the length of a travelling body accounts for 1%; 25%.

29.19. What would the length of body be in the direction of motion past a stationary observer at $v = c$?

29.20. Two spaceships are travelling uniformly along

parallel rectilinear trajectories in one direction, their velocity relative to the Earth being $v = 0.6c$. In the first ship two events occur in succession in $\tau_0 = 8$ h. What time has elapsed between these two events by the clock of an observer on the second ship? By the clock of an observer on the Earth?

29.21. Cosmic rays produce μ-mesons in the upper strata of the atmosphere. At a velocity of $0.995c$ they cover a distance of $6.0 \cdot 10^3$ m before decay. Determine: τ—the lifetime of a μ-meson for an observer on the Earth; τ_0—the proper lifetime of μ-meson; l_0—the proper length of the path travelled by the μ-meson before decay.

29.22. Consider a spaceship travelling uniformly at a velocity v (v = 3 000 km/s, 100,000 km/s, and 250,000 km/s) relative to an inertial frame. What time will elapse by the clock in the spaceship, if the clock associated with the inertial frame measures one hour?

29.23. What time will elapse on the Earth, if 10 years pass on a spaceship travelling at $v = 0.99c$ relative to the Earth?

29.24. Astronauts travel to a star and back in a spaceship flying at $0.99c$. The distance (for an observer on the Earth) to the star is 40 light years. How long will the trip take for the observer and the astronauts?

29.25. A μ-meson born in the upper strata of the atmosphere travels before decay through a distance of 5.00 km. If the proper lifetime of the particle is $2.21 \cdot 10^{-6}$ s, determine its velocity.

29.26. The proper lifetime of a μ-meson is $2.21 \cdot 10^{-6}$ s. Determine, if μ-mesons observed at the surface of the Earth come from space or are generated in the Earth's atmosphere. The velocity of the μ-meson in relation to the Earth is taken to be $0.99c$.

29.27. Two bodies are approaching each other at velocities $v_1 = v_2 = 2.0 \cdot 10^5$ km/s with respect to a stationary observer. Compare their approach velocities as calculated by the classical and relativistic velocity-composition relations.

29.28. Two electrons move along the same straight line with velocities $0.9c$ and $0.8c$ with respect to a stationary observer. What is the approach velocity of the electrons if they travel in the same direction? In the opposite directions?

29.29. An aircraft travels at a velocity v in the direction of a stationary light source. What is the approach velocity u of the aircraft and photons?

29.30. What is the approach velocity of two photons, each of which travels at the velocity c in reference to a stationary observer? What result will be obtained using the classical velocity-composition relation?

29.31. Is it possible for an electron to move faster than the velocity of light in a given medium?

29.32. Will the acceleration of a body under a constant force be permanent at velocities close to c?

29.33. Taking into account the formula relating mass to velocity, determine the velocity range within which the mass of a body may be regarded as constant.

29.34. A particle travels at a velocity $v = 0.75c$ for a stationary observer. Find the ratio of the mass of the particle to its rest mass.

29.35. A body of rest mass 1.00 kg travels at $2.00 \cdot 10^5$ km/s. Determine the mass of the body for a stationary observer.

29.36. If for a stationary observer the mass of a body is 4.0 kg and its rest mass is 2.4 kg, what is the velocity of the body?

29.37. A body travels with a velocity v with respect to a stationary observer. For the observer, what is the change in dimensions of the body in the direction of motion? Of its mass? Of the density of matter? Will these quantities change for an observer travelling with the body?

29.38. What is the momentum of an electron travelling with a velocity $v = 0.8c$, if the rest mass of electron is $m_0 \approx 9.1 \cdot 10^{-31}$ kg?

29.39. Calculate the momentum of a proton whose mass is equal to the rest mass of an alpha-particle. Through what accelerating potential difference would a proton pass to obtain the momentum?

29.40. Two bodies of rest mass m_0 each approach each other with equal velocities with respect to a stationary observer. Find the momentum of each of the bodies in a frame stationary relative to one of them.

29.41. Determine the rest energy of an electron and proton. Express it in terms of joules and electron-volts.

29.42. A body accelerated from rest so that it mass increases by $2m_0$. What is the kinetic energy of the body? Its total energy? Its momentum?

29.43. At what velocity is the kinetic energy of a particle equal to its rest energy?

29.44. The kinetic energy of an unstable particle is 35 MeV. If the rest mass of the particle is 0.15 amu (see Appendix I), by how many times will its half-life increase?

29.45. An accelerator accelerates protons to a kinetic energy of $70 \cdot 10^9$ eV. What is the velocity of the protons? By how many times has their mass increased?

29.46. The mass of a moving electron is 11 times its rest mass. Determine the kinetic energy of the electron and its momentum.

29.47. Through what accelerating potential difference should an electron travel from rest for its kinetic energy to become 10 times its rest energy?

29.48. Through what accelerating potential difference must a proton travel from rest for its total energy to become 11 times its rest energy? By how many times will its mass increase?

29.49. A proton and alpha-particle are accelerated from rest through the same potential difference U, with the result that the mass of the proton becomes one third of the alpha-particle mass. Calculate the potential difference.

29.50. Compute the momentum of a $5.0 \cdot 10^{14}$ Hz photon. What is the mass of the photon?

29.51. Find the momentum of a photon of wavelength 600 nm and its mass.

29.52. Find the wavelength and frequency of radiation consisting of photons with momentum $1.65 \cdot 10^{-23}$ kg·m/s.

29.53. What energy would be liberated in complete transformation of 1.00 g of substance in field matter?

29.54. To what change in mass corresponds the energy produced in 1 hour by a $2.5 \cdot 10^3$ MW power station?

29.55. At the boundary of the Earth's atmosphere the solar energy input per square metre normal to solar rays is $1.37 \cdot 10^3$ J. Determine: the energy radiated by the Sun per second; the mass lost by the Sun per second. The distance R from the Sun to the Earth is taken to be $1.5 \cdot 10^{11}$ m.

CHAPTER V
ATOMIC NUCLEUS

SEC. 30. ATOMIC STRUCTURE

Example 83. Using Bohr's theory, determine the radius of the hydrogen atom when the electron is in the first Bohr orbit, and the velocity of the electron is in this orbit.

Given: $e_- = -1.6 \cdot 10^{-19}$ C is the electron charge, $e_+ = 1.6 \cdot 10^{-19}$ C is the proton charge, $m_e = 9.11 \cdot 10^{-31}$ kg is the electron mass, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the electric constant, $h = 6.62 \cdot 10^{-34}$ J·s is Planck's constant.

Determine: r_1—the radius of the hydrogen atom;
v_1—the velocity of the electron in the first orbit

Selution. The nucleus of the hydrogen atom (proton) and the electron moving about it interact so that the centripetal force is provided by the Coulomb force $F_{el} = e^2/4\pi\varepsilon_0 r^2$, where e is the elementary electric charge. This force makes the electron revolve in the orbit with radius r (Fig. 83), i.e.

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}$$

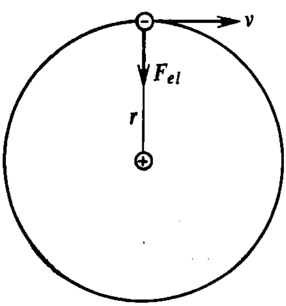


Fig. 83

This is the equation in two unknowns, r and v, where v is the velocity of electron in the orbit. To solve the problem one more equation in the same unknowns is required. It is provided by a Bohr postulate. According to the postulate, for an electron only those orbits are allowed for which the momentum of the electron $m_e v_k r_k$ is equal to an integral multiple of $h/2\pi$ (Bohr's

quantization of orbits), i.e.

$$m_e v_k r_k = kh/2\pi$$

where k is an integer.

For the first orbit $k = 1$. Therefore,

$$m_e v_1 r_1 = h/2\pi$$

From the equation we obtain v_1 and substitute its value into the first equation, and then calculate the radius of the orbit, r_1, and the velocity v_1:

$$v_1 = \frac{h}{2\pi m_e r_1}$$

$$\frac{e^2}{4\pi\varepsilon_0 r_1^2} = \frac{m_e}{r_1} \frac{h^2}{4\pi^2 m_e^2 r_1^2} \qquad \frac{e^2}{\varepsilon_0} = \frac{h^2}{\pi m_e r_1}$$

$$r_1 = \frac{h^2\varepsilon_0}{\pi m_e e^2} = \frac{6.62^2 \cdot 10^{-68}\ \mathrm{J^2 \cdot s^2} \cdot 8.85 \cdot 10^{-12}\ \mathrm{F/m}}{3.14 \cdot 9.11 \cdot 10^{-31}\ \mathrm{kg} \cdot 1.6^2 \cdot 10^{-38}\ \mathrm{C^2}} = 5.3 \cdot 10^{-11}\ \mathrm{m}$$

$$v_1 = \frac{6.62 \cdot 10^{-34}\ \mathrm{J \cdot s}}{6.28 \cdot 9.11 \cdot 10^{-31}\ \mathrm{kg} \cdot 5.3 \cdot 10^{-11}\ \mathrm{m}} = 2.0 \cdot 10^6\ \mathrm{m/s}$$

Answer. The radius of the hydrogen atom is 0.053 nm; the velocity of the electron in the orbit is 2 000 km/s.

Example 84. The electron moves in the first orbit in the hydrogen atom, the radius of the orbit being 0.053 nm. Determine the energy of the electron. What energy should be added to the hydrogen atom for the electron to come over to the next allowed orbit?

Given: $e_- = -1.6 \cdot 10^{-19}$ C is the electron charge, $e_+ = 1.6 \cdot 10^{-19}$ C is the proton charge, $r_1 = 5.3 \cdot 10^{-11}$ m is the radius of the first orbit, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the electric constant.

Determine: E—the energy of the electron in the first orbit of the hydrogen atom;

$E_2 - E_1$—the change in the energy of the atom in transition of the electron from the first orbit to the second one.

Solution. The energy of the electron in an orbit, E_k, consists of the potential energy $E_{p,k}$ and kinetic energy $E_{k,k}$, where k is the orbit serial number

$$E_k = E_{p,k} + E_{k,k}$$

The potential energy is determined by the relation $E_{p,\,k} = \varphi_k e_-$. As $\varphi_k = e_+/(4\pi\varepsilon_0 r_k)$, and e_+ and e_- are equal in magnitude but opposite in sign, we have

$$E_{p,\,k} = \frac{e_+e_-}{4\pi\varepsilon_0 r_k} = -\frac{e^2}{4\pi\varepsilon_0 r_k}$$

By virtue of the Bohr postulates the radius r_k is given by (see the last example)

$$r_k = \frac{h^2k^2\varepsilon_0}{\pi m_e e^2}$$

The kinetic energy of the electron is

$$E_{\kappa,\,k} = \frac{m_e v_k^2}{2} \quad \text{where} \quad v_k = \frac{hk}{2\pi m_e r_k}$$

Now we proceed to calculate the energy of the electron in the first orbit. Since

$$E_k = -\frac{e^2}{4\pi\varepsilon_0 r_k} + \frac{m_e h^2 k^2}{8\pi^2 m_e^2 r_k^2} = -\frac{e^2}{4\pi\varepsilon_0 r_k} + \frac{m_e h^2 k^2 \pi e^2}{8\pi^2 m_e r_k h^2 k^2 \varepsilon_0} = -\frac{e^2}{8\pi\varepsilon_0 r_k}$$

and $r_k = r_1 k^2$, we obtain

$$E_k = -\frac{e^2}{8\pi\varepsilon_0 r_1 k^2}$$

From the condition $k = 1$ we get

$$E_1 = -\frac{e^2}{8\pi\varepsilon_0 r_1} = \frac{1.6^2\cdot 10^{-38}\ \mathrm{C}^2}{8\cdot 3.14\cdot 8.85\cdot 10^{-12}\ \mathrm{F/m}\cdot 5.3\cdot 10^{-11}\ \mathrm{m}} = -2.17\cdot 10^{-18}\ \mathrm{J}$$

Next we find E_2. We obtain for $k = 2$

$$E_2 = -\frac{e^2}{8\pi\varepsilon_0 r_1\cdot 4} = \frac{1}{4}E_1$$

Then the change in the energy of the atom is $E_2 - E_1 = -\frac{3}{4}E_1$

$$E_2 - E_1 = -\frac{3}{4}(-2.17\cdot 10^{-18}\ \mathrm{J}) = 1.63\cdot 10^{-18}\ \mathrm{J}$$

Answer. The energy of the electron in the first orbit of the hydrogen atom is $-2.17 \cdot 10^{-18}$ J; the change in energy in the transition of the electron to the second orbit is $1.63 \cdot 10^{-18}$ J.

Example 85. Compute the Rydberg constant using the Bohr theory.

Given: $e = 1.6 \cdot 10^{-19}$ C is the proton charge, $h = 6.62 \cdot 10^{-34}$ J·s is Planck's constant, $c = 3 \cdot 10^{8}$ m/s is the velocity of light, $\varepsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the electric constant, $m_e = 9.11 \cdot 10^{-31}$ kg is the electron mass.

Determine: R—the Rydberg constant.

Solution. The energy of the electron in the kth orbit of the hydrogen atom is (see the last example)

$$E_k = -\frac{e^2}{8\pi\varepsilon_0 r_1 k^2}$$

For the nth orbit we have

$$E_n = -\frac{e^2}{8\pi\varepsilon_0 r_1 n^2}$$

Since

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{k^2}\right)$$

and

$$\frac{1}{\lambda} = \frac{1}{hc}(E_k - E_n)$$

then

$$R\left(\frac{1}{n^2} - \frac{1}{k^2}\right) = \frac{1}{hc}(E_k - E_n)$$

Substituting for E_k and E_n we find R:

$$R\left(\frac{1}{n^2} - \frac{1}{k^2}\right) = \frac{1}{hc}\left(-\frac{e^2}{8\pi\varepsilon_0 r_1 k^2} + \frac{e^2}{8\pi\varepsilon_0 r_1 n^2}\right)$$
$$= \frac{1}{hc}\frac{e^2}{8\pi\varepsilon_0 r_1}\left(\frac{1}{n^2} - \frac{1}{k^2}\right)$$

Taking into account that $r_1 = h^2\varepsilon_0/\pi m_e e^2$, we obtain

$$R = \frac{1}{hc}\,\frac{e^2\pi m_e e^2}{8\pi\varepsilon_0 h^2\varepsilon_0} = \frac{e^4 m_e}{8h^3 c\varepsilon_0^2}$$

$$= \frac{1.6^4\cdot 10^{-76}\,\mathrm{C}^4\cdot 9.11\cdot 10^{-31}\,\mathrm{kg}}{8\cdot 6.62^3\cdot 10^{-102}\,\mathrm{J}^3\cdot\mathrm{s}^3\cdot 3\cdot 10^8\,\mathrm{m/s}\cdot 8.85^2\cdot 10^{-24}\,\mathrm{F}^2/\mathrm{m}^2} =$$

$$= 1.1\cdot 10^7\ \mathrm{m}^{-1}$$

Answer. The Rydberg constant is equal to $1.1\cdot 10^7\ \mathrm{m}^{-1}$.

30.1. Why did Rutherford use a luminescent screen in his experiment? Was it a transparent or opaque screen?

30.2. What is the principle of the Geiger counter?

30.3. Compare the action of the Wilson and bubble chambers. Which one should be employed to study the properties of high-energy particles?

30.4. What is the contradiction between the Rutherford nuclear model and classical laws?

30.5. What was the quantization quantity in Bohr's nuclear model?

30.6. Is there any connection between the frequency of revolution of the electron about the nucleus in the hydrogen atom and the frequency of its radiation?

30.7. In Bohr's theory, what determines the frequency of radiation of the hydrogen atom?

30.8. What states of an atom are termed excited? What is their difference from the normal state?

30.9. Can an atom in transition to an excited state absorb an arbitrary amount of energy?

30.10. How many quanta of different energy can be emitted by hydrogen atoms, if the electrons are in the third orbit?

30.11. How does Bohr's theory explain the coincidence of absorption and emission spectra for vapours and gases?

30.12. What is the arrangement of electrons in the sodium atom? Lithium atom?

30.13. What radiation is produced by excited atoms in the transition of their electrons in the outer shells? In the inner shells?

30.14. Can X-rays be emitted by helium atoms? Strontium atoms?

30.15. What variation occurs in characteristic X-ray spectra as the atomic number Z grows?

30.16. Is the emission spectra of atoms dependent on their ionization?

30.17. What radiation is emitted by hydrogen atoms in the transition of electrons from the outer orbits to the first one? To the third one?

30.18. Determine the frequency and period of revolution of the electron in the hydrogen atom for the first and second orbits.

30.19. Determine the radiation frequency of the hydrogen atoms in the transition of electrons from the second orbit to the first one.

30.20. Determine the wavelength of the radiation of the hydrogen atom in the transition of electrons from the fourth orbit to the second one. What is the colour of the radiation?

30.21. What is the energy of the electron in the third orbit of the hydrogen atom?

30.22. What is the ionization potential of the hydrogen atom in the normal state?

30.23. Determine the wavelength of the radiation produced in the transition of an ionized hydrogen atom to the normal state.

SEC. 31. ATOMIC NUCLEUS

Example 86. There are $25 \cdot 10^6$ atoms of radius. How many of them will decay in a day, if the half-life for radium is 1 620 years?

Given: $N_0 = 25 \cdot 10^6$ is the initial number of radium atoms, $T = 1\,620$ years is the half-life of radium, $t = 1$ day is the time during which the decay of radium atoms is observed.

Determine: ΔN—the number of decayed radium atoms.

Solution. If the number of atoms of a radioactive substance at a given time is denoted by N_0, the number of undecayed atoms of the same substance in time t is denoted by N, and the half-life of the substance, by T, then these quantities will be related by the following formula:

$$N = N_0 e^{-\frac{0.693t}{T}}$$

where e = 2.71828... is the base of the natural logarithms. It is often called the number "e" or Naperian number.

Having calculated N from this relation, we can find the number of decayed atoms $\Delta N = N_0 - N$.

In the cases, however, where the time t is small as compared with the half-life T, the number of decayed atoms ΔN may be approximately given by

$$\Delta N = \frac{0.693}{T} N_0 t$$

Using the relationship, we calculate ΔN, as here t is small as compared with the half-life T for radium

$$\Delta N = \frac{0.693}{1\,620 \cdot 365 \text{ days}} \cdot 25 \cdot 10^6 \cdot 1 \text{ day} \approx 30$$

Answer. In 24 hours about 30 atoms of radium will decay.

Example 87. The plutonium isotope $^{239}_{94}Pu$ is alpha-radioactive. Its decay process occurs as follows:

$$^{239}_{94}\mathrm{Pu} \rightarrow {}^{235}_{92}\mathrm{U} + {}^{4}_{2}\mathrm{He}$$

The decay results in the energy being released, that consists mostly of the kinetic energy of α-particles. A part of the energy, however, stays with uranium nuclei which release it by emitting γ-rays. Determine the velocity of emitted α-particles in the decay of $^{239}_{94}Pu$, if it is considered that γ-rays have an energy of 0.09 MeV. The masses (in atomic mass units) of atoms involved in the reaction are as follows: $m_{Pu} = 239.05122$ amu, $m_U = 235.04299$ amu, $m_{He} = 4.00260$ amu.

Given: $m_{Pu} = 239.05122$ amu is the mass of atom of the plutonium isotope, $m_U = 235.04299$ amu is the mass of atom of the uranium isotope, $m_{He} = 4.00260$ amu is the mass of atom of the helium isotope, $E_\gamma = 0.09$ MeV is the energy of γ-quanta, $E = 931.3$ MeV/amu is the energy corresponding to one atomic mass unit.

Determine: v_α—the velocity of alpha-particles emitted in the decay of plutonium nuclei.

Solution. We first determine the change in the mass in the decay of plutonium isotope atom

$$\Delta m = m_{Pu} - (m_U + m_{He})$$

Substituting gives

$$\Delta m = 239.05122 \text{ amu} - (235.04299 \text{ amu} + 4.00260 \text{ amu}) = 0.00563 \text{ amu}$$

The energy released in the process is given by the relation $\Delta E = E\,\Delta m$. To obtain the kinetic energy of α-particles we subtract from ΔE the energy carried away by γ-radiation, and hence the velocity of emitted α-particles.

We find the liberated energy

$$\Delta E = 0.00563 \text{ amu} \cdot 931.3 \text{ MeV/amu} \approx 5.24 \text{ MeV}$$

The α-particles carry away the energy

$$E_{\text{He}} = 5.24 \text{ MeV} - 0.09 \text{ MeV} = 5.15 \text{ MeV} = 5.15 \cdot 1.6 \cdot 10^{-13} \text{ J}$$

(Recall that 1 MeV $= 1.6 \cdot 10^{-13}$ J.)

Using the kinetic energy relation $m_{\text{He}} v_\alpha^2/2 = E_{\text{He}}$, we find the velocity of α-particles

$$v_\alpha = \sqrt{2E_{\text{He}}/m_{\text{He}}} = \sqrt{\frac{2 \cdot 5.15 \cdot 1.6 \cdot 10^{-13} \text{ J}}{6.64 \cdot 10^{-27} \text{ kg}}} \approx 1.58 \cdot 10^7 \text{ m/s}$$

Answer. The velocity of α-particles emitted in the decay of plutonium nuclei is about $1.58 \cdot 10^4$ km/s.

31.1. What are isotopes?

31.2. How will the position of a chemical element in the Periodic Table change after the α-decay of nuclei of its atoms?

31.3. How will the position of a chemical element in the Periodic Table change after the β-decay of nuclei of its atoms?

31.4. What is the result of three successive α-decays of the thorium isotope $^{234}_{90}\text{Th}$?

31.5. What is the result of an α-decay and two β-decays of $^{238}_{92}\text{U}$?

31.6. What is the result of three successive β-decays and an α-decay of $^{210}_{81}\text{Tl}$?

31.7. Nuclei of the isotope $^{232}_{90}\text{Th}$ suffer an α-decay, two β-decays, and one more α-decay. What nuclei are produced in the process?

31.8. A nucleus of the isotope $^{211}_{83}Bi$ is obtained from another nucleus as a result of one α- and one β-decay. What nucleus is it?

31.9. A nucleus $^{216}_{84}Po$ is obtained as a result of two successive α-decays. What was the initial nucleus?

31.10. Does the chemical nature of an element suffer any change in emitting γ-rays by its nuclei?

31.11. An α-particle loses energy in travelling through the air. Why?

31.12. In studies of a radiation produced by a radioactive preparation α-particles with two different track lengths have been discovered. What conclusion can be drawn from the fact?

31.13. What characterizes the decay rate of a radioactive substance?

31.14. In a radioactive preparation $6.4 \cdot 10^8$ decays of nuclei per minute are found to occur. Determine the activity of the preparation in becquerels (1 Bc = 1 decay/s).

31.15. How many disintegrations of nuclei per minute occur in a preparation with activity 104 MBc?

31.16. The activity of a preparation is 25 MBc. What is the activity in terms of curies?

31.17. How long does it take for $25 \cdot 10^8$ nuclei to disintegrate in a preparation with a constant activity of 8.2 MBc?

31.18. In a lead capsule there are $4.5 \cdot 10^{18}$ radium atoms. Determine the activity of radium, if its half-life is 1 620 years.

31.19. In a capsule there are 0.15 moles of the plutonium isotope $^{239}_{94}Pu$. Determine the activity of the isotope, if its half-life is $2.44 \cdot 10^4$ years.

31.20. There is a uranium preparation of activity 20.7 MBc. In the preparation determine the mass of isotope $^{235}_{92}U$ with a half-life of $7.1 \cdot 10^8$ years.

31.21. When can the activity of a preparation be considered constant?

31.22. How long will it take for 80 per cent of atoms of the radioactive isotope $^{51}_{24}Cr$ to decay, if its half-life is 27.8 days?

31.23. What fraction of atoms of the radioactive isotope $^{58}_{27}Co$ disintegrates in 20 days, if its half-life is 72 days?

How long will it take for the same portion of atoms of the isotope $^{60}_{27}Co$ to decay, if its half-life is 5.3 years?

31.24. A nuclear explosion contaminates the environment with the many radioactive isotopes of varied half-life. Which of them are the most dangerous for humans travelling through this locality in a certain time after the explosion?

31.25. In observations of Cherenkov radiation, the cone angle is found to be 112°. What is the velocity of electrons causing the radiation?

31.26. In the water electrons travel at a velocity of 265,000 km/s. This produces Cherenkov radiation in the water. What is the cone angle?

31.27. A proton produces a visible track in the Wilson chamber, but a neutron does not. Explain why?

31.28. What neutrons are called thermal neutrons?

31.29. Substances from the middle and end sections of the Periodic Table are not used as neutron moderators. Why?

31.30. What are the uses of the mass-spectrograph?

31.31. Name the hydrogen isotopes. Are there radioactive isotopes among them? If so, describe their decay scheme.

31.32. What nuclear reactions occur in irradiation with α-particles of nuclei of the nitrogen isotope $^{14}_{7}N$? Of beryllium $^{9}_{4}Be$?

31.33. What is the meaning of the separation of tracks in the photographs obtained in the Wilson chamber?

31.34. Describe the path of transformation of $^{238}_{92}U$ nuclei in $^{239}_{94}Pu$ nuclei.

31.35. As $^{7}_{3}Li$ is bombarded with protons, helium results. Write the reaction. Calculate the energy liberated in this reaction. Given that the energy is equally distributed among the two α-particles, find their velocity. The initial kinetic energy of protons and lithium nuclei is taken to be zero.

31.36. In bombarding the nuclei of fluorine $^{19}_{9}F$ with protons the oxygen $^{16}_{8}O$ forms. What is the energy released in the reaction? What other nuclei are produced?

31.37. The boron isotope $^{11}_{5}B$, absorbing a proton, turns into beryllium $^{8}_{4}Be$. What other nuclei are produced in the reaction and what is the energy released?

31.38. When bombarded by α-particles the aluminium isotope $^{27}_{13}Al$ turns into phosphorus $^{30}_{15}P$. Write the reaction and compute the energy released.

31.39. In α-decay of similar nuclei the energies of α-particles are the same, but in β-decay of similar nuclei the energies of α-particles are dissimilar. Account for it.

31.40. Discuss the nuclear reaction that occurs in the absorption of thermal neutrons by $^{235}_{92}U$ nuclei.

31.41. How is the chain fission reaction realized?

31.42. The yield of a nuclear explosion cannot surpass a certain limit. Why? Is there any limit to the yield of a thermonuclear explosion?

CHAPTER VI

ASTRONOMY: BRIEF SURVEY

SEC. 32. ELEMENTS OF ASTRONOMY

Example 88. Determine the mass and average density of lunar matter, if the free fall acceleration on its surface is about 1.63 m/s^2. The Moon's radius is about $1.73 \cdot 10^6$ m.

Given: $R_M \approx 1.73 \cdot 10^6$ m is the Moon's radius, $g_M \approx \approx 1.63$ m/s^2 is the free fall acceleration on the Moon, $\gamma = 6.67 \cdot 10^{-11}$ m^3/kg·s^2 is the gravitational constant.

Determine: M_M—the mass of the Moon;
ρ_M—the average density of lunar matter.

Solution. The problem may be solved using the gravitational law

$$F = \gamma \frac{M_M m}{R_M^2}$$

where γ is the gravitational constant, R_M is the Moon's radius, M_M and m are respectively the masses of the Moon and a body on its surface. Considering that at poles of a planet (and ignoring its rotation about its axis at other latitudes also) the gravitational force is equal to the weight, we obtain

$$mg_M = \gamma \frac{M_M m}{R_M^2}$$

Thus the mass of the Moon, M_M, is

$$M_M = \frac{g_M R_M^2}{\gamma} = \frac{1.63 \text{ m/s}^2 \cdot 1.73^2 \cdot 10^{12} \text{ m}^2}{6.67 \cdot 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)} \approx 7.3 \cdot 10^{22} \text{ kg}$$

The average density of lunar matter is given by

$$\rho_M = M_M / V_M$$

where V_M is the volume of the Moon obtainable from the relation $V_M = \frac{4}{3} \pi R_M^3$. Therefore

$$\rho_M = \frac{g_M R_M^2}{{}^4/_3 \pi\gamma R_M^3} = \frac{3g_M}{4\pi\gamma R_M}$$

$$= \frac{3 \cdot 1.63\ \text{m/s}^2}{4\pi \cdot 6.67 \cdot 10^{-11}\ \text{m}^3/(\text{kg} \cdot \text{s}^2) \cdot 1.73 \cdot 10^6\ \text{m}} \approx 3.3 \cdot 10^3\ \text{kg/m}^3$$

Answer. The mass of the Moon is about $7.3 \cdot 10^{22}$ kg; the average density of lunar matter is about $3.3 \cdot 10^3$ kg/m^3.

Example 89. The satellite period with a circular orbit is $T = 240$ min. The mass of the satellite is $m = 1.2$ t. Determine the orbital altitude above the ground and the kinetic energy of the satellite. The radius of the Earth is $R_E = 6.4 \cdot 10^6$ m.

Given: $T = 240$ min $= 1.44 \cdot 10^4$ s is the satellite period, $m = 1.2$ t $= 1.2 \cdot 10^3$ kg is the mass of the satellite, $g = 9.8$ m/s^2 is the free fall acceleration on the ground, $R_E = 6.4 \cdot 10^6$ m is the radius of the Earth.

Determine: H—the orbital height above the ground;
E_k—the kinetic energy of the satellite.

Solution. As an orbiting satellite is only subjected to the force of attraction to the Earth (ignoring the attractions by all the other celestial bodies), it keeps the satellite in the orbit providing the normal acceleration

$$F = ma_n$$

or

$$\gamma Mm/R^2 = 4\pi^2 Rm/T^2$$

Hence

$$\gamma MT^2/4\pi^2 = R^3$$

where $R = R_E + H$ is the distance from the Earth's centre to the satellite. Multiplying the numerator and denominator on the left-side of the last relation by R_E^2 gives

$$\gamma \frac{M}{R_E^2} \frac{T^2 R_E^2}{4\pi^2} = R^3$$

As $\gamma M/R_E^2 = g$ (see example 88), then

$$\frac{gT^2R_E^2}{4\pi^2} = R^3$$

Hence the distance from the Earth's centre to the satellite is

$$R = \sqrt[3]{\frac{gT^2R_E^2}{4\pi^2}}$$
$$= \sqrt[3]{\frac{9.8\ \mathrm{m/s^2}\cdot(1.44\cdot10^4)^2\ \mathrm{s^2}\cdot(6.4\cdot10^6)^2\ \mathrm{m^2}}{4\pi^2}} = 1.28\cdot10^7\ \mathrm{m}$$

From the above value of R we subtract the Earth's radius R_E to find the orbital altitude H over ground:

$$H = R - R_E = 12.8\cdot10^6\ \mathrm{m} - 6.4\cdot10^6\ \mathrm{m} = 6.4\cdot10^6\ \mathrm{m} = R_E$$

The kinetic energy of the satellite is given by

$$E_k = \frac{1}{2}mv^2$$

where

$$v = 2\pi R/T$$

Therefore

$$E_k = \frac{mv^2}{2} = \frac{m4\pi^2R^2}{2T^2} = \frac{2\pi^2R^2m}{T^2}$$

Substituting gives

$$E_k = \frac{2\cdot9.86\cdot(1.28\cdot10^7\ \mathrm{m})^2\cdot1.2\cdot10^3\ \mathrm{kg}}{(1.44\cdot10^4\ \mathrm{s})^2} = 1.87\cdot10^{10}\ \mathrm{J} \approx 18.7\ \mathrm{GJ}$$

Answer. The satellite travels in a circular orbit with altitude equal to the Earth's radius $R_E = 6.4\cdot10^6$ m; the kinetic energy of the satellite is about 18.7 GJ.

32.1. Using the definition of celestial sphere show that the mutual arrangement of celestial bodies can only be judged by angle measurements.

32.2. What is the direction of diurnal motion of stars, the Sun, the Moon and the planets for an observer in the northern hemisphere if he faces the south?

32.3. By observation, compile a list of constellations that do not set in your locality. Check the answers by a planisphere.

32.4. What is the name of the brightest star on the celestial sphere and the brightest star in the northern hemisphere?

32.5. The position of a star on the celestial sphere is determined by two coordinates: declination δ—the angular separation of the star from the equator (it is analogous to the geographic latitude) and right ascension α, measured along the equator from the vernal equinox to the hour circle of the star (analogous to the geographic longitude, measured in time units). Using a planisphere determine approximately the declination and right ascension of the following stars: α Tauri, Aurigae (Capella), Lirae (Vega). Compare the results obtained with the table and try to find on the sky the constellations to which these stars belong.

32.6. Show on the planisphere the positions and constellations to which the Crab Nebula belongs: $\alpha = 5$ h 35 min, $\delta = +21°$; the Loop Nebula: $\alpha = 20$ h 50 min, $\delta = +31°$.

32.7. Using the planisphere find the Andromeda constellation and the nearest galaxy located near the constellation and seen as a foggy spot.

32.8. The latitude of Moscow is $\varphi = 55°45'$. In Moscow, at what distance from the zenith is the celestial pole?

32.9. What constellation passes through the zenith on November 15, at 10 p.m. at the site of your observation?

32.10. What is ecliptic? What is the angle of inclination of the ecliptic plate to the equator?

32.11. What constellations are called zodiacal? List them.

32.12. What is the midday altitude of the Sun in Moscow at summer solstice? Winter solstice?

32.13. In winter, the Earth in its annual motion is at the shortest Sun's distance. What can be said about the velocity of the Sun travel in ecliptic in this period?

32.14. The horizontal lunar parallax is $p = 57'$ and the Earth's radius is taken to be 6 370 km. Given that radiomagnetic impulses sent by a Moon radar have been received in about 2.56 s, determine the Earth-Moon distance.

32.15. For an observer on the Earth the lunar radius is

seen at an angle of 0.25°. Determine the Moon's radius, if the Earth-Moon distance is taken to be $3.84 \cdot 10^5$ km.

32.16. Determine the lunar horizontal parallax for the case where it is at perigee (the Earth-Moon distance is taken to be $3.63 \cdot 10^5$ km).

32.17. The annual parallax of the Proxima Centauri, a star nearest to the Sun, is 0.762″. Determine the distance to the star in parsecs, light years, and kilometres.

32.18. The solar radius is seen from the Earth at an angle of 0.25°. If the Earth-Sun distance is $1.5 \cdot 10^8$ km, determine the solar radius.

32.19. The distance to Barnard's star is 1.83 pc. What is its annual parallax?

32.20. By how many times is the solar radius larger than the Earth's radius, if the Sun's horizontal parallax is 8.794″, and the mean Sun's angular radius is 16′?

32.21. Determine the local mean solar time at Krasnoyarsk when at Kharkov it is 7 h 13 min 42 s on October 10. (The longitude of Krasnoyarsk is 6 h 11.3 min, and it lies within the sixth time zone; Kharkov belongs to the second zone, and its longitude is 2 h 25.0 min.)

32.22. When at Greenwich it is 8 h 15 min, in Moscow, it is 10 h 45 min 17 s. Determine the longitude of Moscow.

32.23. Determine the local mean solar time at Sverdlovsk (λ_S = 4 h 02.4 min), if in Moscow (λ_M = 2 h 30.5 min) the time is 7 h 28 min 6 s.

32.24. The volume of the Earth can be calculated fairly accurately, assuming that it has the form of a sphere of radius 6 400 km. Determine the average density of the Earth, if its mass is $6 \cdot 10^{24}$ kg.

32.25. The Sun's mass is 333,000 times larger than that of the Earth, and the solar radius is 109 times larger than the Earth's radius. Determine the average density of the Sun. The required data are to be taken from the preceding problem.

32.26. Kepler's third law relates the sidereal periods of planets to their average distances from the Sun as follows: $T_1^2/T_2^2 = a_1^3/a_2^3$, where T_1 and T_2 are the periods of any two planets, and a_1 and a_2 are their average distances from the Sun. If one of the planets is the Earth, for which $T_1 = 1$ year, and $a_1 = 1$ AU, Kepler's law will be $T_2^2 = a_2^3$. Using

the relation, determine the average distance from the Sun for Venus, Pluto, the most distant planet from the Sun, if the sidereal period for Venus is 0.62 year, and for Pluto 248.4 years. Express the distances in astronomical units (AU) and in kilometres.

32.27. Kepler's third law with allowance made for the law of gravitation, may be utilized to determine the ratio between the mass of the Sun and the mass of any planet having a satellite, e.g. the Earth. It has the form

$$\frac{T_E^2}{T_M^2}\cdot\frac{M_S+m_E}{m_E+m_M}=\frac{a_E^3}{a_M^3}$$

Taking into account that the mass of the Earth is small as compared with the mass of the Sun, and the mass of the Moon is small as compared with that of the Earth, the law may be written as follows: $M_S/m_E = (a_E/a_M)^3\,(T_M/T_E)^2$. Determine the ratio of the Sun's mass to the Earth's mass, assuming that the average distance to the Moon is 384,000 km $= 2.56\cdot10^{-3}$ AU, and the Moon revolution period is 27.3 days $= 7.5\cdot10^{-2}$ years.

32.28*. The velocity of light is independent of the fact whether a light source or an observer are at rest or in motion. However, the wavelength seen by the observer with the source at rest (λ_0) and in motion (λ) are different, their difference giving the so-called Doppler displacement ($\Delta\lambda = \lambda - \lambda_0$). At velocities of objects small as compared with the velocity of light the relation $(\lambda - \lambda_0)/\lambda_0 = z = v/c$ is valid, where z is the relative displacement of a spectral line, and v is the velocity of the object radiating the light. The line displacement in the direction of red lines (red displacement) gives $z > 0$, and corresponds to a receding object. The relative red displacement for one of the galaxies is 0.001. Determine whether the galaxy is approaching the observer of the Earth or moving away from it. Determine the absolute displacement for the blue line of hydrogen

* At large velocities, comparable with the velocity of light

$$z=\frac{1+v/c}{\sqrt{1-v^2/c^2}}-1.$$

($\lambda_0 = 486.1$ nm). What is the velocity of the galaxy along the line of sight of the observer?

32.29. Determine the linear velocity of points on the solar equator, given that for the green line of hydrogen with $\lambda_0 = 500$ nm the Doppler displacement is 0.0035 nm.

32.30. In annual motion of the Earth the lines in spectra of stars which the Earth is approaching, are displaced in the violet direction. Determine the magnitude of the velocity of the Earth, if for the green line with $\lambda_0 = 500$ nm the displacement is 0.05 nm.

32.31. Determine the wavelength in the observed spectrum of a star, if the Earth approaches it in its annual motion. The laboratory wavelength of the line of the star is 486.1 nm.

32.32. In 1960 the relative red displacement of radio-galaxy 3C 295* was found to be 0.46. The distance from the galaxy to the Earth is 5 billion light years. Is the galaxy approaching or receding? What is its velocity along the line of sight? Determine the displacement of the green line with $\lambda_0 = 500$ nm.

32.33. In 1963 it was found in measuring the red displacement in the spectrum of quasar 3C 273-B (the brightest and nearest) that it is equal to 0.16. Determine the velocity along the line of sight with which the quasar distance to the Earth changes.

32.34. It was found in 1965 that for quasar 3C 9 the red displacement parameter is $z = (\lambda - \lambda_0)/\lambda_0 = 2$. Why cannot the relation $z = v/c$ be used here? Determine the velocity for this object (see the footnote to problem 32.28).

32.35. What is the velocity of a spaceship, if an astronaut perceives the red light of a laser sent from the Earth to the ship as green. Does the distance between the Earth and the ship increase or decrease? The wavelengths of the red and green light are taken to be respectively 620 and 550 nm.

32.36. The first sputnik launched in the USSR on October 4, 1957, travelled in an orbit with an average altitude above ground $H = 588$ km. Determine the kinetic energy of the orbiting satellite. The mass of the satellite is $m = 83.6 \text{ kg} \approx$ ≈ 84 kg, $R_E = 6\,400$ km, $g_E = 9.8 \text{ m/s}^2$. The orbit is taken to be circular.

---

* Object 3C 295 stands for object 295 in the third Cambridge catalogue.

32.37. The first astronaut Yu.A. Gagarin in his ship "Vostok-1" travelled around the Earth in an orbit with an average altitude above ground of 251 km. Assuming a circular orbit, determine the velocity of the ship in the orbit and the orbital period of the ship.

32.38. What was the average linear velocity of the spaceship "Vostok-2", if its orbital period was $T = 88.6$ min? Assume the orbit to be a circle and $R_E = 6.4 \cdot 10^6$ m.

32.39. A satellite is launched in a circular orbit lying in the equator plane so that all the time it stays above the same point on the equator. Determine: the radius of the satellite orbit; the altitude above the ground; the orbital velocity of the satellite.

32.40. Determine the orbital period and velocity of a satellite travelling around the Moon at $H = 200$ km from ts surface, if $M_M = 7.3 \cdot 10^{22}$ kg and $R_M = 1.7 \cdot 10^6$ m.

ANSWERS *

Sec. 1. Density

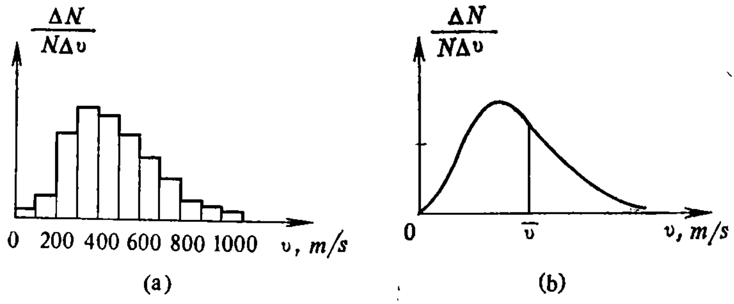
1.1. 77.6 kg; $7.6 \cdot 10^2$ N. 1.2. 68 kg. 1.3. $1.12 \cdot 10^3$ m. 1.4. 1.04 m^2. 1.5. $7.8 \cdot 10^3$ kg/m^3. 1.6. 19.5 kg; 6.75 kg. 1.7. $5.9 \cdot 10^6$ kg. 1.8. Yes; $2.6 \cdot 10^{-3}$ m^3. 1.9. $2.16 \cdot 10^4$ kg/m^3; $4.62 \cdot 10^{-5}$ m^3. 1.10. $8.3 \cdot 10^3$ kg/m^3. 1.11. $7.5 \cdot 10^2$ kg/m^3. 1.12. $1.36 \cdot 10^4$ kg/m^3; mercury. 1.13. $V'/V = \rho_1/\rho_2$. 1.14. $7.8 \cdot 10^3$ kg/m^3. 1.15. $1.00 \cdot 10^3$ kg/m^3.

Sec. 2. Elements of Kinetic Theory of Gases

2.1. The speed of chaotic motion of molecules of the medium grows. 2.2. The pressure in cylinder A grows because the lighter gas passes inside faster, than the air leaves the cylinder; the mercury displaced into the right section of the tube makes the network of the ring. 2.3. The chaotic motion of molecules persists under weightlessness. 2.4. When heating a steel part in a mixture of coal and various salts carbon atoms diffuse into the surface layer of metal, thus adding to the part strength. 2.5. Melted copper diffuses deeper into the surface layer of parts soldered, the longer the diffusion. As a result, the joint becomes stronger. 2.6. Strikes of a hammer provide good contact of "welded" metal pieces. At white heat the interdiffusion occurs faster and deeper. 2.7. Under strong compression, which is accompanied by the softening of surfaces of pieces welded and the increasing of interdiffusion, cohesive forces achieve the level providing fast connection of the pieces. 2.8. Will rise; descend; will bulge; be drawn in. 2.9. The resultant force on a particle changes continually and irregularly in magnitude and direction. The larger the particle, the larger portion of the forces mutually balanced. 2.10. Each segment is the length of straightened path of the particle in a short period of time. Elements of the path could be observed during a shorter period with a larger magnification of a microscope. 2.11. 530 m/s. 2.12. 450 m/s. 2.13. As the velocity ranges decrease without bound, the broken line that bounds the bars of the diagram (see figure *a*) turns into a smooth curvilinear line (see figure *b*). 2.14. $2.15 \cdot 10^{22}$ g^{-1}; $1.37 \cdot 10^{22}$ g^{-1}; $2.7 \cdot 10^{25}$ m^{-3}. 2.15. $1.5 \cdot 10^{23}$ g^{-1}; $4.3 \cdot 10^{22}$ g^{-1}; $2.7 \cdot 10^{25}$ m^{-3}. 2.16. 2.82×10^{22} g^{-1}. 2.17. $3.3 \cdot 10^{-9}$ m. 2.18. $5.33 \cdot 10^{-26}$ kg; $7.3 \cdot 10^{-26}$ kg; $3 \cdot 10^{-26}$ kg; $2.8 \cdot 10^{-26}$ kg. 2.19. $1.3 \cdot 10^{-7}$ m. 2.20. $1.5 \cdot 10^{10}$ s^{-1}; 7.5×10^9 s^{-1}. 2.21. $2.7 \cdot 10^{-10}$ m; $3.7 \cdot 10^{-10}$ m. 2.22. $1.72 \cdot 10^{-5}$ Pa·s; $1.4 \cdot 10^{-5}$ Pa·s. 2.23. 530 rpm; $8.5 \cdot 10^{-6}$ Pa·s. 2.24. The action of the mercury barometer is based of the principle of communicating vessels: the atmospheric pressure is balanced by the pressure of mercury

* As a rule approximate values are given.

column in the tube. 2.25. The total weight of the cup and counterweights is equal to the total weight of the tube with the mercury column in it. Yes, e.g. as the atmospheric pressure builds up, the cup with counterweights goes up. 2.26. Under weightlessness molecules making up the "atmosphere" inside the ship continue to move about chaotically. 2.27. In the mercury barometer mercury will fill up the

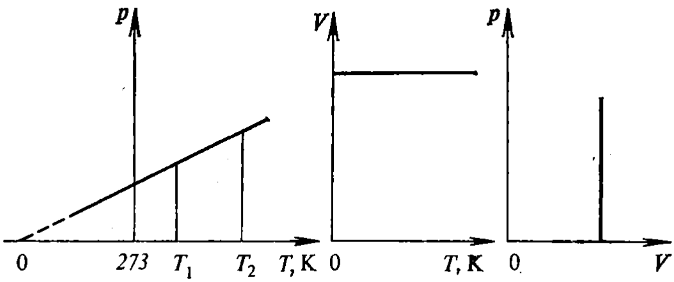


To answer 2.13

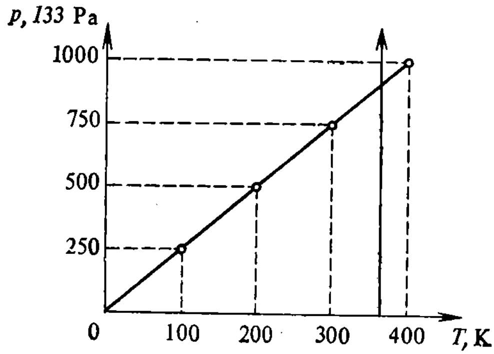
tube completely; the aneroid barometer will function normally; the water-level gauge cannot be used. 2.28. By 10.5 m; by 1920 Pa. 2.29. $8.8\cdot10^4$ Pa; $7.7\cdot10^4$ Pa; $2.54\cdot10^4$ Pa; $4.8\cdot10^3$ Pa. 2.30. 770 N. 2.31. By 1.08 times (the lifting forces can be taken to be equal). 2.32. By 2 times. 2.33. 480 N. 2.34. $9.6\cdot10^5$ Pa. 2.35. $5.4\cdot10^{-21}$ J; $9.0\cdot10^5$ J; $1.5\cdot10^6$ Pa. 2.36. $7\cdot10^{-21}$ J; 300 J; 550 m/s; 2 kg/m³. 2.37. 500 m/s. 2.38. $\alpha = \alpha_{273} = 0.00366$ K^{-1}; $p_{273} = 1.092\cdot10^5$ Pa; $p_{373} = 1.5\cdot10^5$ Pa; $p_{180} = 7.2\cdot10^4$ Pa; 200 K. 2.39. $1.24\cdot10^{-2}$ m/s; $4.38\cdot10^{-3}$ m/s; $4.25\cdot10^{-8}$ m. 2.40. $2\cdot10^{-8}$ m/s. *Hint*: (1) $z/t = N_A m_0/\rho St$; (2) $p = N_A m_0\, \Delta v/St$, where $\Delta v = v$ (inelastic collision); (3) $\varepsilon = m_0v^2/2$. 2.41. $1.1\cdot10^6$ Pa. 2.42. 73 K; will not change. 2.43. $3.3\cdot10^{11}$ m^{-3}; $3.3\cdot10^5$ cm^{-3}. 2.44. $9.72\cdot10^5$ Pa. 2.45. 729 l. 2.46. 240 K. 2.47. 0.68 m³. 2.48. 26 l. 2.49. $4.7\cdot10^4$ Pa; 19.5 g. 2.50. 225 K; 32.8 g. 2.51. Will increase by 8%. 2.52. 2.7 N. 2.53. $2.94\cdot10^5$ Pa. 2.54. By 8.7 m. 2.55. 0.47 kg/m³. 2.56. 2 kg/m³; 0.5 kg/m³. 2.57. 3.3 m/s. 2.58. 1173 K. 2.59. 220. 2.60. 3.6 kg. 2.61. 294 K. 2.62. $R = 8.31$ J/(mol·K). 2.63. 560 K. 2.64. 14. 2.65. $1.3\cdot10^7$ Pa. 2.66. 2.0 kg. 2.67. 1.2 m³. 2.68. 3340 rpm. 2.69. Up to 1270 K. 2.70. 0.17 m. 2.71. 0.116 kg. 2.72. 6.15 kPa; 3.63 kPa.

Sec. 3. Special Applications of Mendeleev-Clapeyron Equation

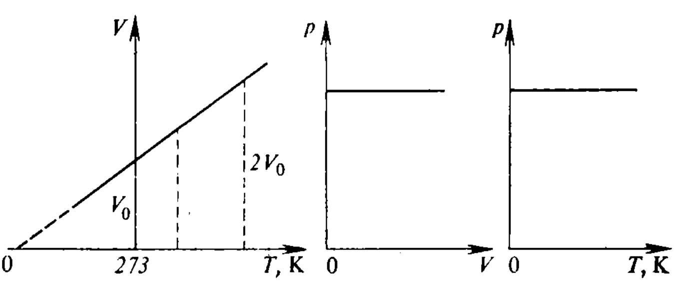
3.1. In the bulb of a switched lamp the gas pressure should not much exceed the atmospheric pressure; in the medical cup applied to the body the air pressure becomes lower than the atmospheric pressure. 3.2. $1.54\cdot10^5$ Pa; $0.91\cdot10^5$ Pa. 3.3. 248 K. 3.4. 633 K. 3.5. —3°C;



To answer 3.9

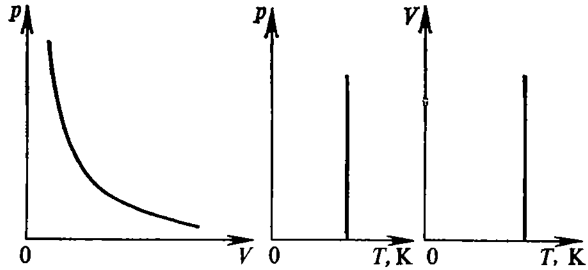


To answe 3.10



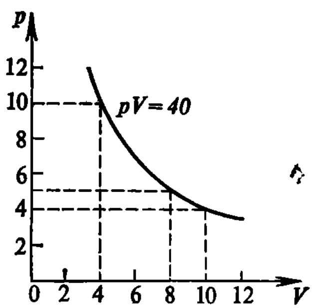
To answer 3.27

1.3 atm. **3.6.** 50°C; —35°C. **3.7.** No. **3.8.** $3.92 \cdot 10^6$ Pa. **3.9.** See the figure. **3.10.** See the figure, $\gamma = (1/373)\ K^{-1}$. **3.11.** Points *A* correspond to larger volume, points *B* to larger density. **3.12.** $1.47 \cdot 10^{-3}$ Pa; $1.76 \cdot 10^{-3}$ Pa. **3.13.** At day time—sea breeze, at night—land breeze. **3.14.** On the lowland where cold air accumulates. **3.15.** It provides the

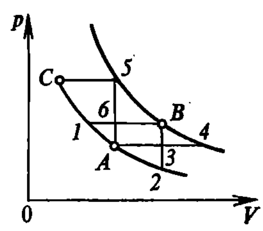


To answer 3.56

required air circulation. **3.16.** At the top. **3.17.** 270 cm³; 225 cm³. **3.18.** 308 K. **3.19.** 1.2 m³. **3.20.** By 20 mm. **3.21.** 303 K. **3.22.** 1400 K. **3.23.** 0.78 kg. **3.24.** 100 l. **3.25.** 2.4 m/s. **3.26.** 10.4 kg; 102 N. **3.27.** See the figure. **3.28.** Point *1*; segments *2B*, *O5*, *OB*; segment *O5*. **3.29.** 77 g. **3.30.** As the volume of a gas increases, its density increases, so does



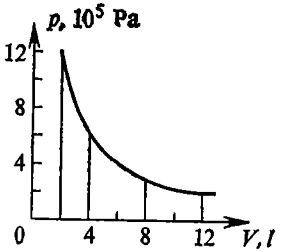
To answer 3.57



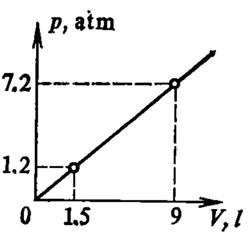
To answer 3.64

the number of molecule impacts per second per unit area of the wall, therefore the gas pressure grows. **3.31.** No. **3.32.** (1) 0.67 and 0.17 kg/m³; (2) 2.23 and 1.80 kg/m³. **3.33.** By about 1.5 times. **3.34.** $3.2 \cdot 10^5$ Pa. **3.35.** 15 l. **3.36.** $1.75 \cdot 10^5$ Pa. **3.37.** $3.8 \cdot 10^5$ Pa. **3.38.** 105 mm; 90 mm. **3.39.** $1.0 \cdot 10^5$ Pa; 147 mm. **3.40.** 95 cm. *Hint*: The air in the tube of the faulty barometer occupies all the space above the mercury surface at a pressure measured by the difference in indi-

cations of the two barometers. 3.41. 10.3 m; 6.4 mm^3. 3.42. 20.7 m. 3.43. 270 mg; 3.9 g; 5.3 kg. 3.44. 0.32 l. 3.45. 3.04·10^6 Pa. 3.46. 1.2 l. 3.47. 84; 72; 60. 3.48. 6.1·10^6 Pa. 3.49. 120. 3.50. 3·10^5 Pa; no. 3.51. 4.1·10^4 Pa; 3.24·10^4 Pa. 3.52. 1·10^5 Pa. 3.53. 8·10^4 Pa; 90 cm^3; 45 cm^3; 65 cm^3. 3.54. 13.6 kg. 3.55. 8·10^{-4} m^3. 3.56. See the figure. 3.57. See the figure. Will decrease by $1/(n+1)$ (by 1/5); will increase by $1/(n-1)$ (by 1/3) of the initial value. 3.58. Points A and B on various hyperbolas may represent the states: of the same quantity



To answer 3.68



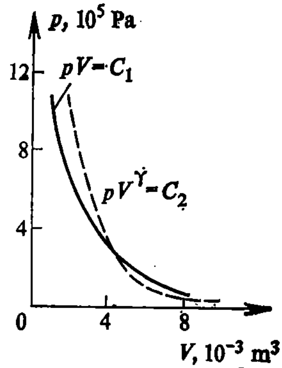
To answer 3.70

of gas at different temperatures ($T_B > T_A$), two different masses of the same gas at the same temperature ($m_B > m_A$). 3.59. 355 m/s; 4.6·10^{-21} J. 3.60. 1,465 m/s, 3.6·10^{-21} J; 1 840 m/s, 5.6·10^{-21} J; 2300 m/s, 8.7·10^{-21} J. 3.61. 49 K; 629 K. By 3.8·10^5 J. 3.62. 50 J; 3.3 kJ; 6 kJ. By 3.3·10^{-21}, 5.5·10^{-21} and 6.6·10^{-21} J. 3.63. 7.2·10^{-21} J; 1.2·10^{-20} J; 300 K. 3.64. Point B. Transition can occur by any of the two isoprocesses: $A1B$, $A2B$, ..., $A6B$. See the figure. 3.65. See the figure to answer 3.64. The gas releases heat in processes $B2$, 21; and absorbs in process $1B$. 3.66. See the figure to answer 3.64. The gas absorbs heat in processes $C5$ and $5B$; and releases in processes $B2$ and $2C$. 3.67. 4.41 kJ; 250 kJ. 3.68. See the figure. To calculate the work done by the gas, the gas volume range should be divided into 12 parts, areas under each segment of the isotherm determined and summed up; $A = 4.35$ kJ. 3.69. 5.1 kJ; 17 MJ. 3.70. The straight line drawn through the origin of the coordinate system in Fig. 3.70; 3.2 kJ; 5·10^6 K/m^6.

Sec. 4. Internal Energy of a Body and Ways of Changing It. Heat and Work. First Law of Thermodynamics

4.1. Seas and oceans have enormous heat capacity, in the coastal region temperature fluctuations are ironed out by the absorption and release of heat by the water. 4.2. The specific heat capacity of sand is small, and the energy liberated by it is insufficient to smooth out diurnal fluctuations of air temperature. 4.3. At high altitudes the air density is fairly small; therefore the amount of heat transferred to

the satellite envelope is insignificant. 4.4. 34°C. 4.5. 30°C. 4.6. 45 l. 4.7. 5.0 m^3. 4.8. 225 and 75 l. 4.9. 40°C. 4.10. 3 kg. 4.11. 0.3 kg. 4.12. Segments *CM*, *KN*, *KL* represent respectively the changes in temperatures of the block, water, and calorimeter during the heat exchange; segments *Om*, *On* and *Ol*—the changes in the internal energy of the same bodies. Segment *pm* represents the heat losses. An equal slope of segments *KL* and *KN* would signify that the ratios of specific heat capacities of water and calorimeter material vary inversely with their masses. 4.13. 8.0 kg. 4.14. 98°C. 4.15. 380 J/(kg·K). 4.16. 168 g. 4.17. 700°C. 4.18. 0.43 kg. 4.19. Up to 1181 K. 4.20. 1.2·10^8 J. 4.21. Up to 26°C. 4.22. 0.078 m^3. 4.23. $c_p > c_V$ as in the first case the energy goes not only into the heating of the gas, but also into the work. 4.24. The metal heats to melting temperature in contact with a rotating disk. 4.25. The convective heat transfer from hot water to environment is accelerated. 4.26. In the first case—the mechanical work performed, in the second—heat transfer. 4.27. The entry friction causes the strong heating of bodies in the atmosphere and adjacent air layers. Small meteorites evaporate in dense layers on the Earth's atmosphere, having not reached the ground. 4.28. By 1.7 K. 4.29. 6.38 km; 14.8 km. 4.30. Copper; by 15 K. 4.31. 0.63 kW; by 20 K. 4.32. By 8.5 K. 4.33. By 38 K. 4.34. 3.3 m/s. 4.35. By 6 K. 4.36. 60 W. 4.37. By 35 K. 4.38. By 17.6 K. 4.39. 6.7 N·m. 4.40. 11.2 kJ; 39 kJ; 27 K; 27.8 kJ. 4.41. 5 kJ; 55 K; 0.08 m. 4.42. 4.05 kJ; 14.1 kJ; 55 mm. 4.43. Quick compression of gas in the cylinder of an engine or pump; the air cooling as it rises to the upper layers of the atmosphere, and so forth. In an adiabatic process all the three parameters of a gas are changing. Yes, if the process occurs sufficiently fast. See the figure. 4.44. 49.3·10^5 Pa; 702 K; 316 kJ. 4.45. 125; 155°C; 11.5 kJ. 4.46. 1.1 kg. 4.47. 2.7·10^7 J/kg. 4.48. 33%. 4.49. 4 l. 4.50. 16 min. 4.51. 27%. 4.52. 310 kg. 4.53. 0.52 t. 4.54. 7.4·10^4 kJ. 4.55. 56 kg. 4.56. 37%; 29%. 4.57. 27%. 4.58. 0.26 kg; 0.33 kg; 7.2 kg. 4.59. 23%. 4.60. 6.3 t. 4.61. 64 t. 4.62. 13.3 l. 4.63. By 61 km. 4.64. 8 h. 4.65. 185 kg. 4.66. 450 km. 4.67. 120 km. 4.68. 35%; 66 t. 4.69. 104 l; 2,450 N; 73.5 kW. 4.70. Coal A-1; 2.05·10^7 J/kg. 4.71. 18.4 t; 4.41 × × 10^4 kW.



To answer 4.43

Sec. 5. Properties of Real Gases and Vapours. Atmospheric Water Vapour

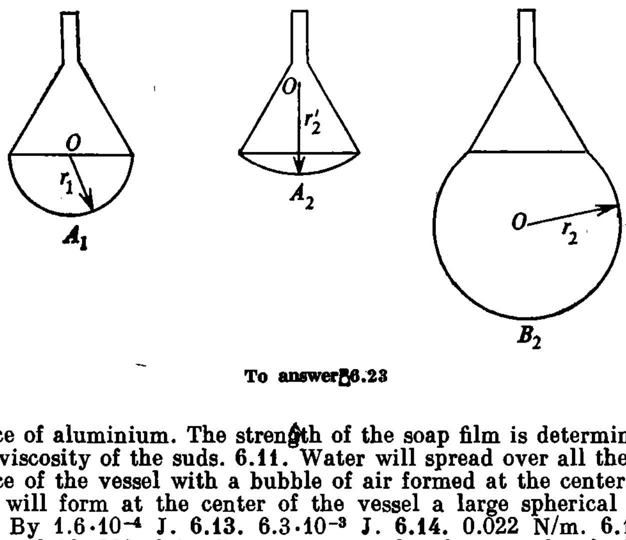
5.1. Without wind the density of water vapour immediately over grass or near fabric is higher, and thus the evaporation is retarded. 5.2. The evaporation occurs due to a reduction in the internal energy

of water that cools harder if in the open. 5.3. For a man just out of water the sensation of cold is caused by the expenditure of the internal energy to evaporate water from the surface of the body; in breezy weather the evaporation is more intensive. 5.4. In rain the evaporating area for a given amount of atomized rain water is great, and the energy is drawn from the ambient air as well. 5.5. In hot weather the sweating protects the body from overheating. In marshy places (and damp localities in general) the water vapour density is higher than in dry ones, and the sweat evaporation occurs slower. 5.6. This dress does not allow the moisture evolved underneath to evaporate into ambient air with the result that the body overheats. 5.7. Can (e.g. ice, naphthalene). 5.8. The water evaporation lowers the temperature of a burning body so as to stop the burning reaction; in addition, vapour envelopes the body keeping oxygen away. Hot water, because it evaporates faster than cold water. 5.9. In a gas-filled lamp the filament is evaporated slower than in a vacuum one. 5.10. *1*—water, *2*—ethyl alcohol; *3*—mercury. 5.11. 200 kJ; 80 kJ. 5.12. 518 kJ. 5.13. 32°C. 5.14. 35 g. 5.15. $2.26 \cdot 10^6$ J/kg; 7.3%. 5.16. Segment BC represents the vapour condensation; segments AB, CM, KL, and KN—variations of the temperature of vapour, condensate, calorimeter, and water that was in it initially. Segments bc, Ob, cm, Ol, On on axis OQ are the amounts of heat released or absorbed in these processes. 5.17. 0°C. 5.18. Up to 46°C. 5.19. 0.8 kg. 5.20. 39 kg. 5.21. Up to 360 K. 5.22. 7 g. 5.23. 40 l. 5.24. 10 l. 5.25. 40%. 5.26. 27 min. 5.27. 70 g. 5.28. 66%. 5.29. $1.2 \cdot 10^5$ Pa. 5.30. No. Yes, if the boiling point of a liquid in outer pan were higher than that of a liquid in the inner pan. 5.31. Tilt the tube in the vertical plane. If there is no air in the tube, the liquid in both sections will be at the same level. 5.32. By heating the saturated vapour without liquid. Overheated; saturated. 5.33. 960 Pa. 5.34. 2 400 Pa; by unsaturated vapour. 5.35. 1 000 Pa; saturated; in cooling to 2°C each cubic metre gives 2.2 g of water. 5.36. The evaporation of water molecules from a dried body into a vacuum is more intensive than into a gas-filled space; 33 g. 5.37. $9.4 \cdot 10^{-3}$ kg/m^3; $28.7 \cdot 10^{-3}$ kg/m^3; 0.2 kg/m^3. 5.38. 2 070 Pa; 3 530 Pa; 12,400 Pa. 5.39. 31.4 l. 5.40. At 298 K. 5.41. 8 l; 0.5 g. 5.42. Vapour, water; water, vapour; vapour, vapour. 5.43. Liquid; gaseous; liquid. 5.44. Yes; no; yes. 5.45. Liquid air is made to boil (T_b = 78-81 K) under lower pressure with continually pumping out the produced vapour. The air components evolve in the process in increasing order of their boiling points: He, Ne, N, Kr, Ar, O, Xe (see Table 9). 5.46. The amount of water vapour in 1 m^3 of room air is much larger than that of air outdoors. 5.47. As the temperature lowers the vapour condensation sets in. 5.48. By the cooling of air (normally by the morning) down to the temperature at which the relative humidity is 100%. 5.49. On a hot day more water evaporates. 5.50. The layer of clouds prevents the ground from cooling. 5.51. $8.55 \cdot 10^{-3}$ kg/m^3; 52%. 5.52. $10.7 \cdot 10^{-3}$ kg/m^3; 62%. 5.53. $9.27 \times \times 10^{-3}$ kg/m^3; 10°C. 5.54. In markedly lowering the temperature. 5.55. In the second case. 5.56. 54%; 44%; 70%; 83%; the results are identical. 5.57. 285 K. 5.58. 279 and 277 K; 30 and 26°C. 5.59. 2.5 g. 5.60. At 13°C. 5.61. No; yes; 1.64 g. 5.62. No; yes; 0.2 g. 5.63. 70%.

5.64. 60.5 g. By 5°C. 5.65. By 568 Pa; by 320 Pa. 5.66. 2.1 and 4.7 kg. 5.67. $1.34 \cdot 10^5$ Pa; 58%. *Hint*: The vapour pressure $p_v = p_n \varphi$. The pressure of dry air $p_{1a} = p_1 - p_{v1}$; $p_2 = p_{1w} T_1/T_2 + p_{2v}$. 5.68. Add and evaporate 5.4 kg more water. 5.69. 2.88 kPa; 40°C; 39%. 5.70. 0.31 g; 21.7 kPa. *Hint*: Use Boyle's law. 5.71. 61%.

Sec. 6. Properties of Liquids

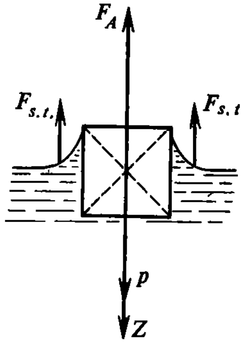
6.1. $4 \cdot 10^{-10}$ m. 6.2. $2.4 \cdot 10^{-9}$ m. 6.3. $3.3 \cdot 10^{22}$ g^{-1}; $6.15 \cdot 10^{21}$ g^{-1}; $1.1 \cdot 10^{22}$ cm^{-3}; $1.65 \cdot 10^{16}$. 6.4. $3.1 \cdot 10^{-10}$ m. 6.5. $3 \cdot 10^{-26}$ kg; 1.63×10^{-25} kg; $3.3 \cdot 10^{-25}$ kg. 6.6. $4.3 \cdot 10^{-25}$ kg; $a/l \approx 0.19$. 6.7. $6 \cdot 10^5$ Pa. 6.8. 15 g. 6.9. 265 mm. 6.10. The soldering is due to the fact that melted solder wets the surfaces of parts being soldered and, on hardening, joints them. But tin cannot wet an oxide film covering the



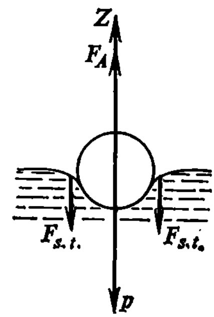
To answer 6.23

surface of aluminium. The strength of the soap film is determined by high viscosity of the suds. 6.11. Water will spread over all the inner surface of the vessel with a bubble of air formed at the center. Mercury will form at the center of the vessel a large spherical drop. 6.12. By $1.6 \cdot 10^{-4}$ J. 6.13. $6.3 \cdot 10^{-3}$ J. 6.14. 0.022 N/m. 6.15. In 24 min. 6.16. 289. 6.17. Water wets wood and gets under the board. Mercury does not wet glass and does not get under the plate; so there is no pressure on the latter from beneath. 6.18. 0.072 N/m. 6.19. 0.08 N. 6.20. 1.4 Pa·s; 2.0 Pa·s. 6.21. $6.3 \cdot 10^{-4}$ Pa·s; $1.19 \cdot 10^{-3}$ Pa·s; $2 \cdot 10^{-3}$ Pa·s; 20 s. 6.22. $4.4 \cdot 10^{-3}$ Pa·s; $1.2 \cdot 10^{-2}$ Pa·s; $5.2 \cdot 10^{-2}$ Pa·s; 52.5 s. 6.23. The volume of the larger bubble will increase; of the smaller, decrease. The equilibrium will come about when the surfaces of both films (sphere and sphere segment) will show the same curvature: $r_2' = r_2$ (see the figure). 6.24. 16 Pa in both cases. 6.25. 1.6 mm. 6.26. 0.78 mm. 6.27. 14.7 cm; 6 cm. 6.28. 4.8 Pa; 60 mm; 120°. 6.29. 7 g. *Hint*: The reaction force $Z = -F_{s.t}$ is directed downward

(see the figure). 6.30. 1.7 mm. *Hint*: Without wetting the reaction force Z is directed upward (see the figure) and equal to $2\sigma(d + l)$. 6.31. Wetting liquid (e.g. water, kerosene) is drawn into capillaries—the pores of fabrick, paper, etc. 6.32. No, soil should be loosened to



To answer 6.29

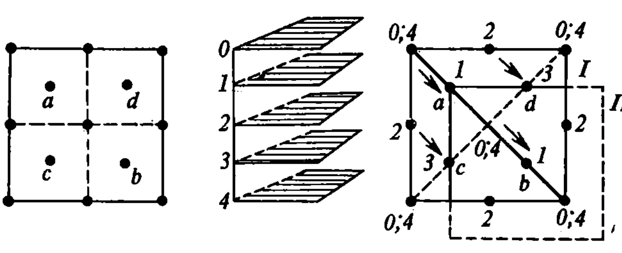


To answer 6.30

prevent soil water from raising along capillaries formed in the upper hard crust. 6.33. 0.022 N/m. 6.34. 15 cm; 6.3 cm; 7 cm; $9.2 \cdot 10^{-7}$ J; $4.6 \cdot 10^{-7}$ J. 6.35. 1.2 mm; 0.6 mm; 0.36 mm. 6.36. By 1.4 cm; by 1.9 cm. 6.37. 12 cm; 4.9 cm; 5.6 cm; 0.074 N/m. 6.38. $x = 2h$ at $l > h$; $x = h + l$ at $l < h$.

Sec. 7. Properties of Solids. Fusion and Crystallization. Deformations

7.1. $1.03 \cdot 10^{22}$ g^{-1}; $3.8 \cdot 10^{21}$ g^{-1}; $2.5 \cdot 10^{22}$ cm^{-3}. 7.2. $5 \cdot 10^{21}$ g^{-1}; $1.1 \cdot 10^{22}$ g^{-1}; $8.4 \cdot 10^{22}$ cm^{-3}. 7.3. (a) 6; 1; $a = d = 3.35 \cdot 10^{-10}$ m;



To answer 7.5

(b) 8; 2; $2.88 \cdot 10^{-10}$ m; $2.45 \cdot 10^{-10}$ m; (c) 12; 4; $4.94 \cdot 10^{-10}$ m; 3.46×10^{-10} m. 7.4. 2 cm^3; 81 cm^3. 7.5. See the figure. The carbon atoms are arranged in five horizontal planes *0-4*. As lattice *II* shifts, its four atoms a, b, c, d stay within the stationary lattice *I* coming over

from plane O to plane *1* (*a* and *b*), and from plane *2* to plane *3* (*c* and *d*). 4; 8. **7.6.** $3.56 \cdot 10^{-10}$ m; $1.54 \cdot 10^{-10}$ m; $1.18 \cdot 10^{24}$. **7.7.** 8; $1+1$; $4.13 \cdot 10^{-10}$ m; 3,970 kg/m^3; 6; $4+4$; $6.28 \cdot 10^{-10}$ m; 2,000 kg/m^3. The lattice of CsCl consists of two simple lattices, and the lattice of KCl of two face-centered cubic lattices with ions of one type in each. The shift was by 1/2 of diagonal (CsCl) and by 1/2 of cube side (KCl). **7.8.** In still waters the distribution of temperatures in cooling is as follows: +4°C at bottom, and 0°C at the surface. As water freezes its density decreases with the ice remaining on the surface. **7.9.** Heat required for ice melting in ice drift is drawn from the surrounding air. **7.10.** In forming the snow (crystallization) heat is released to environment. **7.11.** As compared with lead tungsten has fairly high melting point. **7.12.** Ice, as its density is higher than that of loose snow. To melt the ice about the body being cooled, by far more heat will be required. **7.13.** In freezing, water expands and may break the tubes of radiators and heating systems. **7.14.** No (cf. problem 5.30). **7.15.** (*a*) 327°C; (*b*) 0°C; no melting point. In fusion, lead and wax expand, ice contracts. In the process the wax temperature varies smoothly. **7.16.** The melting of snow and ice, and then the evaporating of water formed occur slowly, so soil has time to saturate with water. **7.17.** $3.32 \cdot 10^4$ J; $3.74 \cdot 10^4$ J. **7.18.** $2.4 \cdot 10^4$ J; $4.6 \cdot 10^4$ J. **7.19.** $5.2 \cdot 10^6$ J. **7.20.** 278.3 K. **7.21.** $3.3 \cdot 10^5$ J/kg. **7.22.** 2.9 kg. **7.23.** 125 g; 193 K. **7.24.** 570 g. **7.25.** All the ice will melt; the water temperature in the vessel will become 4.4°C. In the second case at 0°C water will contain 20.7 g of ice. **7.26.** 16.4 g. **7.27.** Up to 400 K. **7.28.** 5 h. **7.29.** 0.6 kg. **7.30.** 930 kg. **7.31.** 1.4 cm. **7.32.** 6.3 kg. **7.33.** Segments AB, CM, KL and KN represent respectively the variation of temperature of ice, water formed from it, calorimeter and water contained in it initially, segment BC—the process of ice melting, segments OB, Cm, $Ol = nm$, On—the quantities of heat absorbed or released in these processes. If the ice temperature were 0°C, the curve had no section lying below the abscissa axis. **7.34.** Energy required to melt the crystals is taken from the solvent (water), the temperature of the latter decreasing. Dissolved; begins to grow. **7.35.** The freezing point for NaCl solution is much lower than 0°C. **7.36.** In freezing saline water crystals of pure ice separate out; the salt stays in the solution with the result that its concentration increases. **7.37.** If the alloy contains 8% of zinc, it melts at 200°C. In cast iron carbon atoms occupy more sites in the lattice than in iron. Such an irregularity of the lattice simplifies the changing of cast iron into liquid state. **7.38.** In frying it is water contained in food and not oil that boils. It is only when all the water has evaporated that the temperature rises over 100°C. **7.39.** Ice begins to melt, producing water in which the salt dissolves, then the heat required for melting is taken from the solution. The resultant mixture remains liquid till the temperature is reached at which it solidifies (it is −20°C, if the snow and common salt are taken in the ratio 2 : 1). **7.40.** See the answer to the preceding problem. The melting of snow and dissolving of salt in the water formed occur at a temperature that is much lower than 0°C. Cold is felt more on the snow mixed with salt. **7.41.** Sections *I*, *II*, *III* correspond to different states of aggregation: vapour or gas, liquid, solid (crystal);

curves KN, LN and MN represent the processes in which gas, liquid and crystal occur in pairs in equilibrium; N is the triple point for the substance (for water $t_N \approx 0.01°C$, $p_N \approx 613.2$ Pa). Straight lines *1-1* and *2-2* represent isobaric, and *3-3* and *4-4* isothermal transitions from one state into another. **7.42.** $1.83 \cdot 10^7$ J. **7.43.** 4.1 kg. **7.44.** 0.51 kg. **7.45.** 13 kg. **7.46.** 291 K. **7.47.** 508 g. **7.48.** 60 g. **7.49.** 78%. **7.50.** 0.25 kg. **7.51.** 916 kJ; 70%. **7.52.** 24 km. **7.53.** 12%; 52%. **7.54.** 2.2 km/s. **7.55.** 56%. **7.56.** 2.54 km/s. **7.57.** 90.5 kg; 54 kg. **7.58.** 14.0 t. **7.59.** 420 kg. **7.60.** 27%. **7.61.** 52 kg. **7.62.** 78 K. *Hint*: The heats liberated in the boiling of liquid nitrogen and the melting of ice are proportional to the temperature difference inside and outside the vessels: $Q = k\Delta T$; the factor k in both cases is equal. **7.63.** Longitudinal compression; longitudinal extension; bending; torsion; shear. **7.64.** Will increase; will decrease. **7.65.** Torsion. **7.66.** Compression; compression. **7.67.** Well resistant to compression and extension. **7.68.** 1.9 kN. **7.69.** $2.3 \cdot 10^{-2}$ m. **7.70.** $\sigma = ma/(2S)$. **7.71.** $n = 2.45$. **7.72.** $1.6 \cdot 10^3$ kg; $1.5 \cdot 10^3$ kg. **7.73.** $1.03 \cdot 10^{-4}$ m^2. **7.74.** $4.3 \cdot 10^2$; 9.1. **7.75.** $\sigma = \rho g l$. Does not depend. **7.76.** $7.85 \cdot 10^3$ m; $9.04 \cdot 10^3$ m. **7.77.** $3.53 \cdot 10^5$ Pa. At the foundation of the brick wall the brickwork must be stronger. **7.78.** $4.25 \cdot 10^2$ m. **7.79.** Equal; the second rod. **7.80.** The tensile strain of the first wire is four times, and tensile deformation twice smaller than that of the second one. **7.81.** $3.0 \cdot 10^7$ Pa. **7.82.** The residual deformation appears at $F \geqslant 2.2 \cdot 10^2$ N; here $\Delta l \geqslant 4.0 \cdot 10^{-3}$ m. **7.83.** $2.2 \cdot 10^8$ Pa. **7.84.** At stress $\sigma \leqslant 3 \cdot 10^7$ Pa aluminium is elastic, at $\sigma > 3 \cdot 10^7$ Pa plastic; yes. **7.85.** No; no. **7.86.** $4.0 \cdot 10^7$ Pa; $2.0 \cdot 10^{11}$ Pa; $5.0 \cdot 10^{-2}$ J. **7.87.** $S = 2.0 \cdot 10^{-6}$ m^2; $\sigma = 2.4 \cdot 10^7$ Pa. **7.88.** $6.25 \cdot 10^{-4}$ m; $\sigma = 5.1 \cdot 10^8$ Pa, $\sigma < \sigma_l$, hence the wire is strong enough for this load. **7.89.** $2.83 \cdot 10^{-4}$ m. **7.90.** $1.0 \cdot 10^{-3}$. **7.91.** $4.0 \cdot 10^{-1}$ J. **7.92.** 5.0 J. **7.93.** $4.8 \cdot 10^{-3}$ J. **7.94.** 40 N; $8.0 \cdot 10^{-1}$ J. **7.95.** $\Pi_{max} = 2(mg)^2/k$. **7.96.** $4.8 \cdot 10^{-2}$ J; $9.6 \cdot 10^{-2}$ J; $2.4 \cdot 10^{-2}$ J. **7.97.** The copper wire has higher potential energy, as its tensile strain is larger.

Sec. 8. Thermal Expansion of Bodies

8.1. In summer, railway rails heat stronger, and consequently expand due to poor heat conductivity of wooden sleepers and ambient air, whereas the tramway rails exhibit good heat conductivity and release heat to ground. **8.2.** Yes; the heated arm will be heavier. **8.3.** As temperature is elevated the center of gravity of the rod will descend, and the center of gravity of mercury columns will rise. It is possible to design all the parts of the system so that its center of gravity will retain its position at any temperature, thus not interfering with the clock operation. **8.4.** 2,002 m. **8.5.** 0.55 m. **8.6.** 10,013 m. **8.7.** $1.9 \cdot 10^{-5}$ K^{-1}. **8.8.** 0.027 mm. **8.9.** By 31 mm; by 41 mm. **8.10.** 67.5 mm. **8.11.** 12.011 m; 12.011 m; 12.006 m. **8.12.** Yes; the gap between the wheel and the ring will be 0.46 mm. **8.13.** Up to 803 K. **8.14.** 400.8 mm. **8.15.** By 514 K. **8.16.** 15.16 m. **8.17.** By 53 K. **8.18.** $8.0 \cdot 10^7$ Pa. **8.19.** By 35 K. **8.20.** Because their coefficients of linear expansion are similar. **8.21.** It must have the same coefficient

of linear expansion as glass (platynite). 8.22. By 50 cm^2. 8.23. 12,400 mm^2. 8.24. 125°C. 8.25. 805.8 cm^3. 8.26. By 21.3 cm^3. 8.27. By 3.8 cm^3. 8.28. 12.02 l; 11.98 l. 8.29. 7 740 kg/m^3; 7,820 kg/m^3. 8.30. By 16.2 cm^3; by 150 K. 8.31. By 5 cm. 8.32. 2 mm^2; 8 m. 8.33. By 21.6 mm; 6.3 m. 8.34. $4.3 \cdot 10^{-4}$ K^{-1}. 8.35. By 25 K. 8.36. 6.18 m; 303 K. 8.37. 309 K. 8.38. By 54 m^3. 8.39. $5.31 \cdot 10^{-4}$ K^{-1}; aluminium. 8.40. 3.82 cm^3. 8.41. Up to 423 K. 8.42. $3 \cdot 10^{-5}$ K^{-1}. 8.43. $1.8 \cdot 10^{-4}$ K^{-1}. 8.44. $1.8 \cdot 10^{-4}$ K^{-1}; $1 \cdot 10^{-3}$ K^{-1}. 8.45. 13,240 kg/m^3; 13,680 kg/m^3. 8.46. 750 kg/m^3; 890 kg/m^3. 8.47. 0.42 MJ; 0.15 MJ. 8.48. 7.55 and 7.25 N. *Hint*: The buoyancy force F varies with the density of liquid $\rho = \rho_0 (1 + \beta_l \Delta t)$ and the volume of the solid body $V = V_0 (1 + \beta_s \Delta t)$. 8.49. $5 \cdot 10^{-4}$ K^{-1}; 1 260 kg/m^3. *Hint*: By the formula $F_1/F_2 = [1 + \beta_l (t_2 - t_0)]/[1 + \beta_s (t_2 - t_1)]$ we find β_l.

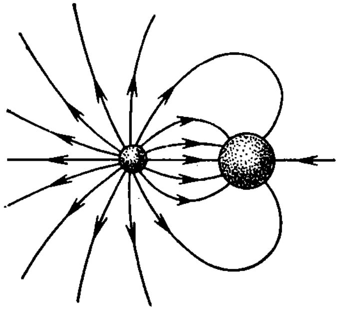
Sec. 9. Interaction Between Charges. Charge Conservation Law. Coulomb's Law

9.1. Yes, if the stick has a handle of insulator. 9.2. The charge sign of the electroscope will be negative, if when touched by the electrified ebonite rod the lobes become separated by a large angle. 9.3. Yes, if a charged conductor is placed inside a hollow isolated conductor and these are brought to contact. 9.4. With unlike charges. 9.5. Electrify one ball, then bring it to contact with the other; electrify the connected balls at a distance and disconnect them in the presence of the inducing charge. 9.6. Under weightlessness the balls will only be subjected to repulsive forces; the balls will be separated by a distance $2l$. 9.7. As the static charge builds up a spark can occur, but colophony makes for the accumulation of the charge; to divert the static charge to ground. 9.8. To divert to ground the accumulated static charge. 9.9. To protect from explosion hazard in triboelectrification. Such gun powder is practically grounded. 9.10. $8.0 \cdot 10^{-14}$ C; $1.5 \cdot 10^5$. 9.11. $1.5 \cdot 10^8$; $-2.4 \cdot 10^{-11}$ C; $9 \cdot 10^{-9}$ N. 9.12. $4 \cdot 10^4$ C. 9.13. Will increase by 64 times. 9.14. $9 \cdot 10^9$ N; $0.1 \cdot 10^9$ N. 9.15. $2.5 \cdot 10^{-6}$ C/m^2. 9.16. $1 \cdot 10^{-6}$ C. 9.17. Will increase. 9.18. $2 \cdot 10^{-5}$ C. 9.19. $7.4 \cdot 10^{-2}$ N; 0.30 m. 9.20. $1 \cdot 10^{-5}$ C; $3 \cdot 10^{-5}$ C; 0.02 m. 9.21. 3.2. 9.22. 1.8×10^{-11} C^2/(N$\cdot$m^2); $2.0 \cdot 10^{-5}$ C. 9.23. $1.0 \cdot 10^{-7}$ C; $-4.0 \cdot 10^{-7}$ C. 9.24. $0.8 \cdot 10^{-3}$ N; $1.44 \cdot 10^{-3}$ N. 9.25. $F_2 > F_1$. *Hint*: The inequality $(q_1 - q_2)^2 > 0$ becomes $\frac{(q_1 + q_2)^2}{4} > q_1 \cdot q_2$ which gives an insight into the force ratio. 9.26. $1.7 \cdot 10^{-6}$ C. 9.27. $1.06 \cdot 10^{-7}$ C. 9.28. At 5.3 cm away from the smaller charge; the equilibrium will not be stable; will not be violated. 9.29. $2 \cdot 10^{-9}$ N; 0.05 m/s^2. 9.30. $6.0 \cdot 10^{-3}$ kg. 9.31. $0.86 \cdot 10^{-13}$ C; as the electric forces are about $0.4 \cdot 10^{43}$ times stronger than gravitational ones. 9.32. $9.8 \cdot 10^{-18}$ C. 9.33. $2.7 \cdot 10^{-6}$ C/kg. 9.34. $F = \frac{Qqh}{4\pi\varepsilon_0 (h^2 + r^2)^{3/2}}$. *Hint*: The resultant force due to the ring on the point charge q may be thought of as the vector sum of forces due to separate elements of the charged ring. In the composition of forces the components are taken into account which are

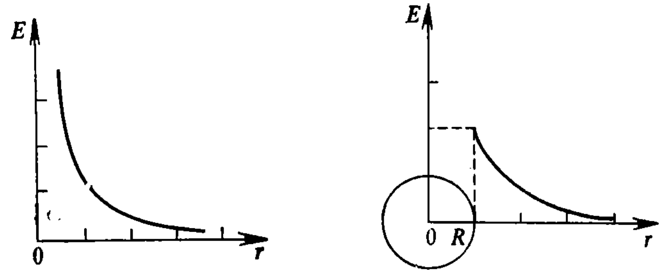
directed along the ring axis; the components directed normal to the axis add up to zero owing to the symmetry. 9.35. $2.8 \cdot 10^{-8}$ C; 8.12×10^{-3} N.

Sec. 10. Electric Field

10.1. If at the point a negative charge is placed. 10.2. Between the wire and tube an electric field is produced that makes the ionized smoke particles settle down on the chimney. 10.3. If the jobs are connected with the other pole of the high-voltage source (see answer 10.2). 10.4. See the figure. 10.5. $E_1/E_2 = \sigma_1/\sigma_2$. 10.6. The electric field strength at the sharp end of the conductor is higher than at the blunt end; under the influence of the strong electric field electrons are removed from gas molecules into ambient air, with the result that the penetrated ions travel from the conductor with the like charge, thus entraining neutral gas molecules to produce the "electric wind". 10.7. The field will be both inside and outside the sphere. The negative charge will be on the inner surface of the sphere; and the positive, on the outer. On moving the charged ball the field inside the sphere will change, and as a charged body is brought near the sphere the electric field outside the sphere will change. 10.8. The absence of field inside the conductor. To screen it from external electric fields. 10.9. On



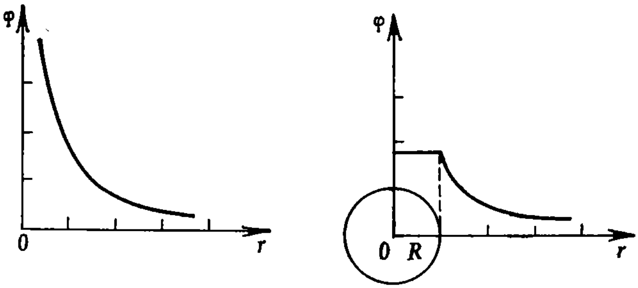
To answer 10.4



To answer 10.16

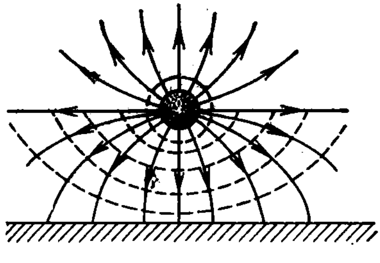
electrons at any points of a uniform electric field equal forces are acting. 10.10. $7.5 \cdot 10^4$ N/C. 10.11. $1.8 \cdot 10^{-3}$ N. 10.12. $8 \cdot 10^5$ N/C. 10.13. $E_1 = 5.52 \cdot 10^6$ N/C. Spherical surfaces with the common

center. 10.14. $4 \cdot 10^4$ N/C; $7.2 \cdot 10^{-5}$ N. 10.15. 0; 0; 245 N/C. 10.16. See the figure. 10.17. $5.1 \cdot 10^{11}$ N/C; $2.2 \cdot 10^6$ m/s. 10.18. 3,300 N/C. 10.19. $3.6 \cdot 10^{-7}$ C; by 2.6 cm. 10.20. Water; $7.2 \cdot 10^{-10}$ C^2/N$\cdot$m^2. 10.21. Unlike the induced ones, the polarization charges cannot be separated, there-



To answer 10.45

fore these are often called "bound". 10.22. $4 \cdot 10^3$ N/C. 10.23. 0; $2q/4\pi\varepsilon_0 a^2$. 10.24. 0; yes. 10.25. $9.2 \cdot 10^5$ N/C. 10.26. $E_1 = E_2 = 2.8 \cdot 10^4$ N/C. 10.27. By 7°. 10.28. 245 N/C; vertically downward. 10.29. $1 \cdot 10^{-8}$ C. 10.30. 0.02 m/s^2. 10.31. By $0.8 \cdot 10^{-3}$ m/s^2. 10.32. $1.8 \cdot 10^5$ m/s^2. *Hint*: As the rod accelerates the charges inside it sepa-



To answer 10.53

rate—electrons are displaced due to inertia until the field formed imparts them an acceleration corresponding to that of the rod. 10.33. $5.3 \cdot 10^8$ m/s^2; $2 \cdot 10^4$ m/s; $0.4 \cdot 10^{-5}$ s. 10.34. $1 \cdot 10^{-7}$ s. 10.35. 11 N/C; $0.25 \cdot 10^{-5}$ s. 10.36. $1.7 \cdot 10^{-4}$ N. 10.37. $1.1 \cdot 10^5$ N/C; $2 \cdot 10^{-6}$ C/m^2. 10.38. $E = \sigma/\varepsilon_0\varepsilon$. 10.39. $E_{in} = 5.6 \cdot 10^3$ N/C; $E_{out} = 3.4 \cdot 10^3$ N/C. 10.40. 11 N. 10.41. 5 V. 10.42. 6,000 V. 10.43. $3 \cdot 10^{11}$. 10.44. 5.4×10^3 V; $5.4 \cdot 10^3$ V; $2.7 \cdot 10^3$ V. 10.45. See the figure. 10.46. 3.6×10^4 N/C; $7.2 \cdot 10^3$ V. 10.47. $1.6 \cdot 10^{-9}$ C; $6 \cdot 10^{-9}$ J. 10.48. $5.9 \cdot 10^5$ C. 10.49. $4 \cdot 10^{-7}$ J. 10.50. 220 V; $2.4 \cdot 10^{-9}$ C. 10.51. Zero. 10.52. To screen from external fields; the body is earthed to charge the potential

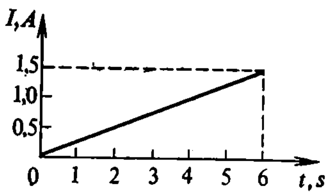
with reference to the Earth. Yes. Equal potential at all points. 10.53. See the figure. 10.54. 0.15 m. 10.55. $2.0 \cdot 10^{-10}$ C. 10.56. 52 V; $2.6 \times \times 10^{-7}$ J. 10.57. 0.1 m; $2.0 \cdot 10^{-8}$ C; 1 800 V. 10.58. 0; 4 500 V/m; 330 V/m; —750 V; —450 V; —125 V; 10.59. $\frac{|q_1|+|q_2|}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{\sqrt{r^2+d^2}}\right)$. 10.60. 65 V. 10.61. 0.27 J. 10.62. 1 000 V; 0.15 N; $6 \cdot 10^{-3}$ J. 10.63. 0.005 m; upward; 0.12 m/s. 10.64. 10. 10.65. $1.1 \times \times 10^7$ m/s; at 260 V. 10.66. $4.2 \cdot 10^3$ kV. 10.67. 0.35 m/s. 10.68. $1.8 \times \times 10^{-6}$ J. 10.69. $3.5 \cdot 10^4$ V/m. 10.70. $2.4 \cdot 10^{-15}$ J. 10.71. 0. 10.72. $1.76 \cdot 10^{11}$ C/kg.

Sec. 11. Capacitance and Capacitor

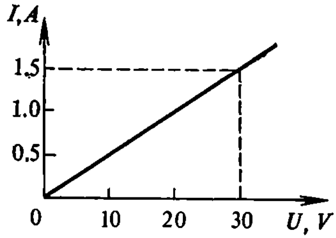
11.1. The potential of the smaller ball is larger; the potential of the ball with larger charge is larger. 11.2. Due to the reduction in capacitance. 11.3. No. In the presence of other conductors their capacitances will change. 11.4. $7.1 \cdot 10^{-4}$ F. 11.5. $9 \cdot 10^6$ km. 11.6. $8 \cdot 10^{-6}$ F. 11.7. $1.7 \cdot 10^{-12}$ F; $1.7 \cdot 10^{-6}$ mkF; 1.7 pF. 11.8. $5 \cdot 10^{-12}$ F; $4.5 \cdot 10^{-12}$ m. 11.9. $4 \cdot 10^4$ V; $4 \cdot 10^{-2}$ m. 11.10. The charges will transfer from the ball with higher potential to the ball with lower potential: 300 V; 900 V; 780 V; 780 V; $1.3 \cdot 10^{-9}$ C; $5.2 \cdot 10^{-9}$ C. 11.11. $8.4 \cdot 10^{-9}$ C; 500 V. 11.12. 0.04 μF. 11.13. When the circuit is touched a discharge current starts; after the circuit containing capacitors is disconnected, the capacitors should be discharged with a rod having an isolating handle. 11.14. $1.2 \cdot 10^{-8}$ F. 11.15. 100 pF; if the separation between the plates is increased. 11.16. $3.2 \cdot 10^{-6}$ C; 320 V; $2.56 \cdot 10^{-4}$ J; $5.12 \cdot 10^{-4}$ J. 11.17. 160 pF; $4.8 \cdot 10^{-8}$ C; $7.2 \cdot 10^{-6}$ J. 11.18. 200. 11.19. $1.44 \cdot 10^{-5}$ J. 11.20. $4.5 \cdot 10^{-3}$ J. 11.21. $1.2 \cdot 10^{-4}$ C; 0.24 J. 11.22. 15 kV. 11.23. 1.5 kV; $4 \cdot 10^{-4}$ J. 11.24. Increased by 1,300 V. 11.25. 0.80 μF; $1.6 \cdot 10^{-3}$ C. 11.26. Yes; in the first case the capacitors should be connected in series; in the second, in parallel. 11.27. From 7.5 to 500 pF. 11.28. 150 V; 150 V; in the first $9 \cdot 10^{-4}$ C, in the second $1.5 \cdot 10^{-3}$ C. 11.29. $1.8 \cdot 10^{-8}$ F. *Hint*: A capacitor with two various dielectrics can be thought of as a bank of capacitors connected in series. 11.30. The oxide film serving as a dielectric in the capacitor is fairly thin. 11.31. Will decrease by ε times. 11.32. $1.6C$. 11.33. The potentials of the spheres before the connection are $q_1/4\pi\varepsilon_0 R_1$ and $q_2/4\pi\varepsilon_0 R_2$, after the connection the charges on the spheres became Q_1 and $Q_2 = q_1 + q_2 - Q_1$, and the total potential will be $Q_1/4\pi\varepsilon_0 R_1 = (q_1 + q_2 - Q_1)/4\pi\varepsilon_0 R_2$; hence $Q_1 = R_1(q_1 + q_2)/(R_1 + R_2)$; $\varphi = (q_1 + q_2)/4\pi\varepsilon_0 (R_1 + R_2)$. 11.34. $2.3 \cdot 10^{-3}$ J. 11.35. 1.25 mC; 0.16 J. 11.36. 5 μF; 1.1 mC. 11.37. 0.8 μF; 44 V; 176 V. 11.38. 0.75 μF; $9 \cdot 10^{-5}$ C; $3 \cdot 10^{-5}$ C; $6 \cdot 10^{-5}$ C; 90 V; 30 V; 30 V. 11.39. $1.2 \cdot 10^{-6}$ C. 11.40. $1 \cdot 10^{-6}$ F; 240 V; 0.03 J. 11.41. Decreased by $1.4 \cdot 10^{-7}$ J. 11.42. $7 \cdot 10^{-8}$ C; $2.1 \cdot 10^{-7}$ C; 3; $1.3 \cdot 10^{-2}$ N. 11.43. 200 V; 150 V. 11.44. 420 V; 0.17 J; no. 11.45. 40,000 V/m; 84 V; 0.021 J. 11.46. $\sqrt{2eE\varepsilon_0 S/(m_e C)}$. 11.47. $2.2 \cdot 10^2$ J/m^3. 11.48. $2\varepsilon/(\varepsilon + 1)$. 11.49. 6.5 V.

Sec. 12. Electric Current in Metals. Direct Current Laws

12.1. 30 mA; $6.2\cdot10^{15}$. 12.2. $2.14\cdot10^{6}$ J; 2.7 A. 12.3. $1.25\cdot10^{-4}$ m/s. 12.4. 600 kJ; 8A; 15.6 Ω. 12.5. See the figure; 4.5 C. 12.6. 0.02 μF. 12.7. 600 C; 132 kJ; 110 Ω. 12.8. 2 A. 12.9. With such a bus the cooling surface is larger than with a round wire. 12.10. 30 V. 12.11. 17 V; 10 V. 12.12. 200 A/m². 12.13. $2\cdot10^{6}$ A/m². 12.14. 4.7 V. 12.15. See the figure. 12.16. 22 Ω. 12.17. 7.2 m; 9 V. 12.18. 0.52 mm. 12.19. 17 Ω; 20 m. 12.20. 2.3 N. 12.21. 750 kg. 12.22. 710 m; 0.71 mm. 12.23.



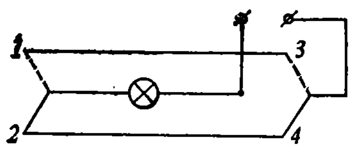
To answer 12.5



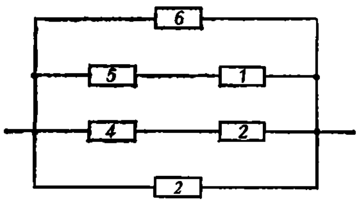
To answer 12.15

0.34 Ω. 16 mm². 12.24. 16; $D_a\rho_m/D_m\rho_a$ (D—density, ρ—resistivity). 12.25. 28. 12.26. $1.1\cdot10^{-6}$ Ω·m; 33 m. 12.27. See the figure. 12.28. 6 Ω; 7.6 m. 12.29. Because the filament resistance in cold condition is smaller, and hence the starting current through the lamp is higher than the operating current. 12.30. To ensure better contact. 12.31. 50 m. 12.32. $5.3\cdot10^{-8}$ Ω·m. 12.33. 2 A; 6 Ω. 12.34. 46 Ω. 12.35. At 118°C. 12.36. 1700°C. 12.37. By 0.48 Ω. 12.38. 1800°C. 12.39. $5\cdot10^{-3}$ K^{-1}. 12.40. -0.0002 K^{-1}. By the negative temperature coefficient of resistance of coal. 12.41. 0.004 K^{-1}. 12.42. 0.0047 K^{-1}. 12.43. 470.2 Ω. 12.44. 60 Ω; 20 V; 40 V; 60 V. 12.45. 100 V; 20 V; 30 V; 50 V. 12.46. 3.2 Ω; 4.8 Ω. 12.47. 44 Ω; 5 A; 50 V; 90 V; 220 V. 12.48. 20 Ω. 12.49. 10 A. 12.50. 4.75 Ω; 45 m. 12.51. 0.5 A; 200 Ω. 12.52. 0.33 A; 0.33 V. 12.53. 13 kΩ. 12.54. 6 kΩ. 12.55. By 10 times. 12.56. 20 kΩ. 12.57. 0.2 V; 0.5 V; 2 kΩ. 12.58. 0.36 A. 12.59. 48 V. 12.60. 120 V; 90 Ω; 150 Ω. 12.61. 5.37 kN; 260 V; no. 12.62. 120 Ω; in parallel. 12.63. 16 Ω. 12.64. By 8. 12.65. Two series groups with two parallel in each, or two parallel groups with two series in each. 12.66. Connect two conductors in parallel and one in series. 12.67. 11 Ω; 10 A. 12.68. $R_1 = R_2 = 10$ Ω. 12.69. 15 Ω; 60 V. 12.70. 46 Ω; 138 Ω; 230 Ω. 12.71. See the figure. 12.72. 18 Ω. 12.73. 10 Ω; 7.5 Ω. 12.74. 6 Ω. 12.75. 6 Ω; $I_1 = 2$ A; $I_2 = I_4 = 1$ A; $I_3 = I_5 = I_6 = 0.5$ A. 12.76. 3.6 Ω; 5 A; 1.7 A, 2.5 A, 0.8 A. 12.77. 25 A; 127 V. 12.78. 32 A; 6.4 V. 12.79. 10 Ω; 22 A. 12.80. 56; 125 V. 12.81. 20 Ω; 2.75 A; 5.5 A; 1.83 A; 11 V; 16.5 V; 27.5 V; 55 V. 12.82. 3.3 Ω. 12.83. 20 A. 12.84. 29 Ω. 12.85. 0.28 m. 12.86. 10 V; 5 Ω. 12.87. 5 A; 15 V. 12.88. 1.55 V; 0.5 Ω. 12.89. 0.2 Ω; 1.9 V. 12.90. 2 Ω;

1.2 V. **12.91.** 1.4 V; 0.25 Ω. **12.92.** 20 A; 132 V. **12.93.** 16.2 Ω. **12.94.** 160 A; 11 V. **12.95.** Will decrease. **12.96.** 45%. **12.97.** 87%. **12.98.** 1.36 V. **12.99.** 2.5 Ω; 0.75 V. **12.100.** 0.5 A; 5.5 V. **12.101.** 0.53 mm. **12.102.** 3 Ω; 0.9 V. **12.103.** 33 A; by 1.8 times. **12.104.** 2 A; 3 Ω. **12.105.** With equal external and internal resistances; 0. **12.106.** 4.8 Ω; 3000 V/m. **12.107.** $3 \cdot 10^{-6}$ C. **12.108.** 2.75 Ω; 0.6 A; 3.5 m. **12.109.** 3 A. **12.110.** 1.2 A. **12.111.** 4 V. **12.112.** 116 V. **12.113.** 2.5 mm^2. **12.114.** 126 V. **12.115.** (1) 5 A, 0, 0; (2) 0, 0, 6 V; (3) 1.4 A, 0.9 A, 4.2 V. **12.116.** 1.8 A. **12.117.** 12. **12.118.** 40 V; 10 Ω. **12.119.** 4.5 V;



To answer 12.27



To answer 12.71

2 Ω. **12.120.** 6.5 m; 11.2 V. **12.121.** 1.4 V; 0.5 Ω; 2.8 A. **12.122.** 2 Ω; 1.5 A; 12 V. **12.123.** 5 A; 120 V; 1 V; 121 V. **12.124.** 149.4 V. **12.125.** 3.3 A. **12.126.** 5 A; 4.8 Ω. **12.127.** $4 \cdot 10^{-6}$ A. **12.128.** 0.37 A; 4.3 V. **12.129.** 0.72 A; 0.43 A. **12.130.** 3.9 Ω; 1.86 m. **12.131.** 2.8 A; 1.4 A; 1.4 A; 3.3 V; 2.5 V; 0.8 V. **12.132.** In series. **12.133.** With series connection. **12.134.** 1.4 V; 0.2 Ω. **12.135.** 5. **12.136.** 0.55 A. **12.137.** 4.5 V; 1.5 A; 0.5 A. **12.138.** 3 A. **12.139.** 12.4 V. **12.140.** 2 V; 0.04 Ω. **12.141.** 1.5 Ω. **12.142.** 1.8 Ω; 0.15 Ω; 55.2 V. **12.143.** 1 Ω. **12.144.** 1.2 V. **12.145.** 0.5 A; 2.5 V; 1.3 V; 1.2 V. **12.146.** 0.91 A; 0.39 A; 0.52 A. **12.147.** 0.47 A; 12 A; 0.59 A. **12.148.** $\mathscr{E}_1 = \mathscr{E}\ (R_1 + R_2)/R_2$. **12.149.** In the first element the internal resistance is three times higher. **12.150.** 1.92 V. **12.151.** 0; 0. **12.152.** 0.4 Ω; 1.5 A; 2 A; 3.5 A.

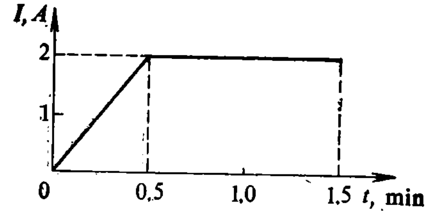
Sec. 13. Work. Power. Heating Effect of Current

13.1. $1.2 \cdot 10^6$ J; 3 A. **13.2.** 0.65 kW·h. **13.3.** 0.8 A; 100 V; 3.6×10^6 J. **13.4.** 48 V; 72 V; 300 Ω; 200 Ω. **13.5.** 20 kW; $3.6 \cdot 10^7$ J. **13.6.** 484 Ω; 0.46 A; 5 kW·h. **13.7.** 2 rub. 16 kop. **13.8.** 6,600 kW; $3.3 \cdot 10^4$ kW·h. **13.9.** $2.9 \cdot 10^6$ J. **13.10.** 5.5 kW·h; 22 kop. **13.11.** 180 W; 1.5 A; 1.44 kW·h. **13.12.** 270 MJ; 252 MJ; 18 MJ. **13.13.** 24.2 Ω; 3.7 Ω. **13.14.** 8.2 A; 6.1 Ω. **13.15.** 103 rub. 60 kop. **13.16.** 760 A. **13.17.** 9 m. **13.18.** 19 W; 25 W; second; 53 V; 67 V. **13.19.** 75 V, 6.25 Ω; 540 W; 900 W. **13.20.** 5 V; 0.13 Ω. **13.21.** 0.96 kW; 7.7 kop. **13.22.** 18 A; 1.8 Ω. **13.23.** 1.25 A; 40%. **13.24.** 9.6 mm^2. **13.25.** 40. **13.26.** 16 V. **13.27.** 150; 7.2 kW. **13.28.** 380 A. **13.29.** 124 W; 86%. **13.30.** 10.5 mm^2. **13.31.** 34 A; 226 V; 0.20 kW. **13.32.** 12 m/s. **13.33.** 17 kW; 77 A. **13.34.** 2.2 kW; 0.6 kW; 72%. **13.35.** 219 kN. **13.36.** 10.3 kN. **13.37.** 8.2 kW; 0.14 kW·h; 37 A. **13.38.** 280 kN. **13.39.** No, the energy consumption will increase. **13.40.** To

protect electric circuits from short circuits; circuit. 13.41. Will double. 13.42. The connection of the heavy-current appliance decreases the total resistance; hence the current grows and so does the voltage drop across the conducting wires. 13.43. The lamp consuming the lower power has the higher resistance and will be brighter. 13.44. The boiler gives away less heat to air than to water with the result that it can burn out. 13.45. Because as the temperature is increased the radiated energy grows too. The filament temperature increases. 13.46. 7 200 J; 1 714 cal. 13.47. 50 Ω. 13.48. 12.6 kJ. 13.49. 300 J. 13.50. 4 Ω. 13.51. 3.3 kg; 600 W. 13.52. 84%; 19 Ω. 13.53. 24.7 min. 13.54. 250 W; 58 Ω. 13.55. 1 800 J; 3 000 J; 700 J; 420 J. 13.56. 0.8 kop; 4.4 A; 77%. 13.57. In parallel. 13.58. 56 min; 7 min. 13.59. 15 Ω. 13.60. 1.3 m. 13.61. 191 kJ; 73%. 13.62. 0.3 mm. 13.63. 22.6 Ω; 5.3 A. 13.64. 0.85 l; 24 Ω. 13.65. By 4.3 K. 13.66. 2.4 min. 13.67. 0.93 kg. 13.68. 533 kJ; 4.6 Ω; 11 m. 13.69. $2.4\cdot10^3$ J/(kg·K). 13.70. 153. 13.71. 0.0062 K^{-1}. 13.72. 1.5 A; 4.5; 6.8 A; 12.8 A; 2.83 kW. 13.73. 6 Ω; 1 A; 24 V; 49.6 W; 48 W. 13.74. 2.5 A. 13.75. The 25 W lamps should be connected in parallel with the third one connected to them in series; 0.23 A; 0.23 A; 0.45 A. 13.76. 1.2 Ω. 13.77. 0.5 A; 2.4 Ω. 13.78. 3 V; 2 Ω. 13.79. 21.5 Ω; 61 m. 13.80. 3 min; 13 min. 13.81. By 0.006°. 13.82. By 1.9°.

Sec. 14. Electric Current in Electrolytes. Galvanic Cells and Storage Batteries

14.1. In the process of dissociation of molecules both positive and negative ions are generated. 14.2. No. 14.3. (1) The amounts of the substance liberated at the electrodes in different baths will be equal. (2) Yes; no. 14.4. The number of ions per unit volume, and their mobility. 14.5. Sulfuric acid takes part in secondary reactions. 14.6. Wet hands. Water on hands contains salts which in dissociation produce ions so that the solution becomes a good conductor. 14.7. Because anhydrous sulfuric acid is not a conductor, however, iron contains impurities, therefore in the presence of conducting diluted acid local currents can occur resulting in the container corrosion. 14.8. 268 C. 14.9. Bivalent nickel. 14.10. $3{,}330\cdot10^{-7}$ kg/C; 1%. 14.11.



To answer 14.11

168 mg. See the figure. 14.12. 33 mg. 14.13. At projections on the surface of a metal the electric field strength is higher than at the smooth surface, therefore when the current is reversed the metal turns to

anode, the projections and irregularities dissolve faster and the metal surface levels out. 14.14. 1.6 g. 14.15. No. 14.16. 6 A. 14.17. In 25 h. 14.18. 30 mg. *Hint*: The problem can be solved by (*a*) plotting the variation of current with time, (*b*) using the integrating procedures. 14.19. 0.6 g. See problem 14.18. 14.20. To achieve smoother cover. Anode dissolution. 14.21. 3.4 kg. 14.22. 6.1 h; 5.3 W·h. 14.23. 200 rub. 14.24. 0.054 kg. 14.25. $16.3\cdot10^6$ J (4.5 kW·h); 9 kop. 14.26. 5.0 V. 14.27. 0.6 V. 14.28. These metals are more resistant to corrosion. 14.29. 0.043 mm. 14.30. 1.9 h. 14.31. 0.6 μm/s. 14.32. $5.6\cdot10^{17}$. 14.33. In 67 min. 14.34. 0.1 A. 14.35. 20 W. 14.36. 35 W. 14.37. 5 A; 6 V; 60 W·h. 14.38. 48 A. 14.39. $2.09\cdot10^{-4}$ m^3; $1.04\cdot10^{-4}$ m^3; 0.02×10^{-3} kg; $1.48\cdot10^{-4}$ kg. 14.40. 1.3 Ω. 14.41. 297 K. 14.42. 0.093×10^{-6} kg/C; 0.024 kg. 14.43. $0.24\cdot10^{-8}$ kg/C; $3.67\cdot10^{-7}$ kg/C; 9.4×10^{-8} kg/C. 14.44. $0.083\cdot10^{-6}$ kg/C; 0.46 mg. 14.45. 176 A/m^2. 14.46. Zinc has higher solution tension than iron, therefore in the voltaic cell formed (zinc, iron, sea water) it becomes the negative electrode and dissolves, whereas hydrogen liberates on the hull. 14.47. Because impurities intensify the discharging of the storage battery. 14.48. 0.5 A; $4\cdot10^{-5}$ m; 88%. 14.49. $2.45\cdot10^5$ J. 14.50. $2.34\cdot10^{21}$. 14.51. $7\cdot10^{17}$. 14.52. 3. 14.53. $1.67\cdot10^{-27}$ kg. 14.54. In 1 min, 0.1 g, 0.8 g. 14.55. $9\cdot10^{-3}$ kg. 14.56. $2.8\cdot10^{-4}$ kg. 14.57. 3 A. 14.58. 158 C. 14.59. $4\cdot10^9$ C. 14.60. $7.9\cdot10^4$ C; $1.6\cdot10^5$ J. 14.61. Distilled water, and not electrolyte, must be added. 14.62. $4.9\cdot10^6$ J. 14.63. 83%. 14.64. 54 A·h; the capacitance does not change. 14.65. 162 W; 81 W; 243 W.

Sec. 15. Electric Current in Gases. Thermionic Emission. Electric Current in Vacuum

15.1. In ionizing the liquid crystals no free electrons are formed, and in gases, apart from ions, free electrons come into being. 15.2. Because along with the ionization there occurs the recombination of ions. 15.3. $3.92\cdot10^{-18}$ J. 15.4. No. The kinetic energy of the ion will be $E_k = A_i = e\varphi_i$, and the collision ionization requires the kinetic energy $E_k = \frac{mv^2}{2} \geqslant A_i\left(1+\frac{m}{M}\right)$, where A_i is the work of ionization, m is the mass of the particle, M is the mass of the atom. 15.5. Ions, as $m_i/M \gg m_e/M$ (see the answer to problem 15.4). 15.6. 5.4 V. 15.7. No. 15.8. $2.75\cdot10^6$ m/s. 15.9. $1.902\cdot10^5$ K; 3.805×10^5 K. 15.10. No. 15.11. Yes. 15.12. Hydrogen atoms will be ionized, neons atoms—not. 15.13. $1.00\cdot10^3$ V. 15.14. No. 15.15. $8.0\cdot10^{-14}$ A. 15.16. The self-sustained gaseous conduction does not depend on the action of an ionizer, whereas the induced one disappears when the ionizer is switched off. At a striking (breakdown) voltage U_s, at which on the average each electron will cause an ionization of at least one molecule (the generation of at least one new electron). 15.17. The heating of the electrodes to a temperature required for an intensive thermionic emission; the voltage sufficient for the impact ionization of molecules in the interelectrode space of the order of

tens of volts). Decreases. 15.18. (1) To maintain across electrodes a voltage required for the arc discharge, (2) to limit the current in the circuit. 15.19. As the cathode cools down, the thermionic emission stops, i.e. the arc goes out. 15.20. Will increase. 15.21. The electrons and ions will be deflected by the magnetic field in the direction determined by the left-hand rule. In a strong magnetic field the electrons and ions will not arrive at the electrodes and the arc will go out. 15.22. The spark discharge occurs at a breakdown voltage sufficient for an ionization by strong field to occur. 15.23. As the spark discharge takes a very short time, its instantaneous power is much higher than the average power developed by the source. 15.24. Because the energy is liberated in a time insufficient for a heat exchange with the environment to take place; at discharge site the temperature grows sharply and the evaporation of the metal occurs. To the positive pole. 15.25. A part to be processed is to be connected to the negative pole of the source, and the electrode to be evaporated, to the positive. 15.26. As the voltage increases the energy losses grow. In damp weather the losses are higher. 15.27. The increase in diameter of wires leads to a decrease in the electric field strength, and decrease in ionization and losses. 15.28. Such a connection results in lower electric field strength (see the answer to problem 15.27). 15.29. The partly ionized plasma. 15.30. The rarefication of a gas increases the mean free path of electrons and ions in the field, i.e. increases their kinetic energy; therefore as the gas rarefication grows the ionization of its molecules occurs at lower voltage. No, in a high vacuum an increase in rarefication leads to higher resistance. 15.31. By means of additional energy supplied to the gas atoms through their collisions with ions and electrons. 15.32. As an electron transfers in the atom from an energy level to a lower one. 15.33. The colour of the glow is governed by the change in atom energy in transition from one state into another, these changes being different for the atoms of various elements. 15.34. During these periods more particles come to the Earth from the Sun than under normal conditions. The magnetic field of the Earth deflects the coming charged particles to the poles. 15.35. The large number of travelling charged particles create the magnetic field of their own that varies continually at the ground. 15.36. Positively charged gas ions produced in collision of the particles in the tube strike the cathode to remove electrons from it. Yes, if thermionic and photoelectron emissions are available. 15.37. From the reader. The pole that is closest to the reader is the north one. 15.38. In vacuum tubes, cathode-ray tubes, X-ray tubes, electron microscopes, electron beam furnaces used to produce super-pure materials. 15.39. 2.49 V; $3.98 \cdot 10^{-19}$ J. 15.40. $3.51 \cdot 10^4$ K. 15.41. No (see the answer to problem 15.40). 15.42. $1.264 \cdot 10^6$ m/s. 15.43. The material, dimensions, and temperature of the cathode. 15.44. By covering the cathode with a metal with smaller work function: barium, thorium, caesium. 15.45. 8 mA. 15.46. $7.5 \cdot 10^{16}$. 15.47. The action of a strong electric field. Autoemission. 15.48. Yes, by changing the cathode temperature but within narrow limits. 15.49. Because in the tube, besides the emission of electrons, electrons of the space charge deposit on the cathode. 15.50. (1) 3 J; (2) 50 V. 15.51. $9.00 \cdot 10^2$ Ω; 1.0 W. 15.52. 6.03 V. 15.53. Plot 2.

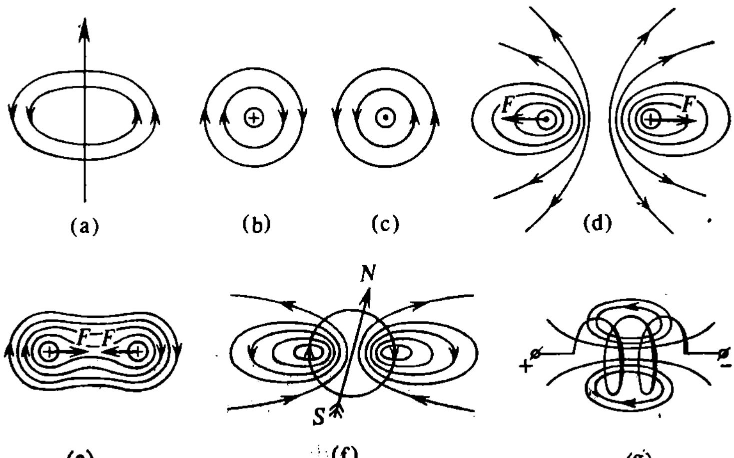
15.54. Curve *1* for $U_c > 0$, *2* for $U_c = 0$, *3* for $U_c < 0$. **15.55.** Plot *2*, because it corresponds to a lower cut-off voltage for the tube. **15.56.** In case (*c*). In case (*a*) there are no current through the circuit. **15.57.** Because in them the number of free charges is much higher, than in vacuum tubes. **15.58.** To do away with the electron scattering and obtain an electron beam. **15.59.** To control the electron beam in the two mutually perpendicular directions. By two coils creating magnetic fields in two mutually perpendicular directions. **15.60.** *1*—cathode, *2*—control electrode, *3* and *4*—focusing and accelerating anodes, *5*—vertical-deflecting plates, *6*—horizontal-deflecting plates, *7*—aquadag (graphite layer to remove electrons from the screen), *8*—luminescent screen; potentiometers: R_1—to regulate brightness, R_3—to focus the electron beam. **15.61.** 300 V.

Sec. 16. Electric Current in Semiconductors

16.1. The resistance of conductors increases, and of isolators, decreases. **16.2.** Decreases in heating. No. **16.3.** In heating the resistance of the thermistor drops sharply resulting in a change in the current in the circuit. **16.4.** Will decrease; will increase. **16.5.** Electrons and holes in equal quantities. **16.6.** Energy must be spent to remove an electron. Heating; radiation. **16.7.** A neutral atom is formed; energy is liberated. **16.8.** Because, along with the formation of electron-hole pairs the recombination occurs of earlier formed electrons and holes. **16.9.** (1) By introducing into a semiconductor impurities of group V of the Periodic Table; by introducing the elements of group III. (2) P, As, Sb—V group elements—predominantly electron conductivity; Ga, B, In—III group elements—predominantly hole conductivity. **16.10.** At lower temperature their resistance grows in heating; at higher temperatures, decreases. **16.11.** Because they are subjected to the electric force that drives them out of the *p-n* junction region. **16.12.** Because the direct current is due to majority carriers; the reverse current, to minority carriers which are not very numerous. **16.13.** Without load the direct voltage across the rectifier will become higher than the contact potential difference across the *p-n* junction and the rectifier will break through. **16.14.** 115 V. **16.15.** In substantially heating the semiconductors, the generation of electron-hole pairs is much intensified. Up to 70°C; up to 120-150°C. **16.16.** For the rectifier in Fig. 16.16 (*a*)—the curve in Fig. 16.16 (*d*). For rectifiers in Fig. 16.16 (*b*) and (*c*)—the curve in Fig. 16.16 (*e*). **16.17.** In order that minority charge carriers passing through the base may not recombine. **16.18.** Because the major contributors to the current through the emitter junction should be the majority charge carriers of the emitter which come over into the base and reach the collector junction. Another part of the current through the emitter junction that consists of the majority carriers of the base is not associated with the collector junction and is useless. **16.19.** $I_e = I_b + I_c$. **16.20.** No. In the first case more. **16.21.** In Fig. 16.21 (*a*) the *p-n-p* transistor, in Fig. 16.21 (*b*) the *n-p-n* transistor.

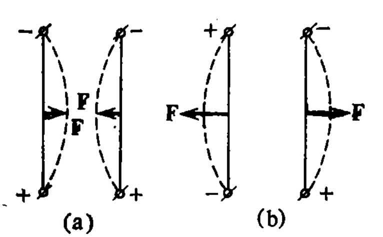
Sec. 17. Electromagnetism

17.1. No; no; yes. 17.2. In the K system—electric field; in the K' system—magnetic field. 17.3. Yes; No. 17.4. No. 17.5. Because its lines are always closed. In such a field the work of magnetic forces over a closed path is not always zero. 17.6. The current flows to the right;

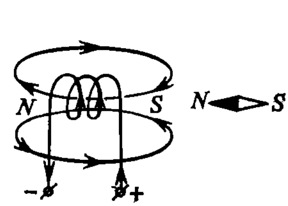


To answer 17.7

the needle will point in the opposite direction. 17.7. See the figure. 17.8. See the figure to the answer to problem 17.7 (d), (e). 17.9. See the figure. 17.10. In the opposite directions. The conductor stretches. 17.11. The spiral will contract. 17.12. See the figure. 17.13. Yes



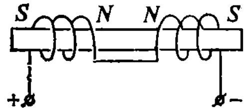
To answer 17.9



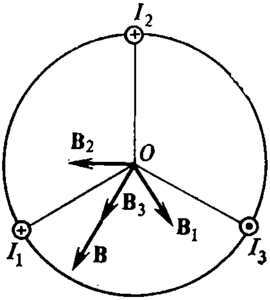
To answer 17.12

(see the figure). 17.14. Yes. Yes; no. 17.15. No, at the magnetic equator only. 17.16. Vertically. 17.17. No, at zero magnetic declination only. 17.18. 12.6 A. The direction of the current flowing through the

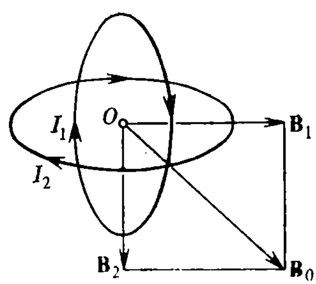
conductors is the same. 17.19. 58 A. 17.20. 1.9 m. 17.21. $3.0 \cdot 10^{-2}$ m. 17.22. $2.0 \cdot 10^{-3}$ N; $8.0 \cdot 10^{-3}$ N; $6.0 \cdot 10^{-3}$ N. 17.23. Vertically downward; normal to the plane of the figure, from the reader. 17.24. 19 N. 17.25. 0.079 T. 17.26. 30°. 17.27. 0.54 m. 17.28. 0.25 N; 10 A. 17.29. 45°. 17.30. $F = k\,(mg \pm BIl)$; 0.148 N, 0.048 N depending on the direction of the current and induction **B**. 17.31. 0.62 N $\leqslant F \leqslant$ 0.88 N. 17.32. 23 A/m; $2.9 \cdot 10^{-5}$ T. 17.33. $2.2 \cdot 10^{2}$ A/m; 63 A. 17.34. $1.6 \cdot 10^{-5}$ T; 0.15 m. 17.35. (1) 0; 0; (2) $I/(4\pi R)$; $\mu_0 I/(4\pi R)$. 17.36. (1) 0; (2) $7.96 \cdot 10^{2}$ A/m; (3) $15.9 \cdot 10^{2}$ A/m; (4) $7.96 \cdot 10^{2}$ A/m. 17.37. A straight line parallel to the conductors and lying with them in one plane at a distance of 0.4 m behind the conductor carrying the smaller current; a straight line between the conductors, 0.08 m away from the conductor carrying the smaller current. 17.38. 37 A/m; 21 A/m. 17.39. $B_1 = 0$; $B_2 = 5.6 \cdot 10^{-7}\, I/x$. 17.40. 44 A/m; 5.5×10^{-5} T (see the figure). 17.41. The conductors repel each other with



To answer 17.13



To answer 17.40



To answer 17.49

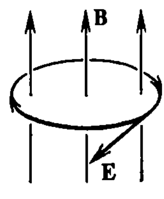
a force of $7.2 \cdot 10^{-3}$ N; attract each other with a force of $2.4 \cdot 10^{-3}$ N. 17.42. A straight line parallel to the conductor at a distance of 5 cm from it; $5 \cdot 10^{-3}$ N. 17.43. Circle. 17.44. If viewed from the magnet, the current flows clockwise. 17.45. 97 A/m; $1.2 \cdot 10^{-4}$ T. 17.46. 1.0×10^{2} A/m; 12 A. 17.47. 0.092 m; $1.5 \cdot 10^{-4}$ T. 17.48. 55 A/m; 5 A/m. 17.49. $1.2 \cdot 10^{-4}$ T; 6.0 A (see the figure). 17.50. $1.7 \cdot 10^{2}$ A/m, 2.1×10^{-4} T; 87 A/m, $1.1 \cdot 10^{-4}$ T. 17.51. $1.3 \cdot 10^{2}$ A/m. 17.52. $4.9 \cdot 10^{3}$ A/m; $6.2 \cdot 10^{-3}$ T. 17.53. 0.74 A. 17.54. 15.2 turns per cm. 17.55. $5.2 \cdot 10^{3}$ A/m. 17.56. 1.4 A/m². 17.57. 15 A; $9.0 \cdot 10^{-2}$ N·m; 90°. 17.58. 5.5 cm. 17.59. $\mu_0 q \sqrt{gl \cos\alpha}/(2\pi l^2 \sin 2\alpha)$; $ql \sqrt{gl \cos\alpha}\,(\sin\alpha \tan\alpha)/2$. 17.60. 11 T; $8.5 \cdot 10^{-24}$ A·m². 17.61. $3.6 \cdot 10^{-1}$ T; $1.7 \cdot 10^{-23}$ A·m². 17.62. $8.8 \cdot 10^{-6}$ Wb. 17.63. $1.8 \cdot 10^{-2}$ Wb. 17.64. By $4.0 \cdot 10^{-3}$ Wb. 17.65. 14 mH. 17.66. 20 turns. 17.67. Will increase by 4 times; will increase by 4 times. 17.68. $5.0 \cdot 10^{-5}$ Wb; $7.0 \cdot 10^{-2}$ Wb; $1.1 \cdot 10^{-2}$ H. 17.69. $\mu_0 \pi U^2 l/(64 N \rho^2 H^3)$; $\mu_0 \pi U^2 l^2/(64 N \rho^2 H^3)$. 17.70. 2.5 J. 17.71. 5.0 J.

17.72. 14 J. 17.73. 0.024 J; 0.048 J; 0. 17.74. Voltage will not change, but induction will grow somewhat; but for practical purposes it may be considered that the induction here is also constant. 17.75. $4.3 \times \times 10^{-4}$ Wb; by 1.08 times; by 1.18 times. 17.76. 0.19 A; will increase by 1.16 times. 17.77. $6.3 \cdot 10^{-13}$ N; 3.1 mm. 17.78. $3.5 \cdot 10^{6}$ m/s. 17.79. No. 17.80. Circle of radius 10 cm; $5.2 \cdot 10^{-4}$ s. 17.81. $8.0 \cdot 10^{3}$ A/m. 17.82. 18.2 kV; $6.0 \cdot 10^{-2}$ T. 17.83. $v = E/B$; $5.0 \cdot 10^{5}$ m/s; $5.0 \cdot 10^{5}$ m/s. 17.84. 7.6 cm; 2.5 m. 17.85. $6.0 \cdot 10^{3}$ m/s; $6.0 \cdot 10^{-7}$ T; 0.18 m. 17.86. 1.8 cm. 17.87. $4.0 \cdot 10^{-16}$ J. 17.88. 36 and 40.

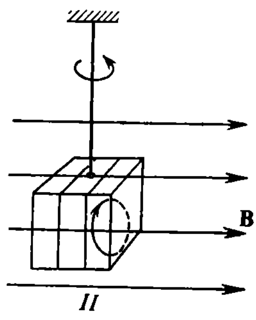
Sec. 18. Electromagnetic Induction

18.1. (1) To the right; (2) Does not deflect, $I = 0$; (3) To the right. 18.2. To the right. 18.3. To the right. 18.4. In the first case the work is larger. 18.5. Second. First is retarded by the magnetic field of induced current. 18.6. No; no. 18.7. No; yes. 18.8. Yes. No, in closed conductors only. 18.9. No; in changing the area of the loop so that the magnetic flux be constant, there are no induced e.m.f. 18.10. 3.5 mV. 18.11. $7.5 \cdot 10^{-3}$ T. 18.12. 13 V. 18.13. In 0.49 s. 18.14. 100 turns. 18.15. $2.5 \cdot 10^{-2}$ T. 18.16. 2.0 C; clockwise (see the figure to problem 18.16), as viewed against **B**; counterclockwise. 18.17. $\mathscr{E}_{av} = \frac{3}{32} \frac{B\pi d^2}{\Delta t} = 5.9 \cdot 10^{-2}$ V; $q = \frac{3}{32} \frac{B\pi d^2}{R} = 2.9 \cdot 10^{-2}$ C. 18.18. $A_2 > A_1$, $A_2/A_1 = \pi - 1$; $A_1 = q\,(d^2/4)\,(\Delta B/\Delta t) = 2.4 \cdot 10^{-2}$ J; $A_2 = q\,(\pi - 1)\,(d^2/4)\,(\Delta B/\Delta t) = 5.1 \cdot 10^{-2}$ J. 18.19. $e = -N\Phi'_t = -5$V; $|e| \neq f(t)$; $I = 2.0$ A. The minus sign signifies that the induced current opposes with its magnetic field the change in the magnetic flux that causes it. 18.20. $\Phi = (2 + 5t^2) \cdot 10^{-4}$ Wb; $\Phi_5 = 1.27 \cdot 10^{-2}$ Wb; $e = -\Phi'_t = -(10t) \cdot 10^{-4}$ V; $e_5 = -5.0 \cdot 10^{-3}$ V. 18.21. (1) $\Psi = (3 + 2t^2) \cdot 2.0 \cdot 10^{-2}$ Wb; $e = -\Psi'_t = -8t \cdot 10^{-2}$ V; (2) $\Psi = 4.06$ Wb ≈ 4.1 Wb; $|e| = 8.0 \cdot 10^{-1}$ V. 18.22. (1) $\Phi = 2.0 \cdot 10^{-4} \cos(4\pi t + \pi/6)$ Wb; (2) $e = -\Phi' = 8.0\pi \cdot 10^{-4} \sin(4\pi t + \pi/6)$ V; (3) $e_4 = 4.0\pi \cdot 10^{-4}$ V $= 1.26 \cdot 10^{-3}$ V. 18.23. (1) $2.0 \cdot 10^{-3}$ C; (2) $2.0 \cdot 10^{-11}$ J; (3) lower. 18.24. The current is directed to the reader. 18.25. South pole. 18.26. Downward. 18.27. No; yes. 18.28. $3.0 \times \times 10^{-3}$ V. 18.29. 29 cm. 18.30. $1.2 \cdot 10^{-3}$ T. 18.31. 23°30′. 18.32. $\mathscr{E} = \mu_0 Hvl \sin\beta = 2.0 \cdot 10^{-2}$ V. 18.33. $\mathscr{E} = Bvl \sin\alpha \cdot \sin\beta = 4.3 \cdot 10^{-1}$ V. 18.34. $\mathscr{E} = \pi Bnl^2/60 = 4\pi \cdot 10^{-2}$ V. 18.35. (1) $2.5 \times \times 10^{-2}$ W; (2) 10^2 rad/s. 18.36. 0; 5.4 V; 2.7 V. 18.37. (1) $I = Bvl/(r + R) = 1.0 \cdot 10^{-1}$ A; (2) $F = B^2vl^2/(r + R) = 2.0 \cdot 10^{-2}$ N; (3) $P = B^2v^2l^2/(r+R) = 2.0 \cdot 10^{-2}$ W. 18.38. $P = \left(kmg + \frac{B^2vl^2}{R+r}\right) v = 1.2 \cdot 10^{-1}$ W. 18.39. $q = BvlC = 2.0 \cdot 10^{-7}$ C; $W = B^2v^2l^2C/2 = 1.0 \cdot 10^{-9}$ J. 18.40. 0.20 A; 0.13 A; 0.07 A; 0.12 W. 18.41. $Q = 2B^2vl^3 \sin^2\alpha/R = 1.3 \cdot 10^{-3}$ J. 18.42. $Q/t = P = (mg/Bl)^2 R = 1.9$ W; $v = mgR/(B^2l^2) = 20$ m/s. 18.43. $I = mg \sin\alpha/(Bl\cos\alpha) = 0.57$ A; $v = mgR \sin\alpha/(B^2l^2 \cos^2\alpha) = 0.65$ m/s. 18.44. 11 m/s. 18.45. Will decrease by 1.1 A. 18.46. 4.0 m/s. 18.47. Yes; when the field is produced by a varying magnetic field. 18.48. Yes; when the

magnetic field is produced by the varying electric field, e.g. in the capacitor. 18.49. With uniformly varying magnetic field; with non-uniform variation $B' \neq$ const. 18.50. According to Lenz's rule and right-hand screw rule (see the figure). 18.51. With alternating current, eddy currents are induced in the object and with direct current, no. 18.52. First. Here the eddy current is smaller. 18.53. Because as cube *II* (see the figure to problem 18.52) is turned, the area through which magnetic induction lines pass grows and the magnetic flux increases, the induced current will oppose with its magnetic field the increasing of the magnetic flux (see the figure). 18.54. In the disk eddy current is produced. In the direction of rotation of the magnet. No. 18.55. The plate repels from the electric magnet; attracts. 18.56. Yes, with fast variation of the current flowing through the magnet. By interaction with the eddy current engendered in the conductor. 18.57. In breaking, as the period taken by the current to fall off to zero is shorter. 18.58. With the source e.m.f. equal to the self-induced e.m.f. the current in the circuit would be zero. There were no variation of the current and self-induced e.m.f. 18.59. 22 V. 18.60. 0.31 H. 18.61. In 91 ms. 18.62. $e = -Li' = 10^{-2}$ V. The minus sign signifies that the self-induced e.m.f. opposes the current increase. 18.63. $1.0 \cdot 10^{-1}$ H. 18.64. 1.0 J. 18.65. 17 mH. 18.66. 2.0 A. 18.67. 0.56 J; 14 V. 18.68. The energy of the power supply turns into the internal energy (the circuit heats) and into the energy of magnetic field. 18.69. $W = \mu_0 N^2 \pi r^2 I^2/2l = 1.26 \cdot 10^{-3}$ J; $e = -Li' = 1.26 \cdot 10^{-4}$ V. 18.70. $L = \Delta W/(I_{av}\, \Delta I) = 1.0 \cdot 10^{-2}$ H. 18.71. $q = 2\,(n-1)\, W/(nIR)$; will decrease by $\Delta W = (n^2 - 1)\, W/n^2$.



To answer 18.50



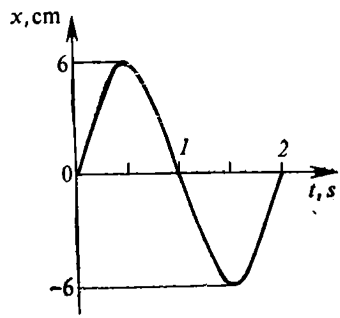
To answer 18.53

Sec. 19. Mechanical Oscillations and Waves. Sound

19.1. $2.0 \cdot 10^{-1}$ s; 5.0 Hz. 19.2. $1.0 \cdot 10^{-4}$ s; $6.0 \cdot 10^{5}$ min^{-1}. 19.3. In the first case the directions of velocities of pendulums at any instant of time are equal, the pendulums oscillating in phase; in the second case the velocities are counterdirected and the oscillations are in antiphase. 19.4. π rad. The pendulums in Fig. 19.4 (*a*) have different periods, for them the phase difference is not constant. In the case shown in Fig. 19.4 (*b*) the phase difference is constant, and the oscil-

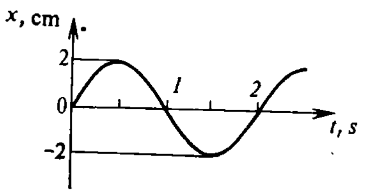
lations are coherent. 19.5. 0; $A/2$; A; 0. 19.6. $T/12$; $T/6$. 19.7. $x_1 = 1.0\cdot10^{-1}\sin\left(2\pi t+\frac{1}{4}\pi\right)$ [m]; $x_2 = 5.0\cdot10^{-2}\sin\left(\pi t+\frac{1}{2}\pi\right)$ [m]; $x_3 = 4.0\cdot10^{-2}\sin(4\pi t+\pi)$ [m]. 19.8. $A = 2.0\times10^{-2}$ m; $\varphi_0 = \frac{1}{2}\pi$ rad; $T = 8.0$ s. 19.9. $x = 5.0\cdot10^{-2}\sin 200\,\pi t$ [m]; $\nu = 100$ Hz; $\omega = 200\,\pi$ rad/s; $v_{max} = 10\pi$ m/s; $a_{max} = -2\cdot10^3\pi^2$ m/s^2; $W = 5\pi^2$ J $= 49$ J. 19.10. $A = 1.0\cdot10^{-1}$ m; $\varphi_0 = \pi/2$; $\omega = 314$ rad/s; $\nu = 50$ Hz; $T = 2.0\cdot10^{-2}$ s; $v_{max} = 31.4$ m/s; $a_{max} = -9.86\times10^3$ m/s^2; $E_{max} = 49.3$ J. 19.11. $1.0\cdot10^{-1}$ m; -394 m/s^2; $x = 1.0\cdot10^{-1}\sin 62.8t$ [m]. 19.12. $x = -6.0\cdot10^{-4}\cos 100t$ [m]; 6.0×10^{-2} m/s; -6.0 m/s^2; $3.6\cdot10^{-4}$ J. 19.13. $|a_{max}| = 4\pi^2\cdot10^{-1}$ m/s^2; 5.0×10^{-2} m; $1.5\cdot10^{-1}$ m. 19.14. $x = -2.0\cdot10^{-1}$ m; $6.0\cdot10^{-1}$ m; $|F_{max}| = 39.4\cdot10^{-2}$ N. 19.15. $1.0\cdot10^{-1}$ m; $-\pi^2\cdot10^{-1}$ m/s^2; -2.0×10^{-2} N; $\pi^2\cdot10^{-3}$ J. 19.16. $a_{max} = g$. From formula $a_{max} = 4\pi^2A/T^2$ we get $A = 6.2\times10^{-2}$ m. 19.17. $A \geqslant kgT^2/(4\pi^2) \approx 0.2$ m. 19.18. $T = 2\pi\sqrt{m/\rho gS}$. 19.19. $T = 2\pi\sqrt{m/(2\rho gS)}$. 19.20. $T = 2\pi\sqrt{m/\rho g\,(S_1+S_2)}$. 19.21. 0.63 s; $8.0\cdot10^{-3}$ J. 19.22. $x = \frac{mg}{k}\sin\sqrt{\frac{k}{m}}\,t$; $W = (mg)^2/2k$. Will not change. 19.23. 2N/m. $E_k = \frac{2\pi^2A^2m}{T^2}\cos^2\frac{2\pi}{T}t = 2.5\cdot10^{-3}$ J. 19.24. $T_{ser} = 2\pi\sqrt{m\,(k_1+k_2)/(k_1k_2)}$; $T_{par} = 2\pi\sqrt{m\,(k_1+k_2)/(4k_1k_2)}$. 19.25. 2.0 s; 9.82 m/s^2. 19.26. 2.05 s; to increase by four times. 19.27. $9.94\cdot10^{-1}$ m; to decrease by four times. 19.28. $l_2/l_1 = 1/4$. 19.29. $T_M = 2.45T_E$. 19.30. 0.27 m; 0.75 m. 19.31. 2.0 s; the phases are equal in every two cycles of the second pendulum or one cycle of the first pendulum. 19.32. $t_1/t_2 = 2\sqrt{2}/\pi \approx 0.9$. The ball falling free from the suspension point will achieve the equilibrium first. 19.33. $T = \pi\sqrt{l/g}\,(1+\sqrt{1/2})$. 19.34. $T_1 = \frac{T}{2\pi}\left(\pi + 2\arcsin\frac{\beta}{\alpha}\right)$. 19.35. $T_P/T_E = 2$. 19.36. $\Delta t = t_1\,(\sqrt{g_p/g_{eq}} - 1)$, where $t_1 = 8.64\cdot10^4$ s; the clock will lag behind in 24 hours by $\Delta t = 3$ min 49 s. 19.37. $\Delta t = th/R_E \approx 54$ s. 19.38. $\Delta t = \left(1 - \sqrt{\frac{1}{1+\alpha\,\Delta T}}\right)t \approx 17.3$ s. 19.39. The tension in the string and the restoring force will grow; the period of oscillations will decrease. 19.40. $T_1 = 2\pi\sqrt{l/(g+a)}$; $T_2 = 2\pi\sqrt{l/(g-a)}$. 19.41. $a = 3g$. 19.42. $T = 2\pi\sqrt{l/(g+a)}$. At the moment the engines are switched off weightlessness sets in, the restoring force becomes zero and oscillations stop; (1) the pendulum will remain in the extreme position, (2) if the way of fastening the pendulum is adequate, the pendulum will move in a circle. 19.43. In both cases $\frac{T_1}{T} = \sqrt{\frac{g}{\sqrt{g^2+a^2}}}$. 19.44. $T = 2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2-2ag\cos\alpha}}}$. 19.45. $v = \sqrt[4]{(n^4-1)\,g^2R^2}$. 19.46.

$T = 2\pi\sqrt{\dfrac{l}{g \pm qU/md}}$ ("+" if the lower plate is charged negatively); $T = 2\pi\sqrt{\dfrac{l}{\sqrt{g^2+(qU/md)^2}}}$. 19.47. $a = 4\pi l/T^2$. 19.48. $W = \frac{1}{2} mgl\alpha^2$; $v_{max} = \alpha\sqrt{gl}$, where α is the angle in radians. 19.49. $\nu = (1/\pi A)\sqrt{W/2m}$; $l = mgA^2/2W$. No. 19.50. $T = \dfrac{2\pi v}{g\sqrt{2(1-\cos\alpha)}} \approx \dfrac{2\pi v}{\alpha g}$. 19.51. 4.0 m. 19.52. 0.50 Hz. 19.53. $T = 2\pi\sqrt{\dfrac{m_1l_1^2+m_2l_2^2}{g(m_1l_1+m_2l_2)}}$; $T = 2\pi\sqrt{\dfrac{m_1l_1^2+m_2l_2^2}{g(m_1l_1-m_2l_2)}}$. 19.54. $x_{res} = x_1 + x_2$, following the rule of the vector composition. 19.55. $x_1 = 2\sin\pi t$ [cm]; $x_2 = 4\sin\pi t$ [cm]; $x_{res} = (A_1+A_2)\times\sin\frac{2\pi}{T}t = 6\sin\pi t$ [cm] (see the figure). 19.56. $x_1 = 2\sin(\pi t - \pi)$ [cm]; $x_2 = 4\sin\pi t$ [cm]; $x_{res} = 2\sin\pi t$ [cm] (see the figure). π. 19.57. $x_1 = 3\sin(\pi t + \pi/2)$; $x_2 = 4\sin\pi t$; $x_{res} = 5\sin(\pi t + 0.64)$ (see the figure). $\pi/2$. 19.58. (1) $A_1 = 0.2$ m; $A_2 = 0.1$ m; (2) $\nu_1 = 200$ Hz; $\nu_2 = 100$ Hz; (3) $v_1 = 2.5\cdot10^2$ m/s; $v_2 = 6.3\cdot10^1$ m/s; (4) $a_1 = -3.2\cdot10^5$ m/s², $a_2 = -3.9\cdot10^4$ m/s²; (5) no; (6) $x_1 = 0.20\sin 400\pi t$ [m]; $x_2 = 0.10\times\sin 200\pi t$ [m]. 19.59. The harmonic oscillations with the same period; $x_1 = (A_1+A_2)\sin\left(\varphi_0 + \frac{2\pi}{T}t\right)$; $x_2 = (A_1 - A_2)\sin\left(\varphi_0 + \frac{2\pi}{T}t\right)$; $x_3 = \sqrt{A_1^2+A_2^2}\sin\left(\arctan\frac{A_2}{A_1} + \frac{2\pi}{T}t\right)$. 19.60. $T_{res} = 1.2$ s; $A_{max} = 7.0\cdot10^{-2}$ m; $\Delta\varphi = 0$; $A_{min} = 3.0\cdot10^2$ m; $\Delta\varphi = \pi$; $x = 3.5\cdot10^{-2}$ m. 19.61. In free oscillations the frequency and amplitude are governed by the construction of the oscillating system. The source of energy is a component of the system. In forced oscillations the frequency is determined by the external influence. 19.62. For pendulums *1* and *3* and for *2* and *4* (see Fig. 19.62). With strong damping. 19.63. In resonance the direction of the force coincides with the direction of motion. The work performed by the external force is positive, which results in the increase in the energy of the system. 19.64. Eliminate the coincidence of frequencies by increasing or decreasing the frequency of the vibrator. 19.65. $v_0 = l/T_0$. 19.66. 1.7 m. 19.67. 1450 m/s. 19.68. 50 m. 19.69. 2π rad; π rad; $\pi/2$ rad; $x_1 = x_2 = 0$, $x_3 = 1.0\cdot10^{-2}$ m. 19.70. $v = 3.6\cdot10^3$ m/s. *Hint.* Use the relation

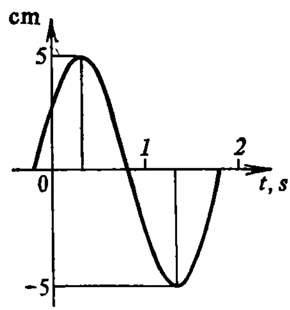


To answer 19.55

$\Delta\varphi = 2\pi(l_2 - l_1)/Tv$. **19.71.** π rad. **19.72.** 2.0 s; 3.0 m/s; 6.0 m. **19.73.** 1.0 Hz; 2.0 Hz. **19.74.** From the formula $x = A\sin[2\pi(t - l/v)/T]$ we get $x = -A = -3.0\times10^{-2}$ m. The minus sign signifies that the point has displaced in the direction of negative values. **19.75.** $v_{\max} = \frac{2\pi A u}{\lambda}$; 0; $x = A\sin\frac{2\pi u}{\lambda}\left(t - \frac{y}{u}\right)$, where y is the coor-

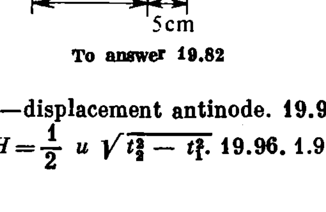


To answer 19.56



To answer 19.57

dinate of the point for which the displacement is determined. The vibrator is at the origin of the coordinate system. **19.76.** $x = A\sin\times$ $\times\frac{v_{\max}}{A}\left(t - \frac{l}{u}\right)$; $v = v_{\max}\cos\frac{v_{\max}}{A}\left(t - \frac{l}{u}\right)$; 0; $-v_{\max}$. **19.77.** $u = \lambda\nu$; $v_{\max} = 2\pi\nu A$; 0. **19.78.** $\Delta l_1 = 2n\cdot50$ m; $\Delta l_2 = (2n-1)\cdot50$ m (n—integer). **19.79.** 5 m; 2.5 m; maximum intensification. **19.80.** 0.20 m. **19.81.** $\Delta\varphi = 0$; in antiphase. **19.82.** 0.40 m (see the figure). **19.83.** 440 Hz. **19.84.** 333 m/s. **19.85.** From 21.5 to 0.017 m. **19.86.** 3.32 m; 3.41 m; 3.44 m. **19.87.** 3.44 m. **19.88.** 883 m. **19.89.** Is independent. **19.90.** u; $u - v \leqslant u' \leqslant u + v$. **19.91.** 17 m. **19.92.** $l = n\lambda/4$, where n—odd number; at the output—displacement antinode. **19.93.** 0.195 m. **19.94.** $5.1\cdot10^{-4}$ m. **19.95.** $H = \frac{1}{2}u\sqrt{t_2^2 - t_1^2}$. **19.96.** 1.9 s; 2.1 s; 2 s. **19.97.** By 142 Hz.

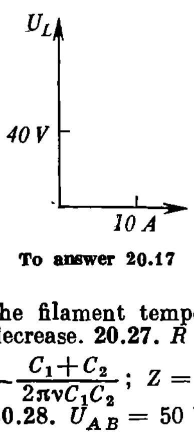


To answer 19.82

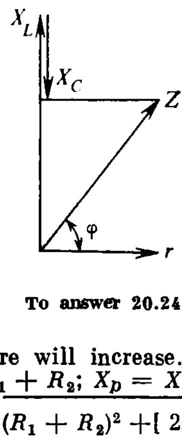
Sec. 20. Alternating Current

20.1. $e = 22\sin 12\pi t$ [V]; 22 V; 15 V. **20.2.** $4.7\cdot10^{-1}$ V; 8.5×10^{-1} V; 0. **20.3.** $e = 6.28\sin 314t$ [V]; 6.28 V; 4.44 V. **20.4.** 100 Hz; 6.28 V; 4.44 V. **20.5.** 84.9 V; 10^{-2} s, $e = 240\sin 1256t$ [V]. **20.6.** 1.5 V. **20.7.** $6.9\cdot10^{-3}$ s; $3.1\cdot10^{-3}$ s. **20.8.** 50 Hz. **20.9.** 24. **20.10.** $e=$

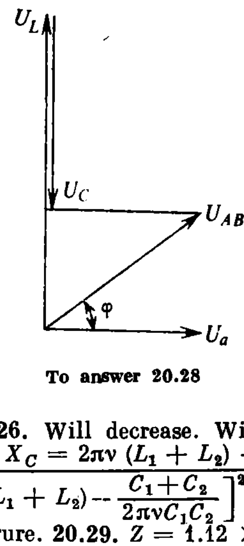
$= 43.4 \sin 24\pi t$ [V]; 31 V. **20.11.** 8.5 A; 0.651 rad; 50 Hz; $i_1 = 5.1$ A; $i_2 = 8.1$ A; 6.0 A. **20.12.** More 311 V. **20.13.** 4.53 A; 82 W; 25.6 V. **20.14.** $u = 25.6 \sin 314t$ [V]; 0. **20.15.** No, it is larger with direct current. **20.16.** 13.2 Ω; 52.8 Ω; 106 Ω. **20.17.** $i = 22.5 \sin 314t$ [A]; $\varphi = \pi/2$ (see the figure). **20.18.** $u = 31.4 \sin (628t + \pi/2)$ V; 314 Ω. **20.19.** 0.8 Hz. **20.20.** 12.7 Ω; 3.2 Ω; 1.6 Ω. **20.21.** No less than 319 V. **20.22.** $i = 1.38 \sin 314t$ [A]; the voltage lags behind by $\pi/2$ rad; $q = -4.4 \cdot 10^{-3} \cos 314t$ [C]. **20.23.** 0.25 rad; $Z = 4.0$ Ω; $\cos \varphi = 0.97$; $R = 3.9$ Ω; $X_L = 0.99$ Ω; $S = 450$ V·A; $P = 436$ W. **20.24.** See the figure. $Z = 5$ Ω; $\cos \varphi = 0.6$. **20.25.** I the three cases



To answer 20.17

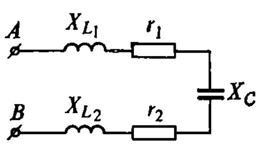


To answer 20.24

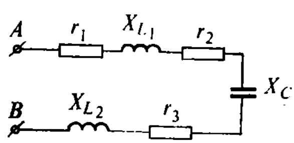


To answer 20.28

the filament temperature will increase. **20.26.** Will decrease. Will decrease. **20.27.** $R = R_1 + R_2$; $X_p = X_L - X_C = 2\pi\nu (L_1 + L_2) - \frac{C_1 + C_2}{2\pi\nu C_1 C_2}$; $Z = \sqrt{(R_1 + R_2)^2 + \left[2\pi\nu (L_1 + L_2) - \frac{C_1 + C_2}{2\pi\nu C_1 C_2}\right]^2}$. **20.28.** $U_{AB} = 50$ V; $\cos \varphi = 0.8$; see the figure. **20.29.** $Z = 1.12 \times$



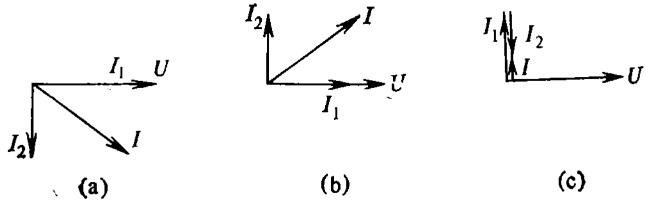
To answer 20.34



To answer 20.35

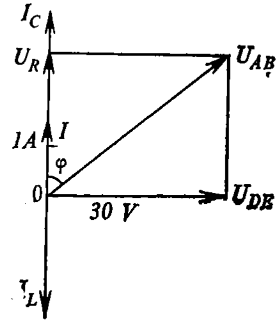
$\times\ 10^2$ Ω. $\varphi = 89°$; $\nu_{res} = 1.42 \cdot 10^2$ Hz; $r_{min} = 2.0$ Ω. **20.30.** Yes. **20.31.** 12.1 A; 2.7 kV·A; 2.2 kW; —1.5 kvar. **20.32.** 135 μF; $U_L = U_C = 590$ V. Across the capacitance and inductance voltages may develop that are much higher than the voltage in the circuit, thus leading to insulation breakdown and short-circuiting. **20.33** $1.4 \cdot 10^2$ V; 22 kW. **20.34.** See the fig-

ure; $U_{AB} = \sqrt{(U_2 + U_4)^2 + (U_1 - U_3 + U_5)^2}$; $\cos\varphi = \frac{\sum U_a}{U_{AB}} = \frac{U_2+U_4}{\sqrt{(U_2+U_4)^2+(U_1-U_3+U_5)^2}}$. 20.35. See the figure. To voltage resonance. 20.36. 20 V; 5.0 Ω; $\cos\varphi = 1$; 80 W. 20.37. Fig. 20.37, (*a*), 5 A; Fig. 20.37 (*b*), 5 A; Fig. 20.37 (*c*), 1 A. 20.38. 0.



To answer 20.37

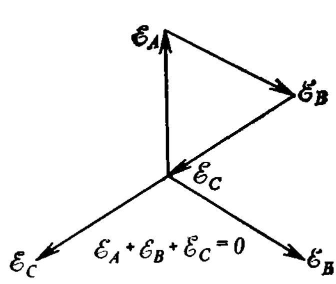
20.39. $\nu = 1/2\pi\sqrt{LC} = 291$ Hz. 20.40. No. 20.41. 0. 20.42. 148 V; see the figure. 20.43. 12 J. 20.44. $P = U^2\cos^2\varphi/R$. 20.45. $\cos\varphi = 0.9$. 20.46. As the capacitance decreases the reactance $X_C = \frac{1}{\omega C}$ increases. The impedance decreases; the filament temperature increase attaining a maximum at $X_C = X_L$ (voltage resonance), then as the capacitance decreases further, the resistance grows and the filament temperature decreases. 20.47. It goes into heating; 90°. 20.48. The transformer winding will fail; the transformer has no losses to friction. 20.49. $k = 0.04$; $n_2 = 2250$. 20.50. $n_1 = 880$; $n_2 = 144$; $k = 6.11$. 20.51. 244 turns. 20.52. Secondary. As $k > 1$, the transformer is a step-up one, i.e. $I_2 > I_1$, and to reduce thermal losses with large current the winding resistance is to be reduced. This is achieved by an increase in the wire section. 20.53. The filament temperature will grow. The power will increase that is consumed by the second winding P_2, and hence the power consumed by the primary winding P_1, which at a constant voltage U_1 results in higher current I_1, and the higher current will cause an increase in the filament temperature. 20.54. 90%. 20.55. 44. 20.56. 0.12 Ω; 7.6 A. 20.57. 110 V; 4.0 A; 98%. 20.58. 38 V; 132 turns; 1.0 A; 95%. 20.59. 200 V. 20.60. $\eta = \frac{k^2P_2}{I_2(kU - I_2R_1)} = 98\%$. 20.61. By $\sqrt{n}$ times.



To answer 20.42

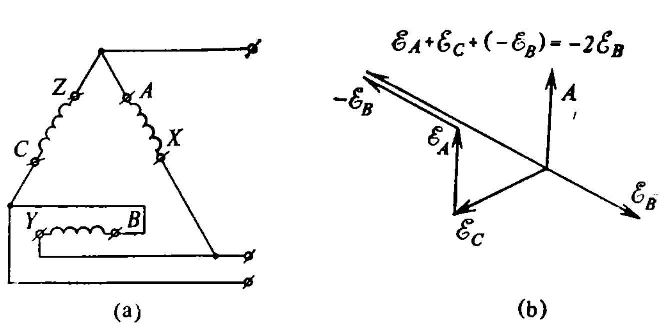
Sec. 21. Three-Phase Current

21.1. An a.c. circuit with three e.m.f.'s o lequa]frequency and initial phases shifted by 1/3 of period. 21.2. A three-phase system with equal e.m.f. amplitudes and frequencies, and 120° phase shift. 21.3. $e_2 = \mathscr{E}_0 \sin\left(314t - \frac{2}{3}\pi\right)$; $e_3 = \mathscr{E}_0 \sin\left(314t + \frac{2}{3}\pi\right)$. 21.4. A system in which each winding of the generator is connected to a separate consumer. An incoherent system requires 6 wires. 21.5. Y-connection. Here the ends of windings X, Y, Z are connected at zero point of the generator, and to the beginning of the windings A, B, C lines are connected; the neutral 'being connected to the zero point (generator neutral). Delta-connection. The end of the first winding X is connected with the beginning of the second winding B, the end of the second winding Y with the beginning of the third winding C, and the end of the third winding Z with the beginning of the first winding A. To points A, B, C wires are connected which lead to the consumer.



To answer 21.9

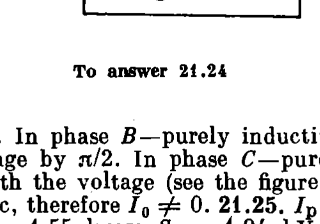
21.6. The voltage between the beginning and end of a phase is called the phase voltage U_p and denoted as U_A, U_B, U_C. The voltages across the beginnings of windings (or between the lines) are called line voltages U_l and denoted by U_{AB}, U_{BC}, U_{CA}. In Y-connection of the



To answer 21.10

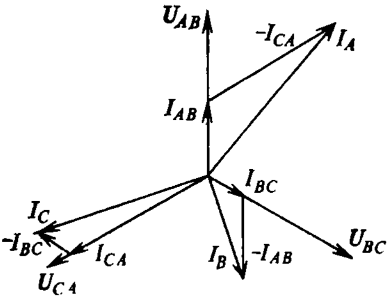
generator windings with a neutral the phase voltages a.e always equal regardless of the load and $U_l = \sqrt{3}\, U_p$. In delta-connection of the

generator windings the phase voltages are equal to the line ones. **21.7.** This connection allows two different voltages to be used: phase voltage—with the consumer connected between the neutral and lines, and line voltage—with the consumer connected across two lines. **21.8.** 220 V; 380 V. **21.9.** See the figure; $\mathscr{E}_A + \mathscr{E}_B + \mathscr{E}_C = 0$. **21.10.** Here the resultant e.m.f. will be equal to twice the phase e.m.f., which, considering the small resistance of the windings, is equivalent to short-circuiting. See the figure to answer 21.10 (*a*), (*b*). **21.11.** The consumer connection does not depend on the connection of the generator windings and is governed by the voltage required. **21.12.** No. These data are insufficient for unique determination. **21.13.** Yes. **21.14.** $I_l = 10$ A. With symmetric load the current in the neutral $I_0 = 0$. **21.15.** If the neutral breaks (the fuze in it blows), with asymmetric load, the voltage across phases with lower resistance drops; and with higher resistance, increases, which is to be avoided, especially for lighting purposes, because this may ruin the lamps. **21.16.** No, as here $I_0 = 0$. **21.17.** $P = P_1 + P_2 + P_3$; $Q = Q_1 + Q_2 + Q_3$; $S = \sqrt{(P_1 + P_2 + P_3)^2 + (Q_1 + Q_2 + Q_3)^2}$. The sign of the reactance is indicative of its character: the plus sign signifies a predominantly inductive or purely inductive load, the minus sign means predominantly capacitive load. **21.18.** $S_p = 500$ V·A; $\cos\varphi_p = \cos\varphi = 0.80$; $P = 1200$ W; $Q = -900$ var; $S = 1500$ V·A. **21.19.** $P = \sqrt{3}\, U_l I_l \cos\varphi = 10$ kW. **21.20.** 10 A. **21.21.** $U_p = 200$ V; $I_1 = 0.45$ A; $I_A = 9.1$ A; $I_B = 7.3$ A; $I_C = 11.7$ A; $P_A = 2.0$ kW; $P_B = 1.6$ kW, $P_C = 2.6$ kW; $P = 6.2$ kW. **21.22.** $I_p = I_l = 6.82$ A; $W = 36$ kW·h. **21.23.** In all the phases the load is active, as the current is in phase with the voltage. If the load is mixed, the capacitive and inductive reactances in each phase are equal; $I_0 = 0$ as the load is symmetrical. **21.24.** In phase A—purely capacitive load as in the vector diagram the current is ahead of the voltage by $\pi/2$. In phase B—purely inductive, as the current lags behind the voltage by $\pi/2$. In phase C—purely active, as the current is in phase with the voltage (see the figure to answer 21.24). The load is asymmetric, therefore $I_0 \neq 0$. **21.25.** $I_p = I_l = 8.8$ A; $P_p = 1.16$ kW; $Q_p = 1.55$ kvar; $S_p = 1.94$ kV·A; $\cos\varphi_p = 0.6$; $P = 3.5$ kW; $Q = 4.65$ kvar; $S = 5.8$ kV·A. **21.26.** Delta-connection. Y-connection with symmetric load. **21.27.** 10.5 A; 6.1 A. **21.28.** 52 A; 15.8 kW. **21.29.** $I_l = 38.6$ A; $R_p = 8.35$ Ω; $Z_p = 9.85$ Ω; $U_l' = 380$ V. **21.30.** The line current and motor power are trebled. **21.31.** 7.86 A. **21.32.** (1) $I_{AB} = 10$ A, $I_{BC} = 5.0$ A, $I_{CA} = 20$ A; (2) $P_{AB} = 2.2$ kW, $P_{BC} = 1.1$ kW, $P_{CA} = 4.4$ kW; (3) $P = 7.7$ kW; (4) (See the figure to answer 21.32.) $I_A = 26.5$ A, $I_B = 13.2$ A, $I_C = 22.9$ A. **21.33.** 44 A; 76.12 A; 0.6; 5.8 kW, 17.4 kW. **21.34.** 10 Ω; 0.8; 22 A, 38.1 A; 220 V, 220 V; 4.84 kV·A;



To answer 21.24

11.62 kW; 14.52 kV·A. **21.35.** In a three-wire three-phase system regardless of the load type and way of its connection the instantaneous



To answer 21.32

values of line current add up to zero. **21.36.** No; no. **21.37.** 3 000 rpm; 1 500 rpm; 1 000 rpm. **21.38.** One should interchange any two lines connected to the motor.

Sec. 22. Electrical Oscillations and Electromagnetic Waves

22.1. The inductance produces the magnetic field and the capacitance produces the electric field. In the process of oscillations there is a continual exchange of energy between the inductance and capacitance. **22.2.** The presence of the resistance in the oscillatory circuit leads to the damping and changing the period of free oscillations. **22.3.** The frequency of oscillations will decrease, and their damping will increase. **22.4.** If the energy lost to heat and radiation is made up. **22.5.** To heating the wires of the oscillatory circuit and to producing the electromagnetic waves in the environment. **22.6.** In order to make it possible to change the frequency (period) of oscillations in the circuit. **22.7.** To connect an aerial to the circuit and increase the frequency of oscillations. **22.8.** The alternative electric field produces the vorticity magnetic field; the alternative magnetic field produces the vorticity electric field. **22.9.** The vectors of electric and magnetic field strengths oscillate in phase and in mutually perpendicular planes. **22.10.** When the circuit is in resonance with the oscillations of the wave. **22.11.** The natural oscillations in the circuit will not change. **22.12.** The period of oscillations will increase by $\sqrt{8}$ times, and the frequency will decrease by $\sqrt{8}$ times. **22.13.** 0.24 ms; 4 200 Hz. **22.14.** 1 550 Hz; will double. **22.15.** 0.38 ms; will double. **22.16.** $L = 6.5 \cdot 10^{-4}$ H; $\pi/2$ rad. **22.17.** $\omega = 2\pi$ s^{-1}; $\nu = 1$ Hz; $T = 1$ s; $\varphi_0 = \pi$ rad; $I_m = 2\pi \cdot 10^{-2}$ A. **22.18.** $\nu = I_m/2\,CU_m$. **22.19.** $q_m = 1.0 \cdot 10^{-3}$ C; $I_m = 10$ A; $i = 1.0 \cdot 10^{1} \sin 10^4 t$ [A]. **22.20.** $T =$

$= 4\pi \cdot 10^{-5}$ s; $q_m = 2.0 \cdot 10^{-6}$ C; $U_m = 0.50$ V (across each capacitor). **22.21.** $i = U_m \sqrt{3C/4L}$. **22.22.** 10.0 V. **22.23.** $4.7 \cdot 10^{-2}$ J. **22.24.** 200 m. **22.25.** 10.5 m. **22.26.** 25.4 pF. **22.27.** From 31 to 260 pF. **22.28.** From 630 to 1,900 m. **22.29.** 485 m. **22.30.** $\lambda = 2\pi v L I_m / U_m$. **22.31.** $\lambda = 2\pi v q_m / I_m$. **22.32.** Over longer distances only short waves can propagate. Therefore low-frequency oscillations are superimposed on the high-frequency oscillations (modulation). It is these latter which carry the low-frequency oscillations to the receiver. **22.33.** So that on the grid no electrons would settle which might lead to signal distortions. **22.34.** Due to a source of electric energy included into the anode circuit. **22.35.** Because the bridge reflects electromagnetic waves. **22.36.** The shorter the electromagnetic waves, the easier are they to be directed. **22.37.** For the reflected impulses to be recorded. **22.38.** The cathode-ray tube. **22.39.** By 16 times. **22.40.** 24 μs. **22.41.** There are no free oscillations in the circuit. **22.42.** 710 m. **22.43.** $1.4 \cdot 10^4$ pF.

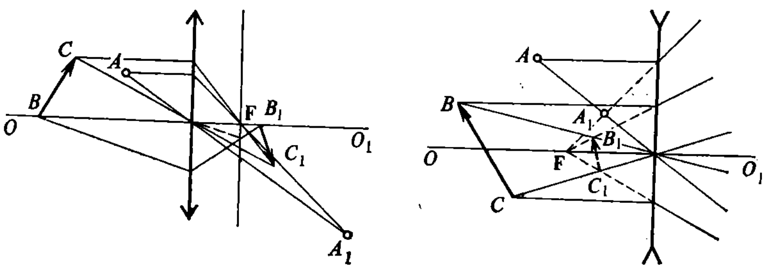
Sec. 23. Velocity of Light. Nature of Light

23.1. $3 \cdot 10^8$ m/s. **23.2.** 8 min 20 s. **23.3.** 6 640 km. **23.4.** From 400 to 750 nm. **23.5.** $5.1 \cdot 10^{14}$ Hz. **23.6.** No; 316 nm. **23.7.** The phases are equal. **23.8.** λ changes, ν does not change. **23.9.** No. **23.10.** Will increase by 103 nm. **23.11.** In a vacuum, no; in a substance, yes. **23.12.** $9.7 \cdot 10^{14}$ Hz; 310 nm; no. **23.13.** $2.76\ 10^{-19}$ J. **23.14.** By 2.3×10^{-19} J. **23.15.** No. **23.16.** By 5 900 times. **23.17.** $26 \cdot 10^{14}$.

Sec. 24. Geometrical Optics

24.1. Will increase by 30°. **24.2.** By 21°; by 42°. **24.3.** By 82 cm. **24.4.** 91 cm. **24.5.** 40°. **24.6.** Owing to a change in the light velocity. **24.7.** 20.5 W/m²; will increase. **24.8.** 0.1. **24.9.** Because over the fire the refractive index of the air changes as it is temperature dependent. **24.10.** Because a man sees in the water the displaced virtual image. **24.11.** If the refractive index of both media is the same or the angle of incidence is zero. **24.12.** Direct a ray at an angle to the interface. The velocity of light is lower in the liquid in which the ray makes a larger angle with the interface. **24.13.** By 1.55 times. **24.14.** 20°. **24.15.** 26°. **24.16.** 34°. **24.17.** 33°. **24.18.** 33.5°. **24.19.** 20°. **24.20.** 37°. **24.21.** 33.5°. **24.22.** 53°. **24.23.** 25°. **24.24.** The angle of refraction will be 41°. **24.25.** For the observer under the water. **24.26.** 65 cm. **24.27.** 124 cm. **24.28.** 45°. **24.29.** By 11°. **24.30.** By 9°; by 28°. **24.31.** 12 cm. **24.32.** Because the rays deviate from the perpendicular to its surface as they travel from it. **24.33.** $2 \cdot 10^5$ km/s. **24.34.** 49°; 40°; 24°. **24.35.** The ray will not go into the air, as it will suffer total internal reflection. **24.36.** 40°. **24.37.** Yes. **24.38.** Light scattering by glass particles. **24.39.** The fibers of the paper scatter the light, though they are transparent. The oil fills up the pores between the fibers, thus reducing the scattering. **24.40.** No. **24.41.** 51°30′; 38°. **24.42.** 47°19′.

24.43. 34°53'. 24.44. 2.0 m. 2 .45. 1.9 m. 24.46. 1.5 m. 24.47. 2.2 m. 24.48. 45 cm. 24.49. 70 cm. 24.50. By 1.9 cm. 24.51. 4.5 cm. 24.52. By 1.6 cm. 24.53. 22°. 24.54. By 1°. 24.55. 19°. 24.56. At the principal focus of the mirror, i.e. at a distance of 60 cm from the mirror. 24.57. At a distance of 54 cm from the mirror. 24.58. At a distance of 37 cm from the mirror. 24.59. 200 cm. 24.60. $F = 41.5$ cm; $R = 83$ cm. 24.61. $f = 46$ cm; the image is real reversed and reduced.

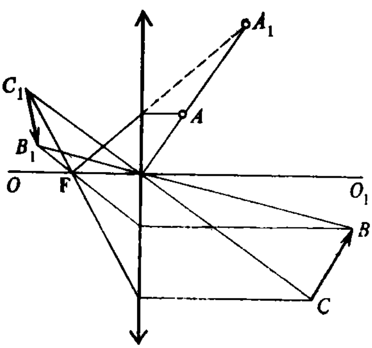


(a) (b)

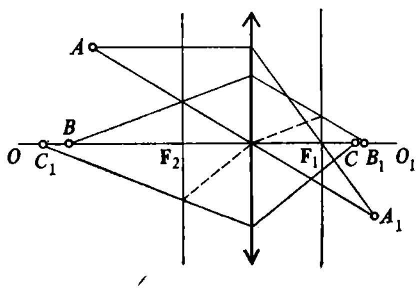
To answer 24.105

24.62. Two-fold. 24.63. —29 cm. 24.64. 40 cm. 24.65. At a distance of 12 cm from the place where the mirror was initially, on the other side of it. 24.66. Rays reflected from the first mirror will pass through the focal point of the second mirror, and the film will ignite. 24.67. These mirrors give a reduced image, therefore the driver sees more objects than in a flat mirror of the same size. 24.68. 0.15 m; 0.3 m. 24.69. 1.33 diopters. 24.70. —2.0 diopters. 24.71. 50 cm; 6.25 cm; —25 cm; —8.3 cm. 24.72. 20 cm. 24.73. —4.8 diopters. 24.74. 6.6 cm. 24.75. 39 cm. 24.76. 2.7 diopters; —0.25 diopters. 24.77. —51 cm; —260 cm; 4.0 m. 24.78. By locating the lens normal to the sun rays measure the distance from the center of the lens to a point through which the rays will come. 24.79. $f = 30$ cm; the image is real, reversed and reduced. 24.80. Near the surface of the ball. 24.81. $f = 300$ cm; $H = 6$ cm. 24.82. 5.0 diopters. 24.83. $\times 25$. 24.84. 36 cm. 24.85. 32 cm; the image is real, reversed and magnified. 24.86. $\times 32$. 24.87. 5 diopters. 24.88. 15 cm. 24.89. $f = 86$ cm. 24.90. The image will become nearer to the lens by 1.5 m. 24.91. By 267 cm. 24.92. By 6.5 cm. 24.93. $d_1 = 84.5$ cm; $d_2 = 35.5$ cm. 24.94. $d = 110$ cm. 24.95. $F = -12.5$ cm. 24.96. 30 cm. 24.97. 6.4 diopters. 24.98. 1.4×2.1 m^2. 24.99. At a distance of 21 cm. 24.100. 20 diopters. 24.101. $F = 120$ cm. 24.102. $d = -40$ cm. 24.103. $F = -60$ cm. 24.104. At a distance of 75 cm from the place where the lens was initially. 24.105. See figure (*a*)—the converging lens, (*b*)—the diverging lens. 24.106. See the figure. 24.107. See the figure. 24.108. Will decrease by 14 cm. 24.109. Will increase by 20 cm. 24.110. Will increase by 1.5 times. 24.111. By 3 times. 24.112. 11.5 cm. 24.113.

20 cm; 5.0 diopters. 4.114. 12 cm; 8.33 diopters. 24.115. 0.24 m. 24.116. 2 diopters. 24.117. 16.8 diopters; —4.7 diopters; —7.6 diopters. 24.118. $F_1 = 27$ cm; will decrease by 0.62 diopters. 24.119. (1) $h = 7$ cm; (2) nearer to the lens by 29 cm; $h = 2.5$ cm. 24.120. $f_2 = 85$ cm; the image is real, reversed and magnified. 24.121. First find the power of the converging lens (see problem 24.78), then put the lenses together and determine the power of the system. By subtracting from the last result the first one we arrive at the power of the diverging



To answer 24.106



To answer 24.107

lens. 24.122. At the principal focus of the lens; real image. 24.123. $f_1 = 140$ cm; $f_2 = 75$ cm; in both cases the image is real, on the same side of the lens as the source. 24.124. $f_1 = -22$ cm; $f_2 = -19.2$ cm; in both cases the image is virtual, the source and its image being on different sides of the lens. 24.125. Because the angle of view decreases with distance. 24.126. At a distance of more than 170 m. 24.127. 0.045 rad (2°35′). 24.128. About 63 m. 24.129. 1.2 mm. 24.130. By 2.9 mm. 24.131. 3.3. 24.132. $F = 2.1$ cm; will decrease. 24.133. 3.6 cm; 28 diopters. 24.134. $\times 13.5$. 24.135. $\times 500$. 24.136. $F = 2.5$ cm. 24.137. $\times 200$. 24.138. $\times 50$. 24.139. $\times 400$. 24.140. $F_{\text{e.p.}} = 6.0$ cm; $F_{\text{ob}} = 15$ m.

Sec. 25. Phenomena Arising from Wave Nature of Light

25.1. Destructive (the number of fringes is $m = 5.3$); destructive ($m = 6.7$); constructive ($m = 10$). 25.2. Constructive ($m = 4$); destructive ($m = 5.3$); constructive ($m = 6$). 25.3. 14.4 mm; 19.5 mm. 25.4. The width of fringes will decrease by 1.33 times. 25.5. 160 μm. 25.6. 3.4 m. 25.7. 457 nm. 25.8. $0.30 \cdot 10^{-3}$ m. 25.9. At the middle of the screen there will be a bright fringe, and to the left and to the

right of it—interference fringes. The picture is the brightest near point O, as here the greatest luminous flux will come. **25.10.** $7.2\cdot10^{-3}$ m. **25.11.** 0.17 mm. **25.12.** 0.12 μm; 0.24 μm. **25.13.** Black; the film will appear now yellow, and now black. **25.14.** The film colour will gradually change from green to blue, and violet. **25.15.** The difference in film thickness. **25.16.** Irregular film thickness. **25.17.** Owing to differences in thickness of the oil film. **25.18.** No, as one and the same colour of the film is obtained at thickness kd, where d is the minimal thickness at which the film has a given colour, k is any integer. **25.19.** No (see the answer to problem 25.18). **25.20.** No, the film is wedge-like. **25.21.** $1.8\cdot10^{-5}$ rad (3.7″). **25.22.** 8.1 mm. **25.23.** 0.06 mm. **25.24.** 9.7. **25.25.** Owing to the running off of the soapy water the thickness of the bubble changes continually. **25.26.** 5.0 mm; 4.3 mm. **25.27.** 11 m. **25.28.** 590 nm. **25.29.** Bright fringe; 4.5 mm. **25.30.** 20°. **25.31.** 0.005 mm. **25.32.** 550 nm. **25.33.** 653 nm. **25.34.** Second order. **25.35.** 590 nm. **25.36.** 13 cm. **25.37.** 400 nm. **25.38.** 0.002 mm. **25.39.** 53°. **25.40.** Aniline.

Sec. 26. Photometry

26.1. 25 cm. **26.2.** 0.48 m². **26.3.** 16 lm. **26.4.** 173 cd; 2180 lm. **26.5.** $15\cdot10^{3}$ lm. **26.6.** 755 lx. **26.7.** 124 cd. **26.8.** 10 lm/W. **26.9.** $6.0\cdot10^{5}$ cd/m². **26.10.** 1 cd. **26.11.** Does not change. **26.12.** 67 lx; 49 lx. **26.13.** By 1.73 times. **26.14.** 20 m. **26.15.** 31 lm. **26.16.** 48×10^{3} lx. **26.17.** 160 cd. **26.18.** 2.3 m. **26.19.** By 22 cm. **26.20.** 53 lx; 34 lx. **26.21.** 7.2 lx. **26.22.** 30 lx. **26.23.** 25 lx. **26.24.** 42 lx. **26.25.** At 1.4 m away from 100 cd lamp. **26.26.** 450 cd. **26.27.** 1.4 m. **26.28.** 100 lx; will decrease by 10 lx. **26.29.** 50 cm. **26.30.** 61 lx; will decrease by 11 lx.

Sec. 27. Radiation and Spectra

27.1. $2.26\cdot10^{8}$ m/s; $2.24\cdot10^{8}$ m/s; by $0.02\cdot10^{8}$ m/s. **27.2.** 1.51; 1.53. **27.3.** In glass; the spectrum obtained using the glass is wider. **27.4.** No; the shorter the wavelength, the faster varies the refractive index, therefore the spectrum is compressed in the red part and stretched in the violet part. **27.5.** The prism spectrum is stretched in its shorter-wave part, whereas the diffraction spectrum is uniformly distributed over all the wavelength range. **27.6.** Glass absorbs both ultraviolet, and infrared radiation; infrared spectra are obtained using prisms of rock salt, and ultraviolet ones using quartz prisms. **27.7.** As the atmosphere strongly absorbs ultraviolet radiation, in mountains the solar radiation contains more ultraviolet radiation than in valleys; therefore the medical lamp producing much ultraviolet radiation is called "mountain sun". **27.8.** The colour is determined by reflected light, but its composition depends on the composition of incident light. **27.9.** Daylight lamps. **27.10.** The solar radiation that comes into the green-house is absorbed by the soil, whereas the infrared radiation of the soil does not come through the glass. **27.11.** The

spectral analysis is based on the line spectra of atoms in vapour of a substance which become excited by a flame or arc. 27.12. The percentage content of various components of alloy. 27.13. The higher the voltage, the shorter the wavelength of radiation emitted. 27.14. The energy of X-ray quanta cannot be higher than the energy liberated in deceleration of electrons; by the voltage across the X-ray tube as $W = Ue$. 27.15. Yes; yes. 27.16. The higher the temperature of a body, the more thermal radiation it emits. 27.17. Black, as it radiates more. 27.18. The higher the absorptive power of a body, the higher is its emissive power. 27.19. Owing to energy exchange by thermal radiation. 27.20. By 3.5 times. 27.21. 2.5 kJ. 27.22. 1 270 nm; in the far infrared range. 27.23. 5 300°C. 27.24. 96 nm; $4.6 \cdot 10^{10}$ W/m².

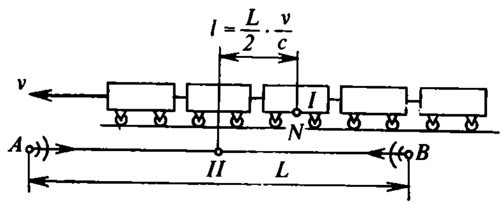
Sec. 28. Phenomena Arising from Quantum Nature of Light

28.1. $5.9 \cdot 10^8$ N. 28.2. By 2.0 kg; the radiation of the Earth reduces its mass. 28.3. Yes; $J_S \sim 1/r^2$. 28.4. 600 W/m². 28.5. 2 680 W/m². 28.6. By 7,200 times; will decrease. 28.7. No; the pressure of light tends to rotate the cross-head in the opposite direction. 28.8. 300 J; $8.3 \cdot 10^{20}$. 28.9. $4.6 \cdot 10^{-6}$ Pa. 28.10. It ensures the synthesis from CO_2 and H_2O of energy-rich compounds. 28.11. Shorter-wave radiation; longer-wave radiation (infrared). 28.12. In external photoelectric effect electrons are ejected from a substance, and in internal effect they remain inside it. 28.13. The metal plate will be charged positively, and the semiconductor one will remain neutral. 28.14. 2.34 eV. 28.15. 260 nm. 28.16. 265 nm. 28.17. 291 nm. 28.18. In caesium, yes; in magnesium, no, as $\lambda_{Cs} = 643$ nm, $\lambda_{Mg} = 337$ nm. 28.19. No. 28.20. $2.13 \cdot 10^{-19}$ J. 28.21. 2.13 eV; 582 nm. 28.22. $6.2 \cdot 10^5$ m/s. 28.23. $\lambda = 367$ nm. 28.24. $\lambda = 223$ nm. 28.25. To produce latent image; by electron beam. 28.26. Because silicon is more sensitive to wavelengths accounting for the most part of the solar spectrum. 28.27. Onto the set-scene covered by luminescent substance when required ultraviolet radiation is directed. 28.28. $8.6 \cdot 10^{-3}$ nm. 28.29. 4 000 km/s; $3.68 \cdot 10^{-24}$ kg·m/s.

Sec. 29. Fundamentals of Special Relativity Theory

29.1. $x' = x - v_x t$, $y' = y$, $z' = z$; $x_1' = 200$ m, $y_1' = 5$ m; $z_1' = 15$ m; $x_2' = 100$ m, $y_2' = 5$ m, $z_2' = 15$ m; $x_3' = -100$ m, $y_3' = 5$ m, $z_3' = 15$ m. 29.2. $x' = 2.0 \cdot 10^3$ m $-$ 100 m/s$\cdot t$, $y' = 0$, $z' = 0$, $x_1' = 2.0 \cdot 10^3$ m; $x_2' = 1.0 \cdot 10^3$ m; $x_3' = 0$; $x_4' = -1.0 \cdot 10^3$ m. 29.3. $x = x' + vt$, $y = y'$, $z = z'$; $x_1 = 300$ m, $y_1 = 0$, $z_1 = 0$; $x_2 = 0$, $y_2 = 0$, $z_2 = 0$; $x_3 = -100$ m, $y_3 = 20$ m, $z_3 = 15$ m. 29.4. $v = v_x = 5.0$ m/s. 29.5. $\mathbf{v}' = \mathbf{v} + \mathbf{u}$; $v' = v_1 + v_2$, $v' = 50$ m/s; $v' = v_1 - v_2$, $v' = 10$ m/s. 29.6. 5.0 m/s. 29.7. $v = \sqrt{v_1^2 + v_2^2 + 2v_1v_2 \cos\alpha} \approx 5.82$ m/s. 29.8. Yes; no. 29.9. (1) $x' = 0$, $y' = 0$, $z' = 0$, $t' = 0$; (2) $x' \approx 10^8$ m, $y' = 0$, $z' = 0$, $t' \approx 0.33$ s; (3) $x' \approx -1.2 \cdot 10^8$ m, $y' = 0$, $z' = 0$, $t' \approx 7.0$ s. 29.10. $x = 1.8 \cdot 10^4$ m,

$y = 2.0 \cdot 10^2$ m, $z = 15$ m, $t = 6.0 \cdot 10^{-5}$ s. **29.11.** No. For observer *II* the lightning struck at point *A* first; for observer *III* the lightning struck first at point *B*. For observers at equal distances from points *A* and *B*. **29.12.** No. As the train moves to point *A*, light from *A* should cover smaller distance than from *B*; hence for the travelling observer *A* will be on first. **29.13.** Light *B*; light *B*, light *A*. Observer *I* should



To answer 29.13

be at point *N* at such a distance l from the middle of the platform so that by the instant the light from *A* and *B* arrives at the middle of the platform he would be opposite it: $l = Lv/2c$ (see the figure). **29.14.** No. **29.15.** The proper length of the rod l_0, is its length in the frame in which the rod is at rest; the length is dissimilar in different frames of reference. No. **29.16.** 0.80 m. **29.17.** $\Delta l = 0.4 l_0$. **29.18.** $42.3 \cdot 10^6$ m/s; $2 \cdot 10^8$ m/s. **29.19.** From the formula $l = l_0/\sqrt{1 - v^2/c^2}$ it follows that at $v \to c$, $l \to 0$; hence at $v = c$ the length of the body would become zero, which is impossible. **29.20.** 8 h; $\tau = 10$ h. **29.21.** $\tau \approx 2 \cdot 10^{-5}$ s; $\tau_0 \approx 2 \cdot 10^{-6}$ s; $l_0 = 6 \cdot 10^2$ m. **29.22.** 59 min 59.8 s; 56 min 34 s; 33 min 7 s. **29.23.** 71 years. **29.24.** $\tau \approx 80.8$ years; $\tau_0 \approx$ ≈ 11.3 years. **29.25.** $0.99c$. **29.26.** In the frame connected with the observer on the Earth the path covered by the μ-meson before decay is about $5 \cdot 10^3$ m, therefore μ-mesons observed on the ground cannot come from space. **29.27.** $u_{cl} = 4.0 \cdot 10^5$ km/s; $u_{rel} = 2.8 \cdot 10^5$ km/s; by $1.2 \cdot 10^5$ km/s. **29.28.** $u_1 = 0.36c$; $u_2 = 0.99c$. **29.29.** $u = c$. **29.30.** $u_{rel} = c$; $u_{cl} = 2c$, which contradicts relativity theory. **29.31.** Yes, but its velocity is less than the velocity of light in a vacuum. This produces electromagnetic radiation (Cherenkov effect). **29.32.** At velocities close to c the changes in mass cannot be ignored. Therefore the acceleration $a = F/m$ at $F = \text{const}$ will not be constant. **29.33.** It follows from the formula $m = m_0/\sqrt{1 - v^2/c^2}$, that $m \approx m_0$ at $v \ll c$. Newton's mechanics can be considered valid at $v \ll c$. **29.34.** By 1.51 times. **29.35.** 1.34 kg. **29.36.** $0.80c$. **29.37.** Longitudinal dimensions of a body in the direction of motion are reduced by $\sqrt{1 - v^2/c^2}$ times, the transverse dimensions being constant; the mass of a body increases by $1/\sqrt{1 - v^2/c^2}$ times; the density increases by $1/(1 - v^2/c^2)$ times. No. **29.38.** $3.64 \cdot 10^{-22}$ kg·m/s. **29.39.** $p = 1.93 \times$ $\times 10^{-18}$ kg·m/s; $U = 2.8 \cdot 10^9$ V. **29.40.** $p_1 = 0$; $p_2 = mu_2$, where

$m = m_0/\sqrt{1 - v^2/c^2}$, $u_2 = \dfrac{v-(-v)}{1-v(-v)/c^2} = \dfrac{2v}{1+v^2/c^2}$ (by velocity-composition theorem). 29.41. $E_{0e} = 8.2 \cdot 10^{-14}$ J $= 0.511$ MeV; $E_{0p} = 1.5 \cdot 10^{-10}$ J $= 938$ MeV. 29.42. $E_k = 2m_0c^2$; $E = 3m_0c^2$; $p \approx 2.82m_0c$. 29.43. $v \approx 0.865c \approx 2.59 \cdot 10^8$ m/s. 29.44. $T/T_0 = 1/\sqrt{1 - v^2/c^2} = 1.25$, where $1/\sqrt{1 - v^2/c^2} = (E_0 + E_k)/E_0$. 29.45. $v = c\sqrt{1 - 1/(1 + E_k/E_0)^2} \approx 3 \cdot 10^8$ m/s; $mc^2/m_0c^2 = 1 + E_k/m_0c^2 \approx 75.6$. 29.46. $E_k = 10E_0 = 5.1$ MeV $= 8.2 \cdot 10^{-13}$ J; $p = 8.65 \times 10^{-22}$ kg·m/s. 29.47. $eU = 10E_0$; $U \approx 5.11 \cdot 10^6$ V. 29.48. 9.38×10^9 V; by 11 times. 29.49. 916 MV. 29.50. $1.1 \cdot 10^{-27}$ kg·m/s; 3.7×10^{-36} kg. 29.51. $1.1 \cdot 10^{-27}$ kg·m/s; $3.7 \cdot 10^{-36}$ kg. 29.52. $4.0 \cdot 10^{-11}$ m; $7.5 \cdot 10^{18}$ Hz. 29.53. $9 \cdot 10^{13}$ J. 29.54. $1.0 \cdot 10^{-7}$ kg. 29.55. $3.87 \cdot 10^{26}$ J; $4.3 \cdot 10^9$ kg.

Sec. 30. Atomic Structure

30.1. To register α-particles by scintillations on the screen; transparent. 30.2. Ionization of gas by charged particles. 30.3. The principle of the Wilson chamber is based on the condensation of vapour on ions; the action of the bubble chamber is based on the boiling of liquid around ions under lower pressure. Bubble chamber because in it particles decelerate faster. 30.4. Classically, the Rutherford atom must be unstable. 30.5. Bohr assumed that for each atom there are a number of distinct values of energy which it can have. To each energy value there corresponds a separate electron orbit. 30.6. No. 30.7. Atom energy change in transition from one allowed state into another. 30.8. The states corresponding to all the allowed energy levels, except the lowest one; in the excited state an atom stays for a limited period of time, and in the normal state—indefinitely. 30.9. No. 30.10. Three. 30.11. An atom can absorb only such quanta which correspond to its transition from one energy level to another. In the reverse process only the same quanta may be emitted. 30.12. Two in K-shell, eight in L-shell and one in M-shell; two in K-shell and one in L-shell. 30.13. Ultra-violet, visible and infra-red; X-rays. 30.14. No; yes. 30.15. The spectrum shifts to red wavelengths. 30.16. Yes. 30.17. Ultra-violet; infra-red. 30.18. $\nu_1 = 6 \cdot 10^{15}$ Hz, $T_1 = 1.67 \cdot 10^{-16}$ s; $\nu_2 = 7.5 \cdot 10^{14}$ Hz, $T_2 = 1.33 \cdot 10^{-15}$ s. 30.19. 2.47×10^{15} Hz. 30.20. 485 nm; green-blue. 30.21. $-2.41 \cdot 10^{-19}$ J. 30.22. 13.5 V. 30.23. 91 nm.

Sec. 31. Nuclear Physics

31.1. Species of an element with the same atomic number and different mass numbers. 31.2. The element shifts two positions to the left. 31.3. Shifts one position to the right. 31.4. Into $^{222}_{84}Po$. 31.5. Into $^{234}_{92}U$. 31.6. Into $^{206}_{82}Pb$. 31.7. $^{224}_{88}Ra$. 31.8. $^{215}_{84}Po$. 31.9. $^{224}_{88}Ra$. 31.10. No. 31.11. Ionization of air. 31.12. Two different species of atoms suffer α-decay. 31.13. Half-life. 31.14. 10.7 MBc. 31.15. 6.2 $\times$

$\times\ 10^9$ decays/min. **31.16.** $6.76 \cdot 10^{-4}$ curies. **31.17.** In 5 min 5 s. **31.18.** 61 MBc. **31.19.** 81.4 GBc. **31.20.** 0.26 kg. **31.21.** When observation time is small as compared with the half-life. **31.22.** In 64.5 days. **31.23.** 17.5%; 1.5 years. **31.24.** Substances with large half-life. **31.25.** $2.72 \cdot 10^5$ km/s. **31.26.** 116°. **31.27.** Neutron produces no ions on its path, but proton does. **31.28.** Neutrons with kinetic energy close to the average kinetic energy of thermal motion of atoms. **31.29.** Because in collision of neutrons with an atom the latter receives the more energy, the smaller its mass. **31.30.** For precision determination of atom masses. **31.31.** ${}^1_1H$, ${}^2_1H$, ${}^3_1H$; isotope ${}^3_1H$ is radioactive and decays as follows: ${}^3_1H \rightarrow {}^3_2He + {}_{-1}^{0}e$. **31.32.** ${}^{14}_7N + {}^4_2He \rightarrow {}^1_1H + {}^{17}_8O$; ${}^9_4Be + {}^4_2He \rightarrow {}^{12}_6C + {}^1_0n$. **31.33.** Ionization of two types of atoms. **31.34.** By absorbing neutrons and two β-decays in succession. **31.35.** ${}^7_3Li + {}^1_1H \rightarrow 2\,{}^4_2He$; 17.4 MeV; $2.0 \cdot 10^7$ m/s. **31.36.** 8.15 MeV; ${}^4_2He$. **31.37.** ${}^4_2He$; 8.6 MeV. **31.38.** ${}^{27}_{13}Al + {}^4_2He \rightarrow {}^{30}_{15}P + {}^1_0n$; -3 MeV (the reaction is possible if α-particles have an energy more than 3 MeV). **31.39.** In α-decay nuclei emit α-particles only, and in β-decay, apart from electrons, neutrinos are produced, which carry away some energy. **31.40.** The nucleus disintegrates into almost equal parts and neutrons are liberated. **31.41.** Using fissible materials and neutrons. **31.42.** Because the explosion occurs only after a certain critical mass of nuclear fuel is achieved; no.

Sec. 32. Elements of Astronomy

32.2. From left to right, clockwise. **32.4.** Sirius; Vega. **32.8.** $z = 90° - \varphi = 34°15'$. **32.10.** The path of apparent annual motion of the Sun among the stars; 23°27′. **32.11.** The constellations traversed by the Sun and Moon: Pisces, Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, 12 constellations all told. **32.12.** $90° - \varphi \pm \delta$; 57°42′; 10°48′. **32.13.** In winter the Sun moves faster over the ecliptic. **32.14.** $3{,}842 \cdot 10^5$ km; $3{,}840 \cdot 10^5$ km. **32.15.** 1,670 km. **32.16.** $p = 60.3'$. **32.17.** $D = \frac{1}{p} = \frac{1}{0.762''}$; 1.3 parsecs; 4.2 light years; $4.02 \cdot 10^{13}$ km. **32.18.** $6.5 \cdot 10^6$ km. **32.19.** 0.543″. **32.20.** By 109 times. **32.21.** 11 h, October 10. **32.22.** 37°34′15″. **32.23.** 9 h. **32.24.** $5.53 \cdot 10^3$ kg/m³. **32.25.** $1.4 \cdot 10^3$ kg/m³. **32.26.** 0.723 AU, $1.082 \cdot 10^8$ km; 39.5 AU,; $5.91 \cdot 10^9$ km. **32.27.** 333,000 Earth's masses. **32.28.** Recedes 0.4861 nm; 300 km/s. **32.29.** 2.1 km/s. **32.30.** 30 km/s. **32.31**/486.051 nm. **32.32.** Recedes; $1.38 \cdot 10^5$ km/s by 230 nm. **32.33.** $4.8 \cdot 10^4$ km/s. **32.34.** At velocities of objects close to the velocity of light c it is necessary to utilize the formula given in problem 32.28; $v = (8/9)c$. **32.35.** $3.8 \cdot 10^4$ km/s decreases. **32.36.** $2.4 \cdot 10^9$ J. **32.37.** $7.7 \cdot 10^3$ m/s; $5.4 \cdot 10^3$ s. **32.38.** 7.8×10^3 m/s. **32.39.** $42.4 \cdot 10^6$ m; $36 \cdot 10^6$ m; $3.1 \cdot 10^3$ m/s. **32.40.** $7.44 \cdot 10^3$ s; $1.6 \cdot 10^3$ m/s.

APPENDICES

Table 1

Fundamental Physical Constants

Gravitational constant	$\gamma = 6.672 \cdot 10^{-11}$ N·m²·kg⁻²
Free fall acceleration (normal)	$g_n = 9.80665$ m/s² ≈ 9.81 m/s²
Normal atmospheric pressure	$p_0 = 101.325$ Pa
Avogadro number	$N_A = 6.022045 \cdot 10^{23}$ mol⁻¹
Volume of 1 mole of ideal gas at STP	$V = RT_0/p_0 = 22.41383 \cdot 10^{-3}$ m³·mol⁻¹
Universal gas constant (molar)	$R = 8.31441$ J·mol⁻¹·K⁻¹
Loschmidt number	$N_L = 2.7 \cdot 10^{25}$ m⁻³
Boltzmann's constant	$k = R/N_A = 1.380662 \cdot 10^{-23}$ J·K⁻¹
Light velocity in a vacuum	$c = 2.99792458 \cdot 10^{8}$ m/s
Magnetic constant	$\mu_0 = 4\pi \cdot 10^{-7}$ H/m $= 12.5663706 \cdot 10^{-7}$ H/m
Electric constant	$\varepsilon_0 = 8.85418782 \cdot 10^{-12}$ F/m
Rest mass of electron	$m_e = 9.109534 \cdot 10^{-31}$ kg $= 5.4858026 \cdot 10^{-4}$ a.m.u.
Rest mass of proton	$m_p = 1.6726485 \cdot 10^{-27}$ kg $= 1.007276470$ a.m.u.
Rest mass of neutron	$m_n = 1.6749543 \cdot 10^{-27}$ kg $= 1.008665012$ a.m.u.
Atomic mass unit	a.m.u. $= 1.6605655 \cdot 10^{-27}$ kg (corresponds to energy 931.3 MeV)
Elementary charge	$e = 1.6021892 \cdot 10^{-19}$ C
Electron charge-mass ratio	$e/m_e = 1.7588047 \cdot 10^{11}$ C·kg⁻¹

Table 1 (continued)

Faraday constant	$F=N_Ae=9.648456\cdot10^4$ C/mol
Planck's constant	$h=6.626176\cdot10^{-34}$ J·s, $\hbar=h/2\pi=1.0545887\cdot10^{-34}$ J·s
Solar constant	$J_S=1370$ W/m^2
Stefan-Boltzmann constant	$\sigma=5.67032\cdot10^{-8}$ W/(m^2·K^4)
Wien constant	$b=0.00289782$ m·K
Rydberg constant	$R_\infty=10,973,731$ m^{-1}
First Bohr radius	$a_0=0.529177006\cdot10^{-10}$ m

Table 2

SI Units

Quantity	Symbol and equation	Unit (abbrev.)	Non-SI Metric Units
		1. Base Units	
Length	l	metre (m)	1 cm $=10^{-2}$ m 1 km $=10^3$ m
Mass	m	kilogram (kg)	1 g $=10^{-3}$ kg 1 q $=10^2$ kg 1 t $=10^3$ kg
Time	t	second (s)	1 min $=60$ s 1 h $=3600$ s 1 day $=86,400$ s
Thermodynamic temperature	T	kelvin (K)	1°C $=1$ K t°C $=(T-273.15)$ K
Electric current	I	ampere (A)	
Luminous intensity	J	candela (cd)	
Amount of substance	μ	mole (mol)	1 kmol $=10^3$ mol

Table 2 (continued)

Quantity	Symbol and equation	Unit (abbrev.)	Non-SI Metric Units
		2. Supplementary Units	
Plane angle	$\varphi = \frac{l}{R}$	radian (rad)	$1° = \frac{\pi}{180}$ rad $1' = \frac{\pi}{108} \cdot 10^{-2}$ rad $1'' = \frac{\pi}{648} \cdot 10^{-3}$ rad
Solid angle	$\Omega = \frac{S}{R^2}$	steradian (sr)	
		3. Derived Units	
Area	$S = l^2$	square metre (m^2)	1 $cm^2 = 10^{-4}$ m^2 1 $km^2 = 10^6$ m^2
Volume, capacity	$V = l^3$	cubic metre (m^3)	1 $cm^3 = 10^{-6}$ m^3 1 l $= 1.000028 \cdot 10^{-3}$ m^3
Density	$\rho = \frac{m}{V}$	kilogram per cubic metre (kg/m^3)	1 $g/cm^3 = 10^3$ kg/m^3
Velocity	$v = \frac{l}{t}$	metre per second (m/s)	1 cm/s $= 10^{-2}$ m/s 1 km/h $= \frac{1}{3.6}$ m/s
Acceleration	$a = \frac{\Delta v}{\Delta t}$	metre per second squared (m/s^2)	1 $cm/s^2 = 10^{-2}$ m/s^2
Force, weight	$F = ma$ $G = mg$	newton (N)	1 kgf $= 9.80665$ N 1 dyn $= 10^{-5}$ N
Momentum	$p = mv$	kilogram-metre per second (kg·m/s)	1 g·cm/s $=$ $= 10^{-5}$ kg·m/s

Table 2 (continued)

Quantity	Symbol and equation	Unit (abbrev.)	Non-SI Metric Units
Impulse of force	$I=Ft$	newton-second (N·s)	1 kgf·s $=9.80665$ N·s
Work, energy: kinetic, potential	$A=FS$ $K=\frac{mv^2}{2}$ $\Pi=mgh$	joule (J)	1 erg $=10^{-7}$ J
Power	$P=\frac{A}{t}$	watt (W)	1 erg/s $=10^{-7}$ W 1 hp $=735.449$ W
Pressure	$p=\frac{F}{S}$	pascal (Pa)	1 at $=9.80665\times$ $\times 10^4$ Pa 1 atm $=101.325$ Pa 1 mm Hg $=$ $=133.32$ Pa
Stress	$\sigma=\frac{F}{S}$	pascal (Pa)	
Stiffness	$k=\frac{F}{\Delta l}$	newton per metre (N/m)	
Period, period of simple pendulum	$T=t/n$, $T=2\pi\sqrt{l/g}$	second (s)	
Frequency	$\nu=\frac{1}{T}$	hertz (Hz)	
Angular velocity	$\omega=\varphi/t$ $\omega=2\pi\nu$	radian per second (rad/s)	
Angular acceleration	$\varepsilon=\frac{\Delta\omega}{\Delta t}$	radian per second squared (rad/s²)	

Table 2 (continued)

Quantity	Symbol and equation	Unit (abbrev.)	Non-SI Metric Units
Phase of oscillation	$\varphi = \omega t + \varphi_0$	radian (rad)	
Quantity of heat	$Q = A$	joule (J)	1 cal = 4.1868 J 1 kcal = 4186.8 J
Specific heat capacity	$c = \frac{Q}{m\,\Delta T}$	joule per kilogram kelvin J/(kg·K)	1 kcal/(kg·°C) = = 4186.8 J/(kg·K)
Molar heat capacity	$C = \frac{Q}{\mu\,\Delta T}$	joule per mole-kelvin (J/mol·K)	
Specific latent heat of change of phase (fusion, boiling)	$\lambda = Q/m$, $r = Q/m$	joule per kilogram (J/kg)	
Coefficient of linear expansion	$\alpha = \frac{\Delta l}{l_0 \Delta T}$	kelvin (K^{-1})	
Coefficient of volume expansion	$\beta = \frac{\Delta V}{V_0 \Delta T}$		
Dynamic viscosity	$\eta = \frac{F}{S\,(\Delta v/\Delta l)}$	pascal-second (Pa·s)	
Surface tension	$\sigma = \frac{F}{l}$	newton per metre (N/m)	1 dyn/cm = = 10^{-3} N/m
Electric charge	$q = It$	coulomb (C)	1 statC = $\frac{1}{3 \cdot 10^9}$ C

Table 2 (continued)

Quantity	Symbol and equation	Unit (abbrev.)	Non-SI Metric Units
Surface density of electric charge	$\sigma = \frac{q}{S}$	coulomb per square metre (C/m^2)	
Electric field strength	$E = \frac{U}{l}$ $E = \frac{F}{q}$ $E = \frac{q}{4\pi\varepsilon_0\varepsilon r^2}$	volt per metre (V/m)	1 statV/cm = $= 3 \cdot 10^4$ V/m
Potential, difference, voltage electromotive force	$\varphi_1 - \varphi_2 = A/q$ $U = IR$ $\mathscr{E} = A/q$	volt (V)	1 statV = 300 V
Capacitance, capacitance of flat capacitor	$C = q/U$ $C = \varepsilon_0\varepsilon S/d$	farad (F)	1 cm = $\frac{1}{9 \cdot 10^{11}}$ F
Energy of charged capacitor	$W_{el} = \frac{CU^2}{2}$	joule (J)	
Energy density of electric field	$\omega_{el} = \frac{\varepsilon_0\varepsilon E^2}{2}$	joule per cubic metre (J/m^3)	
Current density	$j = \frac{I}{S}$	ampere per square metre (A/m^2)	
Electric resistance	$R = U/I$ $R = \rho\frac{l}{S}$	ohm (Ω)	

Table 2 (continued)

Quantity	Symbol and equation	Unit (abbrev.)	Non-SI Metric Units
Electric conductance	$G=\frac{1}{R}$	siemens (S)	
Work, energy	$A=IUt$ $A=I^2Rt$ $A=\frac{U^2}{R}t$	joule (J)	1 MeV= $=1.602\cdot10^{-13}$ J 1 W·h=3 600 J 1 kW·h=$3.6\cdot10^6$ J
Power, electrical output	$P=IU$ $P=I^2R$ $P=U^2/R$	watt (W)	
Electrochemical equivalent	$k=\frac{m}{q}$	kilogram per coulomb (kg/C)	
Magnetic induction	$B=\frac{F}{I\Delta l}$	tesla (T)	1 G=10^{-4} T
Magnetic moment of current-carrying loop	$p_m=IS$	ampere per square metre (A·m²)	
Magnetic flux	$\Phi=BS_\perp$	weber (Wb)	1 Mx=10^{-8} Wb
Magnetic field strength Magnetic field strength produced by: straight current circular current solenoid	$H=\frac{B}{\mu_0\mu}$ $H=\frac{I}{2\pi r}$ $H=\frac{I}{2r}$ $H=\frac{Iw}{l}$	ampere per metre (A/m)	1 Oe=$10^3/4$ A/m

Table 2 (continued)

Quantity	Symbol and equation	Unit (abbrev.)	Non-SI Metric Units
Inductance	$L=\frac{\Psi}{I}$	henry (H)	1 cm $=10^{-9}$ H
Magnetic field energy	$W_{mag}=\frac{LI^2}{2}$	joule (J)	
Volume energy density of magnetic field	$w_{mag}=\frac{\mu_0\mu H^2}{2}$	joule per cubic metre (J/m^3)	
Power of lens	$D=1/F$	diopter (D)	
Luminous flux	$\Phi=J\cdot\omega$	lumen (lm)	
Illuminance	$E=\Phi/S$ $E=J/r^2$	lux (lx)	
Luminance	$B=J/S_{\perp}$	candela per square metre (cd/m^2)	

Table 3

Densities of Substances

Substance	ρ, kg/m^3	Substance	ρ, kg/m^3
	Solids (at 293 K)		
Aluminium	$2.7\cdot10^3$	Coal	$1.4\cdot10^3$
Ammonium chloride	$1.5\cdot10^3$	Constantan	$8.9\cdot10^3$
		Copper	$8.9\cdot10^3$
Brass	$8.5\cdot10^3$	Copper sulphate	$2.2\cdot10^3$
Brick	$1.8\cdot10^3$	Cork	$2.4\cdot10^2$

Table 3 (continued)

Substance	ρ, kg/m³	Substance	ρ, kg/m³
Diamond	$3.5 \cdot 10^3$	Nickel	$8.9 \cdot 10^3$
Ebonite	$1.2 \cdot 10^3$	Nickeline	$8.8 \cdot 10^3$
Germanium	$5.32 \cdot 10^3$	Paraffin	$9.0 \cdot 10^2$
Glass	$2.5 \cdot 10^3$	Platinum	$2.15 \cdot 10^4$
Gold	$1.93 \cdot 10^4$	Polonium	$9.28 \cdot 10^3$
Graphite	$2.1 \cdot 10^3$	Porcelain	$2.3 \cdot 10^3$
Ice (0°C)	$0.9 \cdot 10^3$	Salt, common	$2.1 \cdot 10^3$
Iridium	$2.24 \cdot 10^4$	Silver	$1.05 \cdot 10^4$
Iron, cast iron	$7.4 \cdot 10^3$	Tin	$7.3 \cdot 10^3$
Iron, steel	$7.8 \cdot 10^3$	Tungsten	$1.93 \cdot 10^4$
Lead	$1.14 \cdot 10^4$	Uranium	$1.87 \cdot 10^4$
Manganin	$8.5 \cdot 10^3$	Zinc	$7.1 \cdot 10^3$
Mica	$2.8 \cdot 10^3$	Zinc sulphide	$4.04 \cdot 10^3$
Nichrome	$8.3 \cdot 10^3$		
Liquids (at 293 K)			
Alcohol, ethyl	$7.9 \cdot 10^2$	Oil, olive	$9.2 \cdot 10^2$
Aniline	$1.02 \cdot 10^3$	Petrol	$7.0 \cdot 10^2$
Benzene	$9 \cdot 10^2$	Petroleum	$8\text{-}9 \cdot 10^2$
Copper sulfate, saturated solution	$1.15 \cdot 10^3$	Sulfate	$7.1 \cdot 10^2$
		Turpentine	$8.7 \cdot 10^2$
		Water at 277°K	$1.0 \cdot 10^3$
Glycerin	$1.20 \cdot 10^3$	Water at 373°K	$0.958 \cdot 10^3$
Kerosene	$8.0 \cdot 10^2$	Water, heavy, at 284.23 K (max. density temperature)	$1.106 \cdot 10^3$
Mercury at 0°C	$1.36 \cdot 10^4$		
Nitrobenzene	$1.2 \cdot 10^3$		
Oil, mineral	$9.2 \cdot 10^2$		
Gases (at STP : $p_0 = 1.013 \cdot 10^5$ Pa, $T_0 = 273$ K)			
Acetylene	1.17	Hydrogen	0.09
Air	1.29	Krypton	3.74
Ammonia	0.77	Methane	0.72
Argon	1.78	Neon	0.90
Carbon dioxide	1.98	Nitrogen	1.25
Chlorine	3.21	Oxygen	1.43
Gas, illuminating	0.73	Xenon	5.85
Helium	0.18		

Table 4

Modulus of Elasticity

Substance	*E*, gPa	Substance	*E*, gPa
Aluminium	70	Iron	200
Brass	110	Iron, cast	90
Brick	28	Lead	17
Concrete	20	Steel	220
Copper	130		

Table 5

Specific Heats

Substance	*c*, J/(kg·K)	Substance	*c*, J/(kg·K)
	Solids		
Aluminium	880	Lead	120
Brass	380	Naphthalene	1 300
Brick	750	Paraffin	3 200
Cement	800	Platinum	125
Concrete	880	Sand	970
Copper	380	Silver	250
Glass	840	Sulfur	712
Gold	125	Tin	250
Ice	2 090	Zinc	400
Iron, cast	550	Wood	2 700
Iron, steel	460		
	Liquids		
Alcohol, ethyl	2 430	Oil, machine	2 100
Glycerin	2 430	Oil, transformer	2 093
Iron	830	Sulfate	2 330
Kerosene	2 140	Water	4 187
Mercury	125		
	Gases (at constant pressure)		
Air ($\mu = 0.29$ kg/mol)	1 000	Hydrogen	14 300
Ammonia	2 100	Nitrogen	1 000
Carbon dioxide	830	Oxygen	920
Helium	5 200	Water vapour	2 200

26—01494

Table 6

Specific Heats of Combustion

Substance	q, J/kg	Substance	q, J/kg
	Solid Fuels		
Charcoal	$2.97 \cdot 10^7$	Firewood (dry), straw	$8.3 \cdot 10^6$
Coal:			
brand A-I	$2.05 \cdot 10^7$	Gun powder	$3.0 \cdot 10^6$
brand A-II	$3.03 \cdot 10^7$	Peat	$1.5 \cdot 10^7$
Coal, brown	$9.03 \cdot 10^6$	Wood blocks	$1.5 \cdot 10^7$
Coke	$3.03 \cdot 10^7$		
	Liquid Fuels		
Alcohol, ethyl	$2.7 \cdot 10^7$	Ligroin	$4.33 \cdot 10^7$
Diesel fuel	$4.2 \cdot 10^7$	Mazut	$4.0 \cdot 10^7$
Kerosene	$4.31 \cdot 10^7$	Petrol, petroleum	$4.6 \cdot 10^7$
	Gaseous Fuels (for 1 m^3 at STP)		
Coke-oven gas	$1.64 \cdot 10^7$	Natural gas	$3.55 \cdot 10^7$
Illuminating gas	$2.1 \cdot 10^7$	Producer gas	$5.5 \cdot 10^6$

Table 7

Boiling Points and Specific Heats of Vaporization
(at boiling point)

Substance	T_b, K	t_b, °C	r, J/kg
Acetone	329.2	56.2	$5.2 \cdot 10^5$
Air	81	—192	$2.1 \cdot 10^5$
Alcohol, ethyl	351	78	$8.57 \cdot 10^5$
Ammonia	239.6	—33.4	$1.37 \cdot 10^6$
Freon-12	243.2	—29.8	$1.68 \cdot 10^6$
Iron	3323	3050	$5.8 \cdot 10^4$
Mercury	630	357	$2.85 \cdot 10^5$
Petrol	423	150	$3.0 \cdot 11^5$
Sulfate	308	35	$3.52 \cdot 10^5$
Turpentine	433	160	$2.94 \cdot 10^5$
Water	373	100	$2.26 \cdot 10^6$
Water, heavy	374.43	101.43	$2.06 \cdot 10^6$

Table 8

Variation of Pressure and Density of Water Vapour with Temperature

t, °C	p_s, kPa	ρ, 10^{-3} kg/m³	t, °C	p_s, kPa	ρ, 10^{-3} kg/m³
−10	0.260	2.14	17	1.933	14.5
−5	0.401	3.24	18	2.066	15.4
−4	0.437	3.51	19	2.199	16.3
−3	0.476	3.81	20	2.333	17.3
−2	0.517	4.13	21	2.493	18.3
−1	0.563	4.47	22	2.639	19.4
0	0.613	4.80	23	2.813	20.6
1	0.653	5.20	24	2.986	21.8
2	0.706	5.60	25	3.173	23.0
3	0.760	6.00	26	3.359	24.4
4	0.813	6.40	27	3.559	25.8
5	0.880	6.80	28	3.786	27.2
6	0.933	7.30	29	3.999	28.7
7	1.000	7.80	30	4.239	30.3
8	1.066	8.30	40	7.371	51.2
9	1.146	8.80	50	12.33	83.0
10	1.226	9.40	60	19.92	130.0
11	1.306	10.0	80	47.33	293
12	1.399	10.7	100	101.3	598
13	1.492	11.4	120	198.5	1123
14	1.599	12.1	160	618.0	3259
15	1.706	12.8	200	1554	7763
16	1.813	13.6			

Table 9

Boiling Points and Critical Constants

Substance	Boiling point, t_b, °C	Critical constants	
		Temperature, t_{cr}, °C	Pressure, p_{cr}, 10^5 Pa
Alcohol, ethyl	78	243.1	63
Argon	−186	−122.4	48
Ethylacetate	35	193.8	35.6

Table 9 (continued)

Substance	Boiling point, t_b, °C	Critical constants: Temperature, t_{cr}, °C	Critical constants: Pressure, p_{cr}, 10^5 Pa
Helium	—269	—267.9	2.25
Hydrogen	—253	—241	12.8
Krypton	—193	—63.62	54.27
Neon	—246	—228.7	26.9
Nitrogen	—196	—147.1	33.5
Oxygen	—183	—118.4	49.7
Water	100	374.2	218.5
Xenon	—108	18.76	57.64

Table 10

Melting Points and Specific Heats of Fusion for Solids
(at melting points)

Substance	T_m, K	λ, J/kg	Substance	T_m, K	λ, J/kg
Aluminium	932	$3.8\cdot10^5$	Silver	1233	$8.8\cdot10^4$
Copper	1356	$1.8\cdot10^5$	Steel	1673	$2.1\cdot10^5$
Gold	1337	$6.6\cdot10^4$	Sulfur	385.8	$5.5\cdot10^4$
Iron	1803	$2.7\cdot10^5$	Tin	505	$5.8\cdot10^4$
Iron, cast: white	1473	$1.3\cdot10^5$	Tungsten	3683	$2.6\cdot10^4$
grey	1423	$9.7\cdot10^4$	Water, ice	273	$3.35\cdot10^5$
Lead	600	$2.5\cdot10^4$	Water heavy	276.85	$3.16\cdot10^5$
Mercury	234	$1.25\cdot10^4$	Wood's alloy	—	$3.2\cdot10^4$
Naphthalene	353	$1.51\cdot10^5$	Zinc	692	$1.18\cdot10^5$

Table 11

Surface Tension
(at 293 K)

Substance	σ, N/m	Substance	σ, N/m
Acetone	0.024	Kerosene	0.024
Alcohol ethyl	0.022	Mercury	0.470
Castor oil	0.033	Petrol	0.029
Copper vitriol	0.074	Suds	0.040
Ethyl ether	0.017	Turpentine	0.027
Glycerin	0.059	Water	0.072

Table 12

Coefficients of Linear Expansion of Solids

Substance	α, K^{-1}	Substance	α, K^{-1}
Aluminium, duralumin	$2.3 \cdot 10^{-5}$	Invar	$6 \cdot 10^{-7}$
Brass	$1.9 \cdot 10^{-5}$	Iron, steel	$1.2 \cdot 10^{-5}$
Bronze	$1.8 \cdot 10^{-5}$	Lead	$2.9 \cdot 10^{-5}$
Cast iron	$1.0 \cdot 10^{-5}$	Nickel	$1.28 \cdot 10^{-5}$
Concrete	$(10 \div 14) \cdot 10^{-6}$	Platinite	$9 \cdot 10^{-6}$
Copper	$1.7 \cdot 10^{-5}$	Platinum	$9 \cdot 10^{-6}$
Ebonite	$7.0 \cdot 10^{-5}$	Tin	$2.1 \cdot 10^{-5}$
Glass	$9.0 \cdot 10^{-6}$	Tungsten	$4 \cdot 10^{-6}$
Gold	$1.4 \cdot 10^{-5}$	Zinc	$2.9 \cdot 10^{-5}$

Table 13

Coefficients of Volume Expansion of Liquids

Substance	β, K^{-1}	Substance	β, K^{-1}
Aceton	$1.2 \cdot 10^{-3}$	Sulfuric acid	$5.7 \cdot 10^{-4}$
Alcohol ethyl	$1.1 \cdot 10^{-3}$	Transformer oil	$6.0 \cdot 10^{-4}$
Ethyl ether	$1.6 \cdot 10^{-3}$	Water at 5-10°C	$5.3 \cdot 10^{-5}$
Glycerin	$5.0 \cdot 10^{-4}$	10-20	$1.5 \cdot 10^{-4}$
Kerosene	$1.0 \cdot 10^{-3}$	20-40	$3.02 \cdot 10^{-4}$
Mercury	$1.8 \cdot 10^{-4}$	40-60	$4.58 \cdot 10^{-4}$
Petrol	$1.0 \cdot 10^{-3}$	60-80	$5.87 \cdot 10^{-4}$
Petroleum	$1.0 \cdot 10^{-3}$	80-100	$7.02 \cdot 10^{-4}$

Table 14

Dielectric Constants

Substance	ε	Substance	ε
Air (at 1 atm)	1.0006	Ebonite	2.7
Air (at 100 atm)	1.055	Glass	$5 \div 10$
Amber	2.8	Glycerin	39
Aniline	84	Hydrogen	1.0003

Table 14 (continued)

Substance	ε	Substance	ε
Ice (at $-18°C$)	3.2	Porcelain	4÷7
Kerosene	2.0	Rubber	2÷3
Marble	8÷9	Rutile	130
Mica	6÷9	Sulfur	3.6÷4.3
Oil, transformer	2.2÷2.5	Vacuum	1
Paraffin	2.2	Wax	5.8
Paraffined paper	2.0	Water	81
Petrol	2.3	Water (at 0°C)	88

Table 15

Resistivities

Substance	ρ, Ω·m	Substance	ρ, Ω·m
Aluminium	$2.7\cdot10^{-8}$	Nichrome	$1.05\cdot10^{-6}$
Brass	$6.3\cdot10^{-8}$	Nickel	$7.3\cdot10^{-8}$
Coal	$(4.0\div5.0)\cdot10^{-5}$	Nickeline	$4.2\cdot10^{-7}$
Constantan	$4.7\cdot10^{-7}$	Osmium	$9.5\cdot10^{-8}$
Copper	$1.68\cdot10^{-8}$	Platinum	$1.05\cdot10^{-7}$
Fechral	$1.1\cdot10^{-6}$	Rheotan	$4.5\cdot10^{-7}$
Gold	$2.2\cdot10^{-8}$	Silver	$1.58\cdot10^{-8}$
Iron	$9.9\cdot10^{-8}$	Tin	$1.13\cdot10^{-7}$
Lead	$2.07\cdot10^{-7}$	Tungsten	$5.3\cdot10^{-8}$
Manganin	$3.9\cdot10^{-7}$	Zinc	$5.95\cdot10^{-8}$
Mercury	$9.54\cdot10^{-7}$		

Table 16

Temperature Coefficients of Resistance

Substance	α, K^{-1}	Substance	α, K^{-1}
Constantan	0.000005	Nickeline	0.0001
Fechral	0.0002	Rheotan	0.0004
Manganin	0.000008	Tungsten	0.0050
Nichrome	0.0002		

Table 17

Electrochemical Equivalents

Substance	K, kg/C	Substance	K, kg/C
Aluminium	$9.32 \cdot 10^{-8}$	Magnesium	$1.26 \cdot 10^{-7}$
Calcium	$2.077 \cdot 10^{-7}$	Nickel (bivalent)	$3.04 \cdot 10^{-7}$
Chlorine	$3.67 \cdot 10^{-7}$	Nickel (trivalent)	$2.03 \cdot 10^{-7}$
Copper	$3.294 \cdot 10^{-7}$	Oxygen	$8.29 \cdot 10^{-8}$
Gold	$6.81 \cdot 10^{-7}$	Potassium	$4.052 \cdot 10^{-7}$
Hydrogen	$1.044 \cdot 10^{-8}$	Silver	$1.118 \cdot 10^{-6}$
Lead	$1.074 \cdot 10^{-6}$	Sodium	$2.383 \cdot 10^{-7}$
Mercury	$2.072 \cdot 10^{-6}$	Zinc	$3.388 \cdot 10^{-7}$

Table 18

Refractive Indices

Substance	n	Substance	n
Alcohol, ethyl	1.36	Glycerin	1.47
Alcohol, methyl	1.33	Ice	1.31
Acetone	1.36	Petrol	1.50
Air	1.0003	Quartz	1.54
Aniline	1.59	Salt, rock	1.54
Carbon tetrachloride	1.46	Sugar	1.56
Carbon bisulphide	1.63	Sylvite	1.49
Diamond	2.42	Turpentine	1.51
Glass (crownglass)	1.50	Water	1.33
Glass (flint glass)	$1.6 \div 1.8$		

Table 19

Masses of Isotopes in Atomic Mass Units*

Element	Isotope	Mass	Element	Isotope	Mass
Hydrogen	${}_1^1H$	1.00783	Helium	${}_2^3He$	3.01603
	${}_1^2H$	2.01410		${}_2^4He$	4.00260
	${}_1^3H$	3.01605	Lithium	${}_3^6Li$	6.01513

Table 19 (continued)

Element	Isotope	Mass	Element	Isotope	Mass
	$^{7}_{3}Li$	7.01601	Aluminium	$^{27}_{13}Al$	26.98153
Beryllium	$^{8}_{4}Be$	8.00531	Phosphorus	$^{30}_{15}P$	29.97867
	$^{9}_{4}Be$	9.01219	Radon	$^{222}_{86}Rn$	222.01922
Boron	$^{11}_{5}B$	11.00930	Radium	$^{226}_{88}Ra$	226.02435
Carbon	$^{12}_{6}C$	12.00000	Uranium	$^{235}_{92}U$	235.04299
	$^{13}_{6}C$	13.00335	Uranium	$^{238}_{92}U$	238.05006
Oxygen	$^{16}_{8}O$	15.99491	Neptunium	$^{237}_{93}Np$	237.04706
Fluorine	$^{19}_{9}F$	18.99843	Plutonium	$^{239}_{94}Pu$	239.05122

* 1 a.m.u. is 1/12 of mass of isotope $^{12}_{6}C$.

Table 20

Psychrometric Table

Dry-bulb temperature		Difference in dry- and wet-bulb temperatures											
K	°C	0	1	2	3	4	5	6	7	8	9	10	11
273	0	100	82	63	45	28	11						
	1	100	83	65	48	32	16						
	2	100	84	68	51	35	20						
	3	100	84	69	54	39	24	10					
	4	100	85	70	56	42	28	14					
278	5	100	86	72	58	45	32	19	6				
	6	100	86	73	60	47	35	23	10				
	7	100	87	74	61	49	37	26	14				
	8	100	87	75	63	51	40	28	18	7			
	9	100	88	76	64	53	42	31	21	11			
283	10	100	88	76	65	54	44	34	24	14	4		
	11	100	88	77	66	56	46	36	26	17	8		
	12	100	89	78	68	57	48	38	29	20	11		
	13	100	89	79	69	59	49	40	31	23	14	6	
	14	100	90	79	70	60	51	42	33	25	17	9	
288	15	100	90	80	71	61	52	44	36	27	20	12	5
	16	100	90	81	71	62	54	45	37	30	22	15	8

Table 20 (continued)

Dry-bulb temperature		Difference in dry- and wet-bulb temperatures											
K	°C	0	1	2	3	4	5	6	7	8	9	10	11
	17	100	90	81	72	64	55	47	39	32	24	17	10
	18	100	91	82	73	64	56	48	41	34	26	20	13
	19	100	91	82	74	65	58	50	43	35	29	22	15
293	20	100	91	83	74	66	59	51	44	37	30	24	18
	21	100	91	83	75	67	60	52	46	39	32	26	20
	22	100	92	83	76	68	61	54	47	40	34	28	22
	23	100	92	84	76	69	61	55	48	42	36	30	24
	24	100	92	84	77	69	62	56	49	43	37	31	26
298	25	100	92	84	77	70	63	57	50	44	38	33	27
	26	100	92	85	78	71	64	58	51	45	40	34	29
	27	100	92	85	78	71	65	59	52	47	41	36	30
	28	100	93	85	78	72	65	59	53	48	42	37	32
	29	100	93	86	79	72	66	60	54	49	43	38	33
303	30	100	93	86	79	73	67	61	55	50	44	39	34

Table 21

Induction versus Magnetic Field Strength for Mild Steel

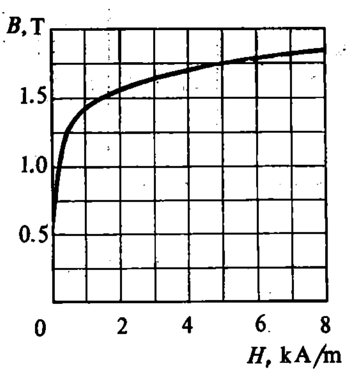


Table 22

Some Astronomical Data*

Earth's radius	$6.37 \cdot 10^6$ m
Earth's mass	$5.98 \cdot 10^{24}$ kg
Sun's radius	$6.95 \cdot 10^8$ m
Sun's mass	$1.98 \cdot 10^{30}$ kg
Lunar radius	$1.74 \cdot 10^6$ m
Lunar mass	$7.33 \cdot 10^{22}$ kg
Sun-Earth distance	$1.49 \cdot 10^{11}$ m
Moon-Earth distance	$3.84 \cdot 10^8$ m
Moon revolution period	27.3 days

* The table only gives average value.

Table 23

Data on Some Stars Visible in the USSR

Star	Magnitude	Right ascension	Declination
α Tauri (Aldebaran)	1.06	4 h 31 min 54 s	+16 22.2
β Orionis (Rigel)	0.34	5 h 11 min 10 s	−8 16.9
α Aurigae (Capella)	0.21	5 h 11 min 31 s	+45 55.7
α Orionis (Betelgeuse)	0.92	5 h 51 min 23 s	+7 23.7
α Canis Majoris (Sirius)	−1.58	6 h 42 min 4 s	−16 37.1
α Geminorus (Castor)	1.99	7 h 30 min 8 s	+32 2.6
α Lurae (Vega)	0.14	18 h 34 min 34 s	+34 43.1
α Cugni (Deneb)	1.33	20 h 39 min 3 s	+45 1.8

Table 24

Sines and Tangents for Angles 0-90°

Grades	Sines	Tangents	Grades	Sines	Tangents
0	0.0000	0.0000	4	0.0698	0.0699
1	0.0175	0.0175	5	0.0872	0.0875
2	0.0349	0.0349	6	0.1045	0.1051
3	0.0524	0.0524	7	0.1219	0.1228

Table 24 (continued)

Grades	Sines	Tangents	Grades	Sines	Tangents
8	0.1392	0.1405	50	0.7660	1.192
9	0.1564	0.1584	51	0.7771	1.235
10	0.1736	0.1763	52	0.7880	1.280
11	0.1908	0.1944	53	0.7986	1.327
12	0.2079	0.2126	54	0.8090	1.376
13	0.2250	0.2309	55	0.8192	1.428
14	0.2419	0.2493	56	0.8290	1.483
15	0.2588	0.2679	57	0.8387	1.540
16	0.2756	0.2867	58	0.8480	1.600
17	0.2924	0.3057	59	0.8572	1.664
18	0.3090	0.3249	60	0.8660	1.732
19	0.3256	0.3443	61	0.8746	1.804
20	0.3420	0.3640	62	0.8829	1.881
21	0.3584	0.3839	63	0.8910	1.963
22	0.3746	0.4040	64	0.8988	2.050
23	0.3907	0.4245	65	0.9063	2.145
24	0.4067	0.4452	66	0.9135	2.246
25	0.4226	0.4663	67	0.9205	2.356
26	0.4384	0.4877	68	0.9272	2.475
27	0.4540	0.5095	69	0.9336	2.605
28	0.4695	0.5317	70	0.9397	2.747
29	0.4848	0.5543	71	0.9455	2.904
30	0.5000	0.5774	72	0.9511	3.078
31	0.5150	0.6009	73	0.9563	3.271
32	0.5299	0.6249	74	0.9631	3.487
33	0.5446	0.6494	75	0.9659	3.732
34	0.5592	0.6745	76	0.9703	4.011
35	0.5736	0.7002	77	0.9744	4.331
36	0.5878	0.7265	78	0.9781	4.705
37	0.6018	0.7536	79	0.9816	5.145
38	0.6157	0.7813	80	0.9848	5.671
39	0.6293	0.8098	81	0.9877	6.314
40	0.6428	0.8391	82	0.9903	7.115
41	0.6561	0.8693	83	0.9925	8.144
42	0.6691	0.9004	84	0.9945	9.514
43	0.6820	0.9325	85	0.9962	11.43
44	0.6947	0.9657	86	0.9976	14.30
45	0.7071	1.0000	87	0.9986	19.08
46	0.7193	1.036	88	0.9994	28.64
47	0.7314	1.072	89	0.9998	57.29
48	0.7431	1.111	90	1.000	∞
49	0.7574	1.150			

Table 25

Multiples, Submultiples and Prefixes

Multiples and Submultiples	Prefixes	Symbols
$1\,000\,000\,000\,000 = 10^{12}$	tera	T
$1\,000\,000\,000 = 10^{9}$	giga	G
$1\,000\,000 = 10^{6}$	mega	M
$1\,000 = 10^{3}$	kilo	k
$100 = 10^{2}$	hecto	h
$10 = 10^{1}$	deka	da
$0.1 = 10^{-1}$	deci	d
$0.01 = 10^{-2}$	centi	c
$0.001 = 10^{-3}$	milli	m
$0.000\,001 = 10^{-6}$	micro	μ
$0.000\,000\,001 = 10^{-9}$	nano	n
$0.000\,000\,000\,001 = 10^{-12}$	pico	p
$0.000\,000\,000\,000\,001 = 10^{-15}$	femto	f
$0.000\,000\,000\,000\,000\,001 = 10^{-18}$	atto	a

Greek Alphabet

Α	α	alpha	Ν	ν	nu
Β	β	beta	Ξ	ξ	xi
Γ	γ	gamma	Ο	ο	omicron
Δ	δ	delta	Π	π	pi
Ε	ε	epsilon	Ρ	ρ	rho
Ζ	ζ	zeta	Σ	σ	sigma
Η	η	eta	Τ	τ	tau
ϴ Θ	ϑ	theta	Υ	υ	upsilon
Ι	ι	iota	Φ	φ	phi
Κ	ϰ	kappa	Χ	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
Μ	μ	mu	Ω	ω	omega

TO THE READER

Mir Publishers would be grateful for your comments on the content, translation, and design of this book. We would also be pleased to receive any other suggestions you may wish to make.

Our address is:

Mir Publishers
2 Pervy Rizhsky Pereulok
I-110, GSP, Moscow, 129820
USSR

Printed in the Union of Soviet Socialist Republics

OTHER MIR PUBLISHERS' BOOKS

Introduction to Physics

A. KITAIGORODSKY, D. Sc.

This book written by a leading Soviet physicist is an educational aid for students of engineering institutions of higher learning as well as for all persons wishing to take a higher-school course in physics independently. The material in it is given in unbroken sequence by a new method. Another feature is that the book is relatively small (the entire physics course is expounded in a single volume). It consists of three parts (thirty-seven chapters) and an Appendix containing the table of electrodynamic formulas. Apart from his monographs and textbooks, Prof. Kitaigorodsky has written many books popularizing science: *Physics for Everyone* (together with the renowned scientist, winner of the Nobel and Lenin prizes, Academician Lev Landau), *I am a Physicist*, *Order and Disorder in the World of Atoms*, etc. which are also translated into English by Mir Publishers.
Translated from the second updated Russian edition. The first edition in English was published in 1963 and reprinted in 1968 and 1976.

Physics. A General Course, Vol. III

I. SAVELYEV, D. Sc.

This third volume of the book concludes the modernized course in physics embodying the experience gained by the author during many years of teaching the subject at the Moscow Physical Engineering Institute. The accent is placed not on imparting information, but on the formation of physical thinking by the students and on their mastering the ideas and methods of physics. Methodically more improved ways of treating a number of questions have been found. This has made the expounding of the material stricter, and at the same time simpler and easier to understand.

The book is intended first of all for students of higher educational institutions with a broad syllabus in physics. The material is arranged, however, so that by omitting separate portions, it can be used by students taking an ordinary course in physics. The first two volumes of the book covering the physical fundamentals of mechanics, molecular physics and thermodynamics (Vol. I), electricity and magnetism, waves, and optics (Vol. II) were published in 1980.

Vector Analysis

M. KRASNOV, Cand. Sc., A. KISELEV, Cand. Sc. and G. MAKARENKO, Cand. Sc.

This text is designed for students of engineering colleges and also for practicing engineers who feel a need to refresh their knowledge of such an important area of higher mathematics as vector analysis. Each section of the text starts out with a brief review of the essentials of theory (propositions, definitions, formulas). This is followed by detailed solutions of examples and problems.

The book contains about 300 problems to be solved by the student. They are all provided with hints and answers. The 41 drawings that accompany the text serve to help the reader in analyzing the theoretical material and problems.